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Peristaltic pumping of a conducting fluid in a channel with a porous peripheral layer

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ABSTRACT

Peristaltic transport of a conducting fluid in a composite region between two flexible walls is investigated under the assumptions of long wavelength and low Reynolds number. The composite region consists of core and peripheral layers. The core layer is a free flow region consisting of a conducting Newtonian fluid and the peripheral layer is a porous region filled with conducting fluid. An infinite train of peristaltic waves is moving on the walls of the channel. The fluid flow is investigated in the wave frame of reference moving with the velocity of the peristaltic wave. Brinkman extended Darcy equation is used to model the flow in the porous layer. A shear-stress jump boundary condition is used at the interface. The physical quantities of importance in peristaltic transport like pressure rise etc. are discussed for various parameters of interest governing the flow like viscosity ratio, magnetic parameter and amplitude ratio. The results found will have applications for understanding the physiological flows in small blood vessels which can modeled as channels bounded by finite permeable layers (Fung and Tang, [5]).

Key words: peristaltic transport, conducting Newtonian fluid, porous peripheral layer, low Reynolds number.

INTRODUCTION

Peristaltic transport of a biofluid through a channel with permeable walls is of considerable importance in biology and medicine. Peristalsis is an inherent property of many of the smooth muscle tubes such as the gastrointestinal tract, bile duct, ureter and other glandular ducts. The fluids present in the ducts of a living body are called biofluids. The biofluid has to be treated as Newtonian or non-Newtonian depending on the physiological situation. Peristaltic pumping through a tube and a channel under the assumptions of low Reynolds number and long wavelength is discussed by Shapiro [18]. Lu [10] studied the influence of two Newtonian fluids with different viscosities on peristaltic pumping. Kavitha et al. investigated the Peristaltic flow of a micropolar fluid in a vertical channel with longwave length approximation. Brasseur et al. [3] discussed the influence of a peripheral layer of different viscosity on peristaltic pumping with Newtonian fluids.

The boundary conditions to be satisfied at the interface of a two fluid system are the matching of tangential velocity, normal velocity, shear stress and normal stress. Beavers and Joseph [2] have studied the fluid flow at the interface between a porous medium and fluid layer experimentally and proposed a slip condition in velocity at the interface. There exist numerous subsequent studies in the literature which suggest different boundary conditions at the interface between porous and fluid layers (Chen and Chen [4], Neale and Nader [11], Poulikakos and Kazmierczak [15], Saffman [17], Vafai and Kim [20]. Ochoa-Tapia and Whitaker [14] introduced a new boundary condition which accounts for the jump in the shear stress at the interface between porous and fluid layer by applying a sophisticated averaging volume technique. Kuznetsov [8,9] discussed the significance of the shear stress jump condition at the interface and applied this condition to investigated the fluid flow in a channel partially filled with a porous medium. Alazmi and Vfai [1] investigated the fluid flow and heat transfer between porous medium and a fluid layer by considering various types of interfacial matching of shear stress conditions reported in the literature.

The mathematical modeling of the two fluid system involves the determination of the interface between the core and peripheral layers. Ramachandra Rao and Usha [16] analyzed the peristaltic transport of two immiscible Newtonian fluids in a circular tube. Mishra and Ramachandra Rao [13] studied the peristaltic transport in a channel with a porous peripheral layer. Most of the physiological fluids (for eg : blood) are observed to be electrically conducting. Further the behavior of such fluids under a magnetic field in various organs of a human/animal body has to be analyzed due to its applications in medical diagnosis.

Motivated by these facts, the peristaltic transport of a conducting fluid in a two-layered system with a porous and core layers is investigated. The fluid flow is studied in the wave frame of reference moving with the velocity of the peristaltic wave. Brinkman extended Darcy equation is used to describe the flow in the porous layer. The interface is obtained as a part of the solution using the conservation of mass in both the porous and fluid regions independently. Ochoa-Tapia and Whitaker [14] shear-stress jump boundary condition is used at the interface. The physical quantities of importance in peristaltic transport like pressure rise etc. are discussed for various parameters of interest governing the flow like viscosity ratio, amplitude ratio and magnetic parameter.

2. Mathematical formulation

Consider the peristaltic transport in a two dimensional channel, with a porous peripheral layer consisting of a conducting Newtonian fluid of viscosity μ_1 and an incompressible conducting Newtonian fluid of viscosity μ_2 in the core region. A uniform transverse magnetic field of strength B_0 is applied perpendicular to the channel walls. We assume that the porous medium is isotropic and homogeneous. The channel wall is flexible and infinite wave train is moving on the walls of amplitude b with wave length λ in the axial direction with a constant speed c. The walls are taken by $y = \pm H\left(X - ct\right)$ in Cartesian coordinate system $\left(X, Y\right)$ with t as the time. The mean width of the channel is 2a and the deformed interface separating the core and the peripheral regions is denoted by $y = H_1\left(X - ct\right)$. Under the assumptions that the tube length is an integral multiple of the wavelength and the pressure difference across the ends of the channel is constant (Shapiro et al. [18]) and an additional condition of periodicity of the interface with the same period as the peristaltic wave (Brasseur et al. [3]), the flow becomes steady in a wave frame of reference (x, y) moving with speed c in the direction of the wave propagation.

