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Advances in Applied Science Research, 2012, 3 (4):2108-2119

ISSN: 0976-8610
CODEN (USA): AASRFC

Peristaltic flow of a Prandtl fluid in a symmetric channel under the effect of a magnetic field

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ABSTRACT

In this paper, we studied the MHD peristaltic flow of a Prandtl fluid in a uniform channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. The effects various emerging parameters on the pressure gradient and pumping characteristics are studied in detail through graphs.

Keywords: Hartmann number; peristaltic flow; Perturbation method; Prandtl fluid

INTRODUCTION

Past five decades considerable attention has been focused on the peristaltic transport of Newtonian and non-Newtonian fluids through tubes/channels. Such flows are significant in both theoretical and industrial perspectives. Peristaltic transport widely occurs in many biological systems for example, food swallowing through the esophagus, intra-urine fluid motion, circulation of blood in small blood vessels and the flows of many other glandular ducts. Several theoretical and experimental studies have been undertaken to understand peristalsis through abrupt changes in geometry and realistic assumptions. A review of much of the early literature is presented in an article by Jaffrin and Shapiro [4]. All the important literature up to 1978 on peristaltic transport has been documented by Rath [11].

Even though the consideration of Newtonian fluid as a representative of blood and other physiological fluids provides a satisfactory understanding of the peristaltic mechanism in the ureter, but it fails to justify a better understanding when peristaltic motion is involved in small blood vessels, intestines and ductus efferentus of the male reproductive organs and in transport of spermatozoa in the cervical canal. Ramachandra Rao and Mishra [10] have studied the peristaltic flow of a power-law fluid in a porous tube. The peristaltic flow of a power-law fluid in an asymmetric channel was investigated by Subba Reddy et al. [14]. Nagendra et al. [7] have studied the peristaltic flow of a Jeffrey fluid in a tube. Recently, Akbar et al. [1] have discussed the peristaltic flow of a Prandtl fluid in an asymmetric channel.

The study of peristaltic flow of a fluid in the presence of magnetic field is of enormous importance with regard to certain problems involving the movement of conductive physiological fluids, e.g., blood and saline water. Stud et al. [13] have first investigated the effect of moving magnetic field on the blood flow. They found that a suitable moving magnetic field accelerates the speed of blood. Mekheimer [6] have studied the peristaltic flow of a blood in a nonuniform channel under the effect of a magnetic field. Peristaltic transport of a Johnson-Segalman fluid in a channel under the effect of a magnetic field was studied by Elshahed and Haroun [2]. Hayat and Ali [3] have discussed the effect of magnetic field on peristaltic transport of a Jeffrey fluid through a tube. Sudhakar Reddy et al. [15] have analyzed the peristaltic motion of a carreau fluid through a porous medium in a channel under the effect of a magnetic field. Pandey and Chaube [8] have investigated the effect of magnetic field on the peristaltic flow of a micropolar fluid through a porous medium in a channel. Peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field was studied by Jayarami Reddy et al. [5].

In view of these, we studied the MHD peristaltic transport of a Prandtl fluid in a uniform channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. The effects various emerging parameters on the pressure gradient and pumping characteristics are studied in detail through graphs.

2. Mathematical formulation

We consider the peristaltic flow of a conducting Prandtal fluid in a two dimensional channel of width $2a$. The walls of the channel are flexible. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The fluid is taken to be of small electrical conductivity, so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The flow is induced by periodic peristaltic wave of length λ and amplitude b with constant speed c along the channel walls. The physical model of the symmetric channel is shown in Fig. 1.

The equation of the wall is given by

$$Y = \pm H(X,t) = \pm a \pm b \sin\frac{2\pi}{\lambda}(X - ct) \tag{2.1}$$

where t is the time, λ is the wavelength and (X,Y) are the Cartesian co-ordinates in laboratory frame of reference.