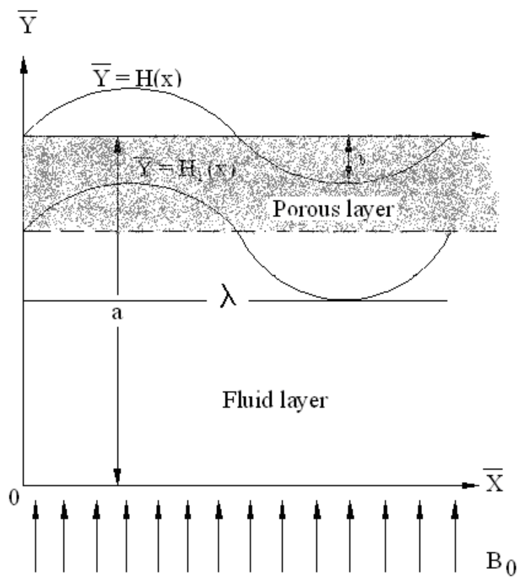


Fig.1: Physical Model

The wave frame is connected to the fixed frame by the following equations:

$$x = X - ct,\ y = Y,\ u_i = U_i - c,$$
$$v_i = V_i,\ p_i(x) = P_i\left(X, t\right) \quad (1)$$

Here $\left(u_i, v_i\right)$ and $\left(U_i, V_i\right)$ are the velocity components in axial and transverse directions p_i and P_i are the pressures in wave and fixed frame of references and the subscripts i takes the value 1 for the core layer and 2 for the peripheral layer .

The governing equations of motion in the fluid region in the core layer

$\left(0 \le y \le H_1\right)$ are

$$\rho_1\left(u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y}\right) = -\frac{\partial p_1}{\partial x} + \mu_1\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial x^2}\right) - \sigma_1 B_0^{\,2} u_1 \quad (2)$$

$$\rho_1\left(u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y}\right) = -\frac{\partial p_1}{\partial y} + \mu_1\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial x^2}\right) \quad (3)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \quad (4)$$

where μ_1, σ_1, B_0 and ρ_1 are viscosity , electrical conductivity, magnetic flux and density in the fluid region. The governing equations of the porous region in the peripheral layer $\left(H_1 \le y \le H\right)$ are the Brinkman extended Darcy equations given by (Alazmi and Vafai, 2001)

$$\rho_2\left(u_2\frac{\partial u_2}{\partial x} + v_2\frac{\partial u_2}{\partial y}\right) = -\frac{\partial p_2}{\partial x} + \frac{\mu_2}{\varepsilon}\left(\frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial x^2}\right) - \frac{\mu_2}{k}u_2 - \sigma_2 B_0^{\,2} u_2 \quad (5)$$

$$\rho_2\left(u_2\frac{\partial v_2}{\partial x} + v_2\frac{\partial v_2}{\partial y}\right) = -\frac{\partial p_2}{\partial y} + \frac{\mu_2}{\varepsilon}\left(\frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_2}{\partial x^2}\right) - \frac{\mu_2}{k}v_2 \quad (6)$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \quad (7)$$

where $\rho_2, \sigma_2, \mu_2, \varepsilon$ and k are the density, electrical conductivity, viscosity, porosity and permeability in the porous region.

Following the experimental investigation of Gilver and Altobelli [6] the effective viscosity in the Brinkman model is taken as $\frac{\mu_2}{\varepsilon}$.

The following non-dimensional quantities are used

$$x^1 = \frac{x}{\lambda},\ y^1 = \frac{y}{a},\ t^1 = \frac{ct}{\lambda},\ p^1{}_i = \frac{a^2 p_i}{\mu_1 c\lambda},\ u^1{}_i = \frac{u_i}{c_1},\ v^1{}_i = \frac{v_i}{c\delta}$$
$$\delta = \frac{a}{\lambda},\ h = \frac{H}{a},\ h_1 = \frac{H_1}{a},\ \phi = \frac{b}{a},\ \mu = \frac{\mu_2}{\mu_1},\ \rho = \frac{\rho_2}{\rho_1} \quad (8)$$
$$\psi_i^1 = \frac{\psi_i}{ac},\ M_i^{\,2} = \frac{\sigma_i B_0^{\,2} a^2}{\mu_1},\ \mathrm{Re} = \rho_1\frac{ca}{\mu_1},\ Da = \frac{k}{a^2}$$

The governing equations of motion (2) - (7) in terms of stream functions ψ_i (where $u_i = \frac{\partial \psi_i}{\partial y}, v_i = -\frac{\partial \psi_i}{\partial x}$) after nondimensionalyzation become

$$\mathrm{Re}\,\delta\left(\psi_{1y}\psi_{1yx}-\psi_{1x}\psi_{1yy}\right)=-p_{1x}+\psi_{1yyy}+\delta^2\psi_{1yxx}-M_1^{\,2}\psi_{1y},$$
$$\mathrm{Re}\,\delta^3\left(\psi_{1y}\psi_{1xx}-\psi_{1x}\psi_{1yx}\right)=p_{1y}+\delta^2\left(\psi_{1xyy}+\delta^2\psi_{1xxx}\right),\qquad 0\le y\le h \tag{9}$$

$$\mathrm{Re}\,\delta\rho\left(\psi_{2y}\psi_{2yx}-\psi_{2x}\psi_{2yy}\right)=-p_{2x}+\frac{\mu}{\varepsilon}\left(\psi_{2yyy}+\delta^2\psi_{2yxx}\right)-\frac{\mu}{Da}\psi_{2y}-M_2^{\,2}\psi_{2y},$$
$$\mathrm{Re}\,\delta^3\rho\left(\psi_{2y}\psi_{2xx}-\psi_{2x}\psi_{2xy}\right)=p_{2y}+\frac{\mu}{\varepsilon}\delta^2\left(\psi_{2xyy}+\delta^2\psi_{2xxx}\right)-\frac{\mu}{Da}\delta^2\psi_{2x},\qquad h_1\le y\le h \tag{10}$$

where Re is the Reynolds number, Da is the Darcy number and M is the magnetic parameter. The subscripts x and y denote the partial differentiation with respect to that variable.

Under the assumptions of negligible inertia ($\mathrm{Re}\to o$) and long wavelength ($\delta<<1$) and eliminating pressures p_1 *and* p_2 from the equations (9) and (10) by cross differentiations, the equations governing the motion are

$$\psi_{1yyyy}-M_1^{\,2}\psi_{1yy}=0 \text{ in } 0\le y\le h_1 \tag{11}$$
$$\psi_{2yyyy}-\beta^2\psi_{2yy}=0 \text{ in } h_1\le y\le h \tag{12}$$

Where $\beta^2=\alpha^2+\dfrac{\varepsilon}{\mu}M_2^{\,2}$ and $\alpha^2=\dfrac{\varepsilon}{Da}$

The corresponding dimensionless boundary conditions are

$$\psi_{2y}=-1,\ \psi_2=q \quad \text{at } y=h(x) \tag{13}$$
$$\psi_{1yy}=0,\ \psi_1=0 \quad \text{at } y=0 \tag{14}$$

At the interface $y=h_1(x)$:

$$\psi_1=\psi_2=q_1 \tag{15}$$
$$\psi_{1y}=\psi_{2y} \tag{16}$$
$$\psi_{1yy}=\mu_\varepsilon\left(\psi_{2yy}-\beta_1\psi_{2y}\right) \tag{17}$$
$$\psi_{1yyy}=\mu_\varepsilon\left(\psi_{2yyy}-\alpha^2\psi_{2y}\right) \tag{18}$$

where q and q_1 in (13) and (15) are the total and core fluxes respectively,

$$\mu_\varepsilon=\frac{\mu}{\varepsilon} \text{ and } \beta_1=\frac{\beta\varepsilon}{\sqrt{Da}}$$

In (13) the first condition at y=h(x) is the no-slip condition. The second at $y=h(x)$ in (13) and the conditions at $y=h_1(x)$ in (15) are the requirement of conservations of mass in core as well as in the peripheral layer independently across any cross section in the wave frame. The conditions in (14) imply that the velocity attains a maximum on the streamline $y=0$. The continuity of velocity across interface is given by (16). The shear stress jump condition (17) is according to Ochoa-Tapia and Whitaker [14]. The continuity of normal stress implies $p_1=p_2$, which means the continuity of the pressure gradient $\dfrac{\partial p_1}{\partial x}=\dfrac{\partial p_2}{\partial x}$ at the interface and it reduces to (18). Hence the pressure remains constant across any cross section of the channel. We observe that the above governing equations and the corresponding boundary conditions reduce to those given by Brasseur et al. [3] in the limit $Da\to\infty$ and $M_i\to 0$.

SOLUTION OF THE PROBLEM

Solving equations (11) and (12) with the use of corresponding boundary conditions (13)-(18), we obtain the stream functions in the core and peripheral layers as

$$\psi_1 = c_2 \sinh M_1 y - \frac{1}{M_1^2} c_1 y, \qquad 0 \le y \le h_1, \tag{19}$$

$$\psi_2 = D_3 \cosh \beta y + D_4 \sinh \beta y - \frac{1}{\beta^2}(D_1 y + D_2), \qquad h_1 \le y \le h, \tag{20}$$

Where

$$c_1 = \frac{1}{L_1}(c_2 L_2 + D_1 L_3 - D_3 L_5 - D_4 L_4),\ c_2 = \frac{1}{L_{19}}(-D_1 L_{20} - D_2 L_{21} - D_3 L_{22} - D_4 L_{23})$$

$$D_1 = \frac{1}{L_{33}}(-qL_1 L_7 - D_4 L_{35} - D_3 L_{34}),\ D_2 = \frac{1}{L_{32}}(-D_1 L_{29} + D_3 L_{30} + D_4 L_{31})$$

$$D_3 = \frac{1}{L_{36}}(qL_1 L_7 L_{24} - D_4 L_{37}),\ D_4 = \frac{L_{24}(qL_1 L_7 L_{27} + L_{36})}{L_{27} L_{37} - L_{28} L_{36}}$$