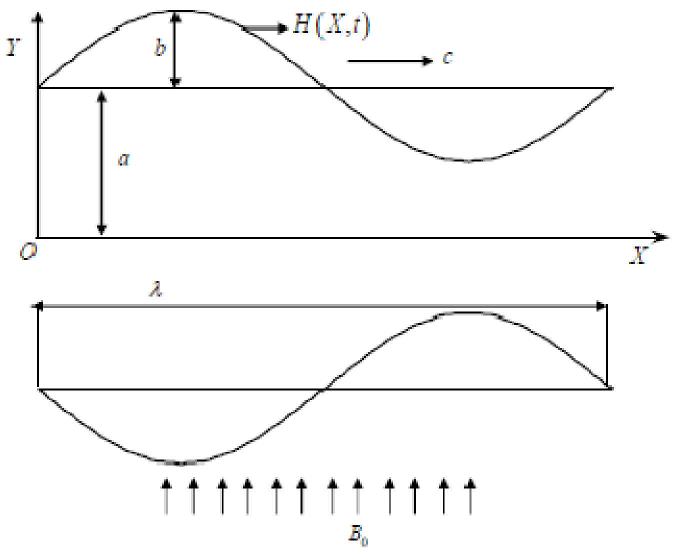

Fig. 1. The physical model

We introduce a wave frame of reference (x, y) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., 1969). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$x = X - c\,t,\; y = Y,\; u = U - c,\; v = V \quad \text{and} \quad p(x) = P(X,\, t), \tag{2.2}$$

where $(u,\, v)$ and $(U,\, V)$ are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The Constitute equations for Prandtal fluid is given by (Patel and Timaol [9])

$$\tau = \frac{A\sin^{-1}\left(\frac{1}{C}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right]^{\frac{1}{2}}\right)}{\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right]^{\frac{1}{2}}}\frac{\partial u}{\partial y} \tag{2.3}$$

in which A and C are material constants of Prandtl fluid model.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \sigma B_0^2\,(u+1) \tag{2.5}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \tag{2.6}$$

where λ_1

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{\lambda},\quad \bar{y} = \frac{y}{a},\quad \bar{u} = \frac{u}{c},\quad \bar{v} = \frac{v}{c\delta},\quad \bar{p} = \frac{pa^2}{\mu c\lambda},\quad \bar{t} = \frac{ct}{\lambda},\; h = \frac{H}{a},$$

$$\bar{\tau} = \frac{a\tau}{\mu c},\quad \phi = \frac{b}{a},\quad \delta = \frac{a}{\lambda}$$

where μ is the constant viscosity, in the Eqs. (2.4) – (2.6), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.7}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \delta\frac{\partial \tau_{xx}}{\partial x} - M^2(u+1) \tag{2.8}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial \tau_{xy}}{\partial x} + \delta\frac{\partial \tau_{yy}}{\partial y} \tag{2.9}$$

where $M = aB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number.

Under the assumptions of long wave length $(d << 1)$ and low Reynolds number $(\text{Re} \to 0)$, the Equations (2.8) and (2.9) become

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} - M^2(u+1) \quad (2.10)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2.11)$$

here $\tau_{xy} = \alpha\frac{\partial u}{\partial y} + \frac{\beta}{6}\left(\frac{\partial u}{\partial y}\right)^3$.

The corresponding boundary conditions in wave frame of reference are given by

$$u = -1 \quad \text{at} \quad y = h = 1 + \phi\cos 2\pi x, \quad (2.12)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0. \quad (2.13)$$

Equations (2.10), (2.11) indicate that p is independent of y. Therefore Eq. (2.10) can be rewritten as

$$\frac{dp}{dx} = \alpha\frac{\partial^2 u}{\partial y^2} + \frac{\beta}{6}\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^3\right] - M^2(u+1) \quad (2.14)$$

The volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u\,dy. \quad (2.15)$$

The instantaneous flux $Q(X,t)$ in the laboratory frame is

$$Q(x,t) = \int_0^h u\,dy = \int_0^h (u+1)dy = q + h. \quad (2.16)$$

The time average flux over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,dt = \int_0^1 (q+h)dx = q+1. \quad (2.17)$$

3. Solution

The Equation (2.14) is non-linear and its closed form solution is not possible. Hence, we linearize this equation in terms of $\beta(<<1)$. So we expand u, p and q as

$$u = u_0 + \beta u_1 + O(\beta^2)$$
$$p = p_0 + \beta p_1 + O(\beta^2)$$
$$q = q_0 + \beta q_1 + O(\beta^2) \quad (3.1)$$

Substituting (3.1) in the Equation (2.14) and in the boundary conditions (2.12) and (2.13) and equating the coefficients of like powers of β to zero and neglecting the terms of β^2 and higher order, we get the following equations:

3.1 System of order zero (β^0)

$$\alpha\frac{\partial^2 u_o}{\partial y^2} - M^2 u_o = \frac{dp_0}{dx} + M^2 \tag{3.2}$$

with the corresponding boundary conditions are

$$u_0 = -1 \quad \text{at} \quad y = h = 1 + \phi\cos 2\pi x, \tag{3.3}$$

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0. \tag{3.4}$$

3.2 System of order one (β)

$$\alpha\frac{\partial^2 u_1}{\partial y^2} - M^2\, u_1 = \frac{dp_1}{dx} - \frac{1}{6}\frac{\partial}{\partial y}\left(\frac{\partial u_o}{\partial y}\right)^3 \tag{3.5}$$

with the corresponding boundary conditions are

$$u_1 = 0 \quad \text{at} \quad y = h = 1 + \phi\cos 2\pi x, \tag{3.6}$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0. \tag{3.7}$$

3.3 Solution of order zero (β^0)

Solving Eq. (3.2) together with the boundary conditions (3.3) and (3.4), we obtain

$$u_o = \frac{1}{M^2}\frac{dp_o}{dx}\left[\frac{\cosh Ny}{\cosh Nh} - 1\right] - 1 \tag{3.8}$$

The volume flow rate q_0 in the moving coordinate system is given by

$$q_0 = \int_0^h u_0 dy = \frac{1}{M^2}\frac{dp_0}{dx}\left[\frac{\sinh Nh}{N\cosh Nh} - h\right] - h \tag{3.9}$$

From Eq. (3.9), we have

$$\frac{dp_0}{dx} = \frac{M^2\left(q_0 + h\right)N\cosh Nh}{\left(\sinh Nh - Nh\cosh Nh\right)} \tag{3.10}$$

3.4 Solution of order one (β)

Solving the Equation (3.5) by using the Equation (3.8) and the boundary conditions (1.6) and (1.7), we obtain

$$u_1 = \frac{1}{M^2}\frac{dp_1}{dx}\left[\frac{\cosh Ny}{\cosh Nh} - 1\right] + \frac{NA^3}{8M^6}\left(\frac{dp_0}{dx}\right)^3\left[\frac{y\sinh Ny}{2\alpha N} - \frac{\cosh 3Ny}{8M^2} - B\cosh Ny\right] \tag{3.11}$$

where $A = \dfrac{N}{\cosh Nh}$, $B = \left(\dfrac{h \sinh Nh}{2\alpha N} - \dfrac{\cosh 3Nh}{8M^2} \right) \dfrac{1}{\cosh Nh}$

and the volume flow rate q_1 is given by

$$q_1 = \int_0^h u_1 dy = \frac{1}{M^2 N \cosh Nh} \frac{dp_1}{dx} \left[\sin hNh - Nh \cosh Nh \right] + C \left(\frac{dp_o}{dx} \right)^3 \tag{3.12}$$

where $C = \dfrac{NA^3}{8M^6} \left[\dfrac{h \cosh Nh}{2\alpha N^2} - \dfrac{\sinh Nh}{2\alpha N^3} - \dfrac{\sinh 3Nh}{24NM^2} - \dfrac{B \sinh Nh}{N} \right]$.