$$L_1 = \frac{1}{M_1^2},\ L_2 = M_1 \cosh M_1 h_1,\ L_3 = \frac{1}{\beta^2}, L_4 = \beta \cosh \beta h_1,\ L_5 = \beta \sinh \beta h_1,\ L_6 = h L_3$$

$$L_7 = M_1^2 \sinh M_1 h_1,\ L_8 = \mu_\varepsilon \frac{\beta_1}{\beta^2}, L_9 = \mu_\varepsilon (\beta\beta_1 \sinh \beta h_1 - \beta^2 \cosh \beta h_1)$$

$$L_{10} = \mu_\varepsilon (\beta\beta_1 \cosh \beta h_1 - \beta^2 \sinh \beta h_1), L_{11} = \beta \sinh \beta h,\ L_{12} = \beta \cosh \beta h$$

$$L_{13} = M_1^3 \cosh \beta h_1,\ L_{14} = \mu_\varepsilon \frac{\alpha^2}{\beta^2}, L_{15} = \mu_\varepsilon \beta(\beta^2 - \alpha^2) \sinh \beta h_1$$

$$L_{16} = \mu_\varepsilon \beta(\beta^2 - \alpha^2) \cosh \beta h_1, L_{17} = h_1 L_1,\ L_{18} = h_1 L_3, L_{19} = L_1 \sinh M_1 h_1 - L_2 L_{17}$$

$$L_{20} = L_1 L_{18} - L_3 L_{17},\ L_{21} = L_1 L_3, L_{22} = L_5 L_{17} - L_1 \cosh \beta h_1, L_{23} = L_4 L_{17} - L_1 \sinh \beta h_1$$

$$L_{24} = L_7 L_{14} - L_8 L_{13}, L_{25} = L_7 L_{15} + L_9 L_{13}, L_{26} = L_7 L_{16} + L_{10} L_{13}, L_{27} = L_3 L_{25} + L_{11} L_{24}$$

$$L_{28} = L_3 L_{26} + L_{12} L_{24}, L_{29} = L_8 L_{19} + L_7 L_{20}, L_{30} = L_9 L_{19} - L_7 L_{22}, L_{31} = L_{10} L_{19} - L_7 L_{23}$$

$$L_{32} = L_7 L_{21} = L_7 L_1 L_3, L_{33} = L_1 L_7 L_6 - L_{29}, L_{34} = L_{30} - L_1 L_7 \cosh \beta h,$$

$$L_{35} = L_{31} - L_1 L_7 \sinh \beta h,\ L_{36} = L_{25} L_{33} - L_{24} L_{34},\ L_{37} = L_{26} L_{33} - L_{24} L_{35}$$

Using the relations (19) and (20) in the momentum equations (9) or (10) we obtain the pressure gradient as

$$\frac{dp}{dx} = c_1 = \frac{1}{L_1}(c_2 L_2 + D_1 L_3 - D_3 L_5 - D_4 L_4) \tag{21}$$

At any axial station, the non-dimensional flux Q in the fixed frame is connected with the flux q in the wave frame by

$$Q = \int_0^h (u+1)\, dy = q + h \tag{22}$$

The mean flow rate over one period ($T=\dfrac{\lambda}{c}$) of the peristaltic wave is given by

$$\begin{aligned}\overline{Q}&=\frac{1}{T}\int_0^T Q\,dt=\frac{1}{T}\int_0^T (q+h(x,t))\,dt\\ &=q+\int_0^1 h\,dx=q+1\end{aligned} \tag{23}$$

The peristaltic wave propagating on the walls in fixed frame is govern by $H(X,t)=a+b\sin\dfrac{2\pi}{\lambda}(X-ct)$, and its non-dimensional form in wave frame is $h(x)=1+\phi\sin 2\pi x$.

The interface between the porous and fluid regions which is not known 'a priori', and it should be a streamline in order to satisfy the conservation of mass in both the regions. From the equations (15) and (20), we find an equation governing the interface $h_1(x)$ as

$$f\left(h_1\right)=q_1-D_3\cosh\beta h_1-D_4\,\sinh\beta h_1+\frac{1}{\beta^2}\left(D_1 h_1+D_2\right)=0 \tag{24}$$

The constants q and q_1 are independent of x. By prescribing $h_1=\gamma$ at $x=0$ in (24), we get

$$q_1=E_3\cosh\beta\gamma-E_4\,\sinh\beta\gamma+\frac{1}{\beta^2}\left(E_1\gamma+E_2\right)$$