From Eq. (3.12), we have

$$\frac{dp_1}{dx} = \frac{M^2 N \cos hNh \left[q_1 - C \left(\frac{dp_o}{dx} \right)^3 \right]}{\sinh Nh - hN \cos hNh} \tag{3.13}$$

Substituting Equations (3.10) and (3.13) into the second Equation of (3.1) and using the relation $\dfrac{dp_0}{dx} = \dfrac{dp}{dx} - \beta \dfrac{dp_1}{dx}$ and neglecting terms greater than $O(\beta)$, we get

$$\frac{dp}{dx} = \frac{NM^2 \cosh Nh}{\left[\sinh Nh - Nh \cosh Nh \right]} \left[q + h - \beta C \left\{ \frac{M^2 (q_0 + h) N \cosh Nh}{\sinh Nh - Nh \cosh Nh} \right\}^3 \right] \tag{3.14}$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \tag{3.15}$$

RESULTS AND DISCUSSION

Fig. 2 illustrates the variation of axial pressure gradient $\dfrac{dp}{dx}$ with β for $\phi = 0.6$, $\alpha = 1.5$ and $M = 1$. It is observed that, the axial pressure gradient $\dfrac{dp}{dx}$ increases with increasing β.

The variation of axial pressure gradient $\dfrac{dp}{dx}$ with M for $\phi = 0.6$, $\alpha = 1.5$ and $\beta = 0.1$ is shown in Fig. 3. It is noted that, the axial pressure gradient $\dfrac{dp}{dx}$ increases with an increase in M.

Fig. 4 depicts the variation of axial pressure gradient $\dfrac{dp}{dx}$ with α for $\phi = 0.6$, $\beta = 0.1$ and $M = 1$. It is found that, the axial pressure gradient $\dfrac{dp}{dx}$ increases on increasing α.

The variation of axial pressure gradient $\frac{dp}{dx}$ with ϕ for $\beta = 0.1$, $\alpha = 1.5$ and $M = 1$ is depicted in Fig. 5. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing ϕ.

Fig. 6 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of β with $\phi = 0.6$, $\alpha = 1.5$ and $M = 1$. It is observed that, the time averaged flux $\overline{Q}$ increases with increasing β in the pumping region $(\Delta p > 0)$, while it decreases with increasing β in both the free-pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions. Further, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid $(\alpha = 1, \beta = 0)$.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of M with $\phi = 0.6$, $\alpha = 1.5$ and $\beta = 0.1$ is depicted in Fig. 7. It is found that, any two pumping curves intersect in a first quadrant, to the left of this point of intersection the time averaged flux $\overline{Q}$ increases with increasing M and to the right of this point of intersection $\overline{Q}$ decreases with increasing M.

Fig. 8 illustrates the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of α with $\phi = 0.6$, $\beta = 0.1$ and $M = 1$. It is noted that, the time averaged flux $\overline{Q}$ increases with increasing α in both the pumping and free-pumping regions, while it decreases with increasing α in the co-pumping region.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of ϕ with $\beta = 0.1$, $\alpha = 1.5$ and $M = 1$ is shown in Fig. 9. It is noted that, the time averaged flux $\overline{Q}$ increases with increasing ϕ in both the pumping and free-pumping regions, while it decreases with increasing ϕ in the co-pumping region.

CONCLUSION

In this chapter, we studied the MHD peristaltic transport of a Prandtl fluid in a uniform channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. It is observed that, the axial pressure gradient increases with increasing β, M, α and ϕ. In the pumping region, time averaged flux $\overline{Q}$ increases with increasing β, M, α and ϕ. Also, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid.

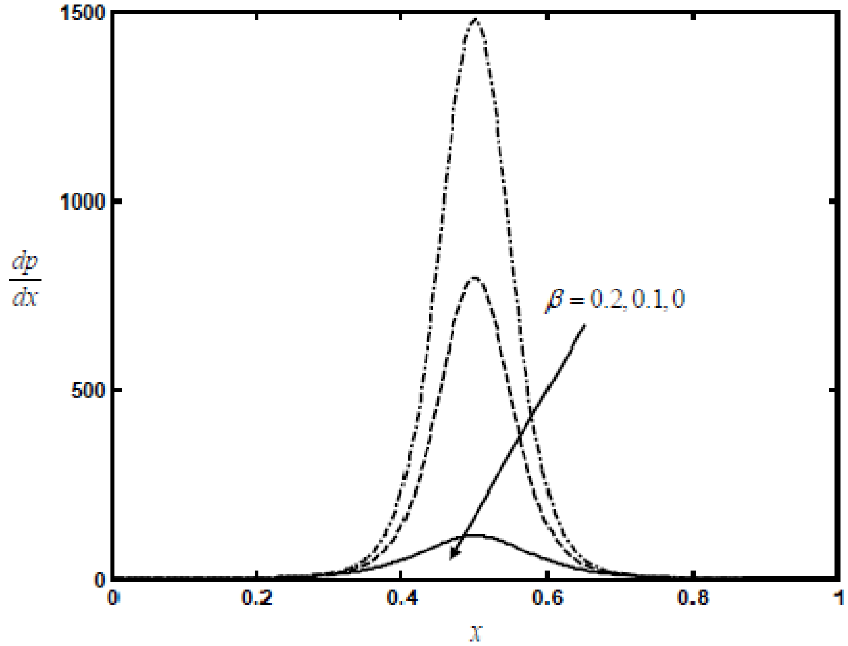

Fig. 2 The variation of axial pressure gradient $\frac{dp}{dx}$ **with** β **for** $\phi = 0.6$, $\alpha = 1.5$ **and** $M = 1$.

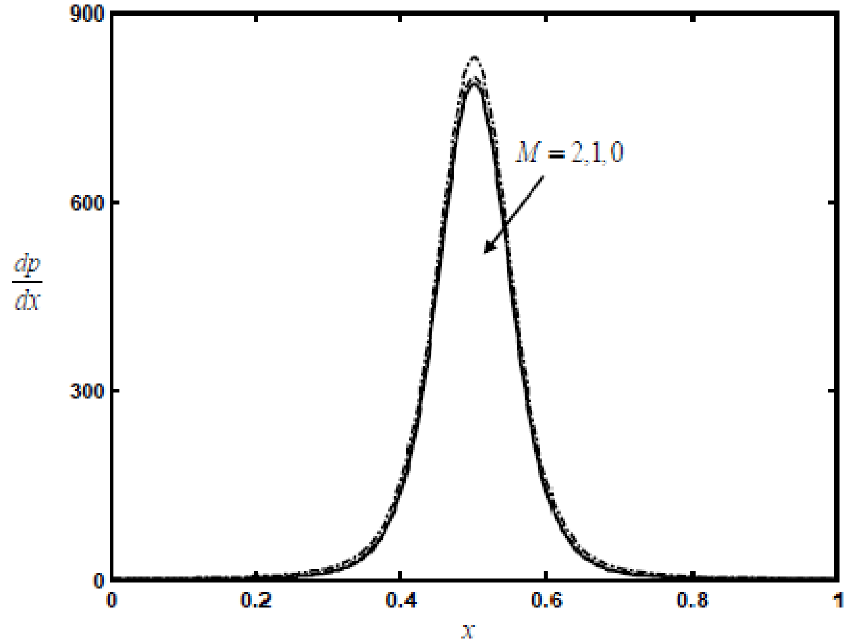

Fig. 3 The variation of axial pressure gradient $\frac{dp}{dx}$ **with** M **for** $\phi = 0.6$, $\alpha = 1.5$ **and** $\beta = 0.1$.