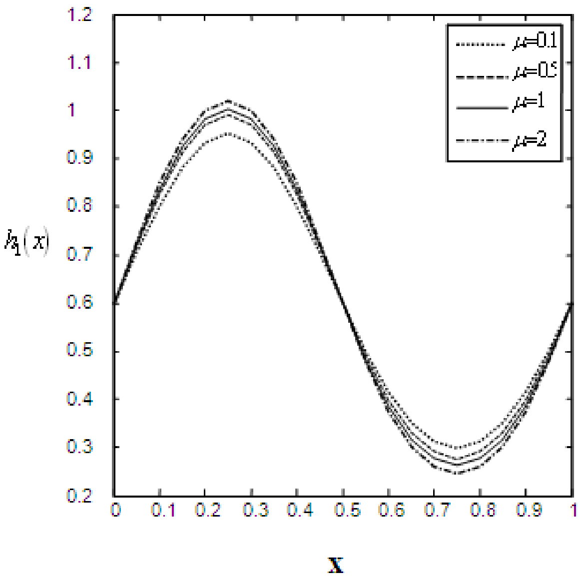
Where

$$E_1=\frac{1}{S_{33}}\left(-\left(\overline{Q}-1\right)S_1S_7-E_4S_{35}-E_3S_{34}\right)$$
$$E_2=\frac{1}{S_{32}}\left(-E_1S_{29}+E_3S_{30}+E_4S_{31}\right)$$
$$E_3=\frac{1}{S_{36}}\left(\left(\overline{Q}-1\right)S_1S_7S_{24}-E_4S_{37}\right)$$
$$E_4=\frac{S_{24}\left(\left(\overline{Q}-1\right)S_1S_7S_{27}+S_{36}\right)}{S_{27}S_{37}-S_{28}S_{36}}$$
$$S_1=L_1,\ S_2=M_1\cosh M_1\gamma,\ S_3=L_3$$
$$S_4=\beta\cosh\beta\gamma,\ S_5=\beta\sinh\beta\gamma,\ S_6=S_3$$
$$S_7=M_1{}^2\sinh M_1\gamma,\ S_8=L_8$$
$$S_9=\mu_\varepsilon\left(\beta\beta_1\sinh\beta\gamma-\beta^2\cosh\beta\gamma\right)$$
$$S_{10}=\mu_\varepsilon\left(\beta\beta_1\cosh\beta\gamma-\beta^2\sinh\beta\gamma\right)$$
$$S_{11}=\beta\sinh\beta,\ S_{12}=\beta\cosh\beta$$
$$S_{13}=M_1{}^3\cosh\beta\gamma,\ S_{14}=\mu_\varepsilon\frac{\alpha^2}{\beta^2}=L_{14}$$
$$S_{15}=\mu_\varepsilon\beta\left(\beta^2-\alpha^2\right)\sinh\beta\gamma$$
$$S_{16}=\mu_\varepsilon\beta\left(\beta^2-\alpha^2\right)\cosh\beta\gamma$$
$$S_{17}=\gamma S_1,\quad S_{18}=\gamma S_3$$

$$S_{19}=S_1\sinh M_1\gamma-S_2S_{17}$$
$$S_{20}=S_1S_{18}-S_3S_{17},\quad S_{21}=S_1S_3$$
$$S_{22}=S_5S_{17}-S_1\cosh\beta\gamma$$
$$S_{23}=S_4S_{17}-S_1\sinh\beta\gamma$$
$$S_{24}=S_7S_{14}-S_8S_{13}$$
$$S_{25}=S_7S_{15}+S_9S_{13}$$
$$S_{26}=S_7S_{16}+S_{10}S_{13}$$
$$S_{27}=S_3S_{25}+S_{11}S_{24}$$
$$S_{28}=S_3S_{26}+S_{12}S_{24}$$
$$S_{29}=S_8S_{19}+S_7S_{20}$$
$$S_{30}=S_9S_{19}-S_7S_{22}$$
$$S_{31}=S_{10}S_{19}-S_7S_{23}$$
$$S_{32}=S_7S_{21}=S_7S_1S_3$$
$$S_{33}=S_1S_7S_6-S_{29}$$
$$S_{34}=S_{30}-S_1S_7\cosh\beta$$
$$S_{35}=S_{31}-S_1S_7\sinh\beta$$
$$S_{36}=S_{25}S_{33}-S_{24}S_{34}$$
$$S_{37}=S_{26}S_{33}-S_{24}S_{35}$$

We assume that $\gamma > \phi$ to avoid the intersection of the interface with x-axis (Mishra and Ramachandra Rao, [12]). Since the equation (24) is a transcendental equation and it may have many positive real roots. But the interface is well defined only when there exists a single root h_1 of $f(h_1) = 0$ in the interval $0 \le h_1 \le h$. In this way for a given x, h_1 (x) is computed. Integrating equation (21) over one wave length, we get the pressure rise as

$$\Delta P = p_1 - p_2 = \int_0^1 \frac{1}{L_1}\left(c_2 L_2 + D_1 L_3 - D_3 L_5 - D_4 L_4\right) dx \qquad (25)$$



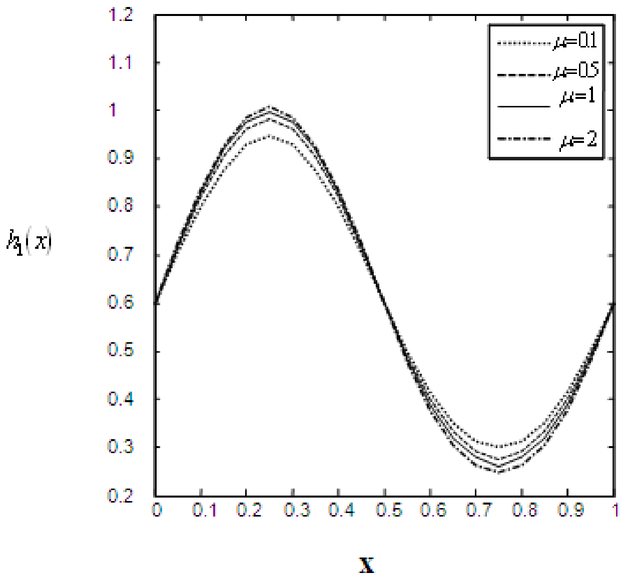
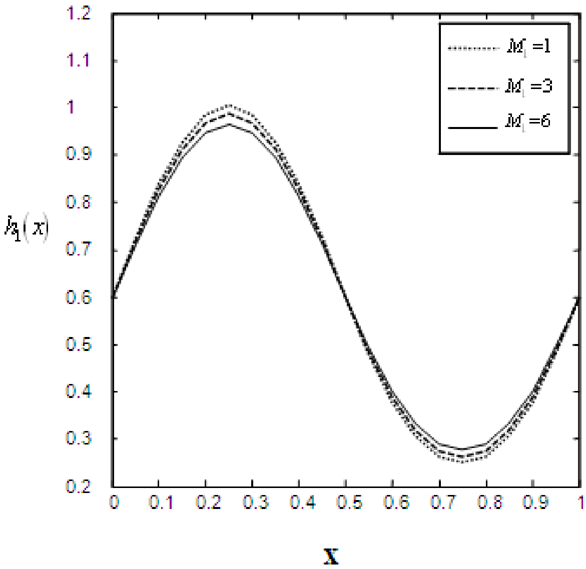


Fig.2: The Shape of the interface for different μ with fixed $M_1=0.5,\ M_2=1,\ \gamma=0.6,\ \phi=0.4,\ \varepsilon=0.5$,**Da=0.5,** $Q=0.5$.

Fig.3: The Shape of the interface for different μ with fixed $M_1=1.5,\ M_2=1,\ \gamma=0.6,\ \phi=0.4,\ \varepsilon=0.5$,**Da=0.5,** $Q=0.5$.



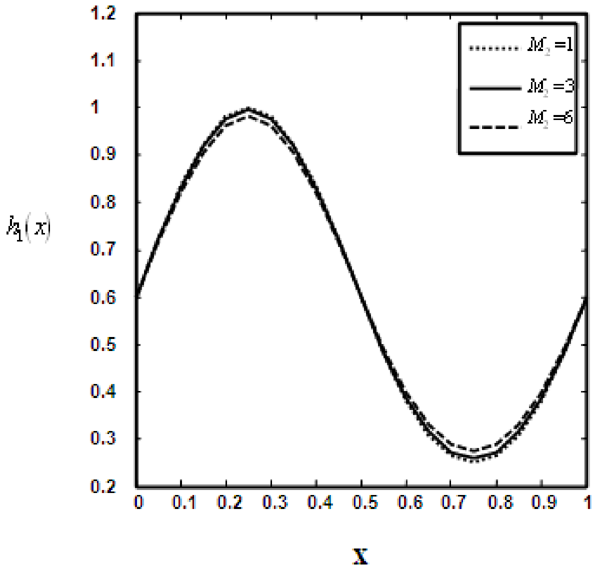
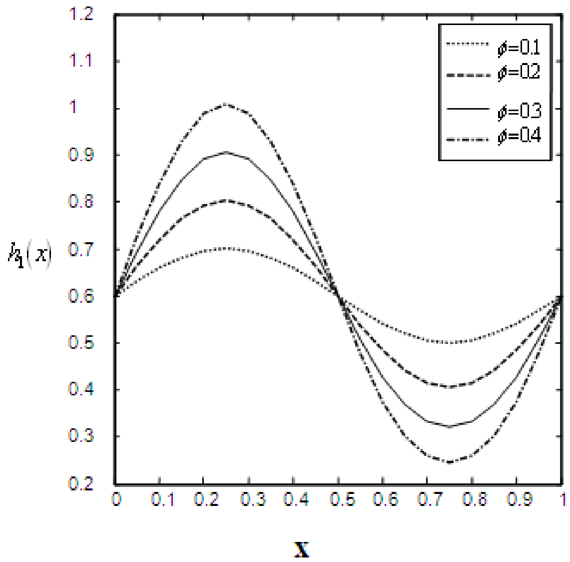


Fig.4: The Shape of the interface for different M_1 **with fixed** $M_2=1,\ \gamma=0.6,\ \phi=0.4,\ \varepsilon=0.5$, **Da=0.5,** $\mu=2,\ Q=0.5$.

Fig.5: The Shape of the interface for different M_2 **with fixed** $M_1=1,\ \gamma=0.6,\ \phi=0.4,\ \varepsilon=0.5$, **Da=0.5,** $\mu=2,\ Q=0.5$.