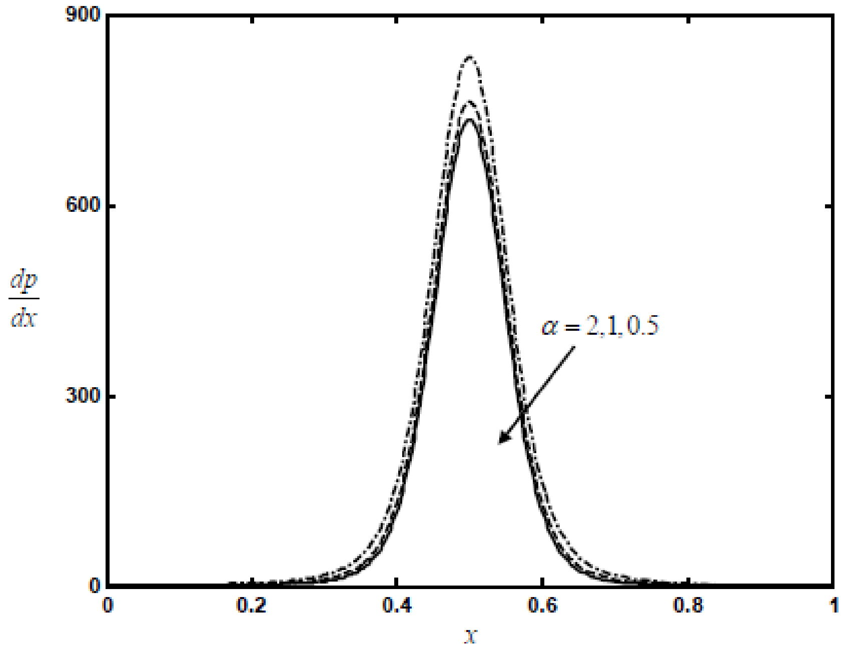

Fig. 4 The variation of axial pressure gradient $\frac{dp}{dx}$ **with** α **for** $\phi = 0.6$, $\beta = 0.1$ **and** $M = 1$.

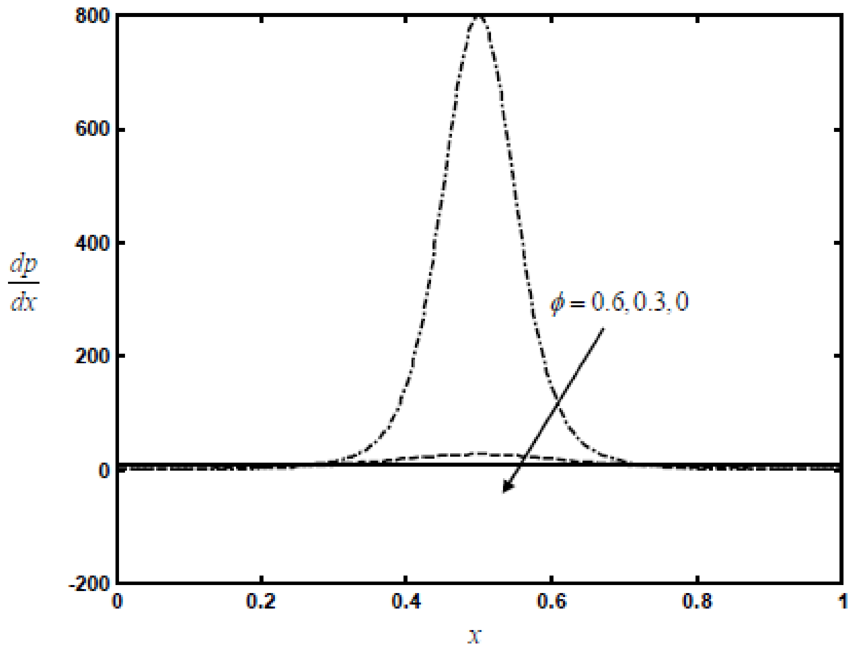

Fig. 5 The variation of axial pressure gradient $\frac{dp}{dx}$ **with** ϕ **for** $\beta = 0.1$, $\alpha = 1.5$ **and** $M = 1$.