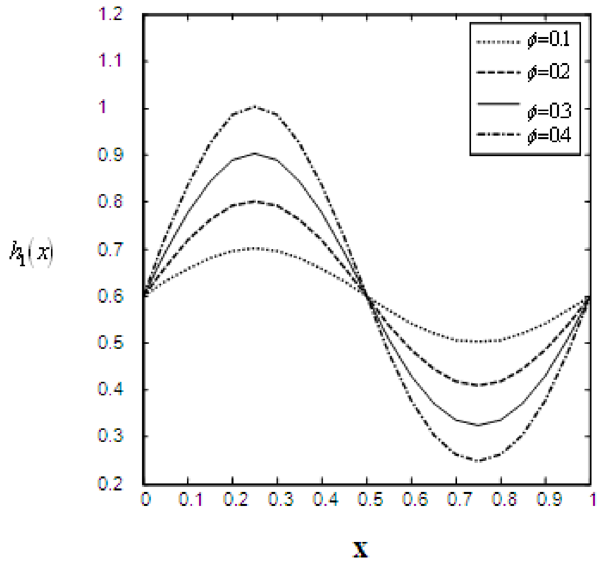
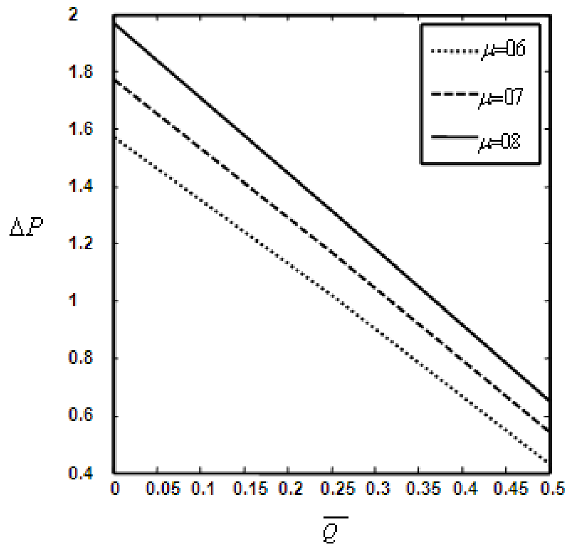


Fig.6: The Shape of the interface for different ϕ **with fixed** $M_1=0.5, M_2=1, \gamma=0.6, \mu=2, \varepsilon=0.5$,**Da=0.5,** $Q=0.5$.

Fig.7: The Shape of the interface for different ϕ **with fixed** $M_1=1.5, M_2=1, \gamma=0.6, \mu=2, \varepsilon=0.5$,**Da=0.5,** $Q=0.5$.



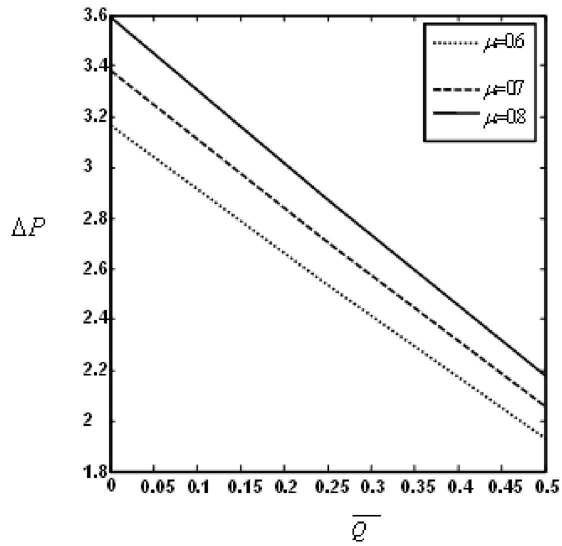


Fig.8: Variation of pressure rise versus mean Flow rate for different μ **(**$M_1=0.5, M_2=1$**).**

Fig.9: Variation of pressure rise versus mean flow rate for different μ **(**$M_1=1.5, M_2=1$**) with fixed** $\gamma=0.4, \phi=0.6, \varepsilon=0.4, Da=0.6$

RESULTS AND DISCUSSION

The interface which is a stream line in the wave frame is obtained from (24) and plotted in figures (2) - (7) to study the effects of different parameters on the shape of interface. From figures (2) and (3) we observe that the thickness of the peripheral layer decreases slightly in the dilated region with the increase in viscosity ratio. From figures (4) and (5) it is noticed that lower magnetic field gives rise to a thin peripheral layer in the dilated region. The uniform shape is never obtained. From (6) and (7) we observe that the thickness of the peripheral layer decreases in the dilated region with the increase in ϕ.

The equation (25) gives the expression for the pressure rise ΔP in terms of the mean flow $\overline{Q}$. Figures (8) and (9) shows that the pressure rise decreases with the increase in $\overline{Q}$. We find that for fixed $\overline{Q}$ pressure rise increases with increasing μ. We also notice that for a given ΔP mean flow rate increases with the increase in μ.

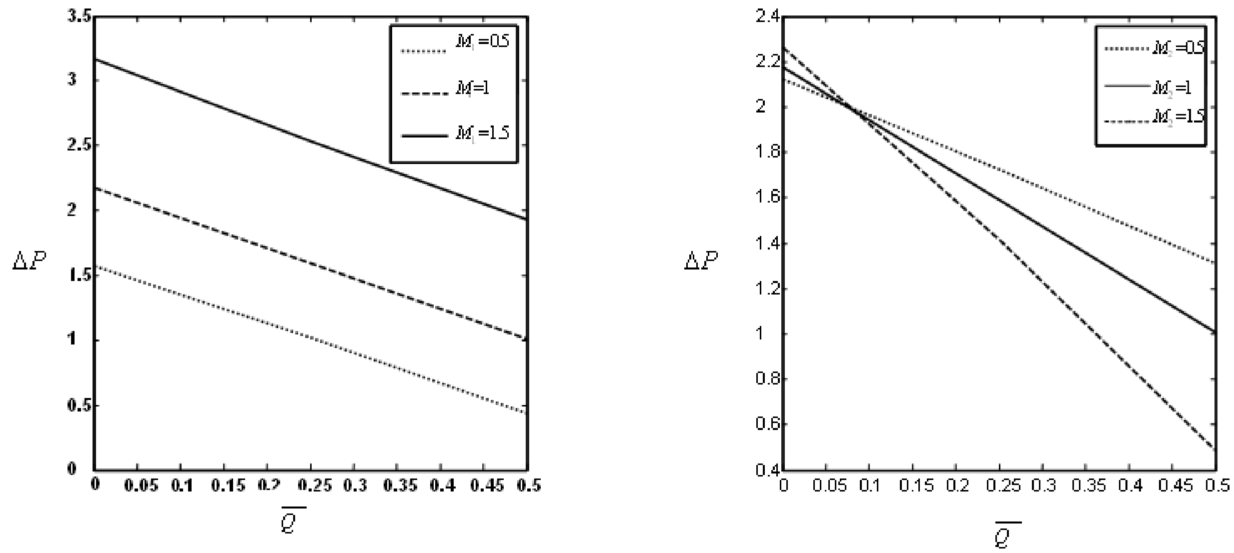


Fig.10: Variation of pressure rise versus mean flow rate for different M_1 (M_2=1).
Fig.11: Variation of pressure rise versus mean flow rate for different M_2 (M_1=1) with fixed $\gamma = 0.4$, $\phi = 0.6$, $\varepsilon = 0.4$, Da=0.6, $\mu = 0.6$.

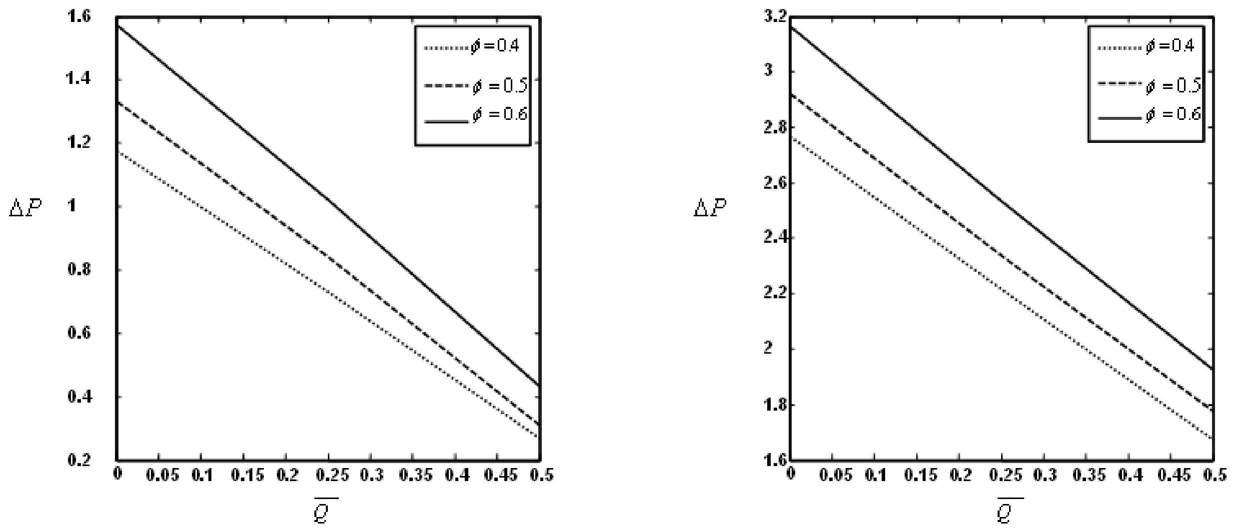


Fig.12: Variation of pressure rise versus mean flow rate for different ϕ
Fig.13: Variation of pressure rise versus mean flow rate for different ϕ with fixed $M_1 = 1.5$, $M_2 = 1$, $\gamma = 0.4$, $\mu = 0.6$, $\varepsilon = 0.4$, Da=0.6.

The variation of pressure rise with the mean flow for different values of M_1 is shown in figure (10). We observe that for fixed $\overline{Q}$ pressure rise increases with the increase in M_1. It is noticed that for a given ΔP mean flow rate increases with increasing M_1.

The variation of pressure rise with the mean flow for different values of M_2 is shown in figure (11). We observe that the pumping curves are intersect at appoint between 0.05 and 0.1, above this point for fixed $\overline{Q}$ pressure rise increases with the increase in M_2 and below the intersecting point opposite behavior can be observed.

The variation of pressure rise with the mean flow for different values of ϕ is shown in figures (12) and (13). We notice that for fixed $\overline{Q}$ pressure rise increases with increasing ϕ. It is observed that for a given ΔP mean flow rate increases with increasing ϕ.

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