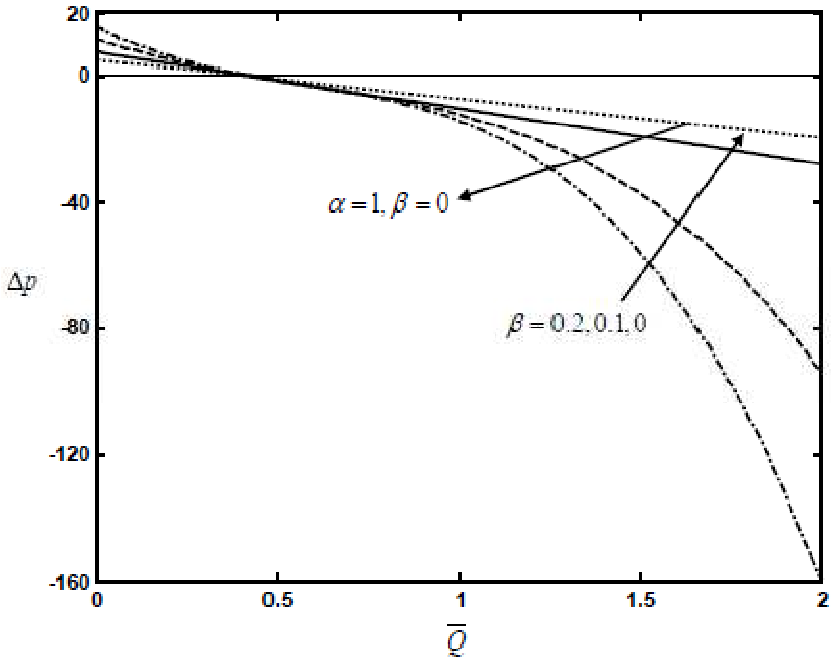

Fig. 6 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of β with $\phi = 0.6$, $\alpha = 1.5$ and $M = 1$.

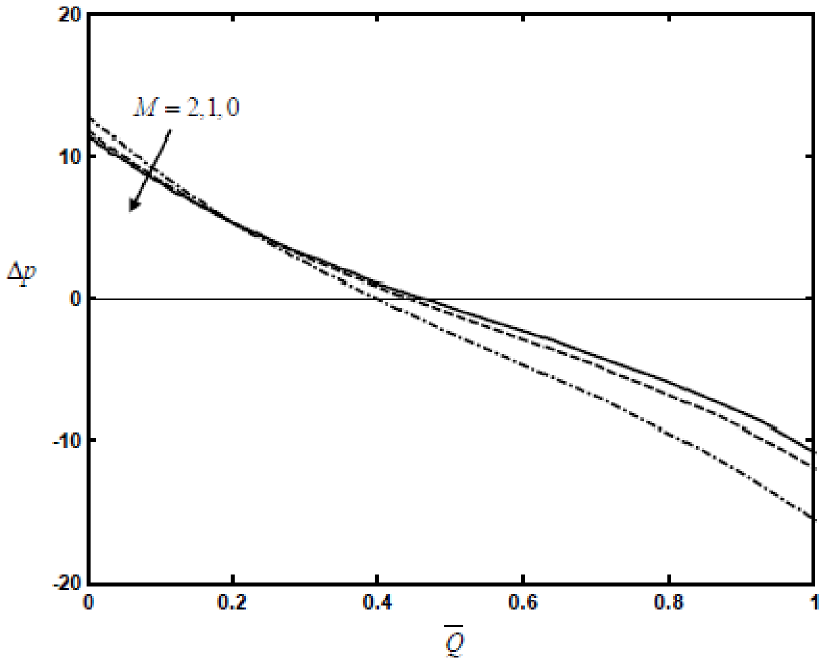

Fig. 7 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of M with $\phi = 0.6$, $\alpha = 1.5$ and $\beta = 0.1$.

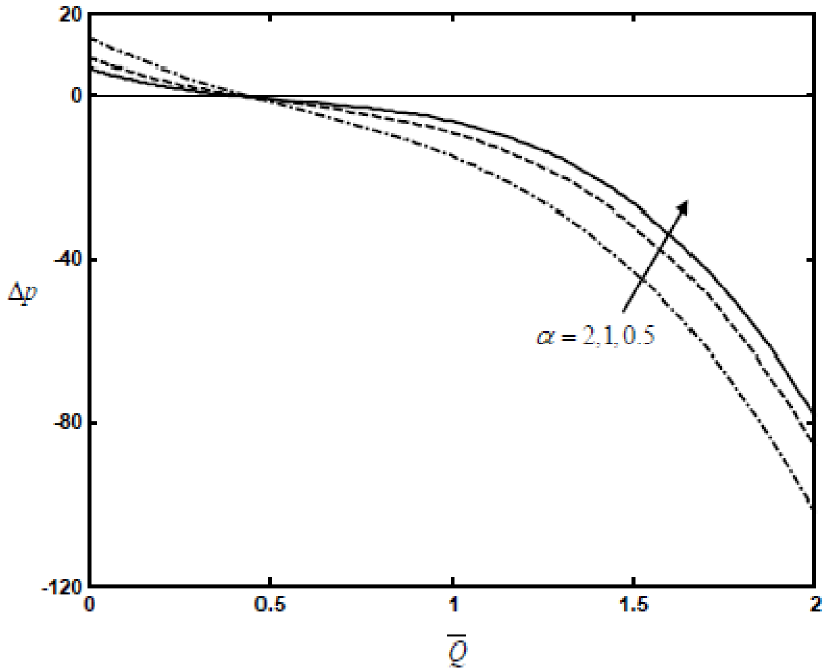

Fig. 8 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of α with $\phi = 0.6$, $\beta = 0.1$ and $M = 1$.

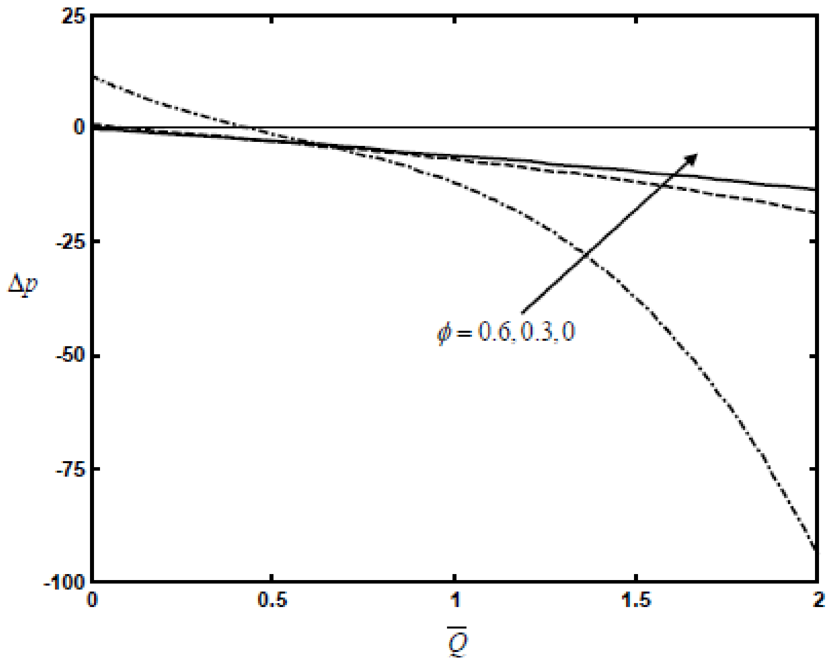

Fig. 9 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of ϕ with $\beta = 0.1$, $\alpha = 1.5$ and $M = 1$.

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