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Peristaltic flow of a micropolar fluid in a vertical channel with longwave length approximation

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ABSTRACT

Peristaltic flow of a micropolar fluid in a channel with longwave length approximation is studied under long wavelength and low Reynolds number assumptions. The velocity, the pressure rise over one cycle of the wave and frictional force are obtained. It is observed that for a given flux $\bar{Q}$ *, the pressure difference* Δp *increases with increasing parameter* η.

Keywords: Peristaltic transport; micropolar fluid; volume flow rate; pressure rise; pumping characteristics.

INTRODUCTION

Peristalsis is the mechanism by which fluid is transported through a distensible tube when contraction or expansion waves propagate along its length. Peristalsis appears to be the mechanism for fluid transport in many physiological situations such as transport of urine through ureter, food mixing and chyme movement in intestines, transport in bile duct, etc. The study of peristaltic transport of fluid is based on the principles of fluid mechanics involving interaction of fluid motion in tubes with flexible boundaries. In such investigations an appropriate mathematical model of the physiological system is made by keeping in view the nature of the physiological fluid (i.e. its Newtonian or non-Newtonian character, its behavior as a two phase mixture, its viscosity), the nature of the tube and other processes involved. Pioneering work in this area has been done by Jaffrin and Shapiro [7, 8], Brasseur et al. [3], Usha and Ramachandra Rao [9, 10, 16 & 17], Shukla et al [11, 12], Vajravelu et al. [18-21] and many others [13-15].

In classical continuum theory a body is assumed to be a dense collection of point masses in which there is no internal structure. In the motion of a volume element Δv it is assumed that the individual motions of material points coincide with the motion of centre of mass of the volume element Δv. In this case the density ρ of the volume element Δv is independent of the size of Δv and independent on it's location in space and the time t.

Eringen [4] reported that this is not true as $\Delta v \rightarrow 0$. The density ρ shows an increasing dependence on the size of the Δv, when Δv is less than a critical value Δv^*. Classical continuum theory cannot explain the mechanical behavior of rheologically complex fluids, such as liquid crystals, colloidal fluids and blood. Due to this fact a new approach was necessitated. There are several approaches to the formulation of microcontinuum theories of fluids such as simple deformable directed fluids, dipolar fluids, polar fluids, simple micro-fluids, micropolar fluids, etc. All these consider the existence of couple stresses and body couples.

Eringen [5, 6] reported the theory of micropolar fluids in which the fluid micro elements undergo rotations without stretching. Micropolar fluids are superior to the Navier-Stokes fluids and they can sustain stresses and body couples. Here the micro particles in the volume Δv rotate with an angular velocity about the centre of gravity of the volume in an average sense and is described by the micro rotation vector $\bar{\Omega}$. The micropolar fluids can support stress and body couples and find their applications in a special case of fluid in which micro rotational motions are important. Ariman and Cakmak [1, 2] discussed three basic viscous flows of micropolar fluids. They are Couette and Poiseuille flows between two parallel plates and the problem of a rotating fluid with a free surface. The results obtained are compared with the results of the classical fluid mechanics. Srinivasacharya et al. [13] made a study on the peristaltic pumping of a micropolar fluid in a tube. The gravitational effects are also important in peristaltic pumping. In view of this, we have considered the peristaltic pumping of a micropolar fluid in an inclined channel. This mathematical model may be useful to have a better understanding of the physiological systems such as blood vessels. The velocity field, the stream function, the volume flow rate and the pressure rise are obtained and results are discussed through graphs.

Mathematical formulation and solution

Consider the peristaltic pumping of a micropolar fluid in a vertical channel of half-width ‘a’. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity we restrict our discussion to the half-width of the channel as shown in figure. (1)

The wall deformation is given by

$$H(X,\ t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \qquad (1)$$

Where b is the amplitude, λ is the wavelength and c is the wave speed.

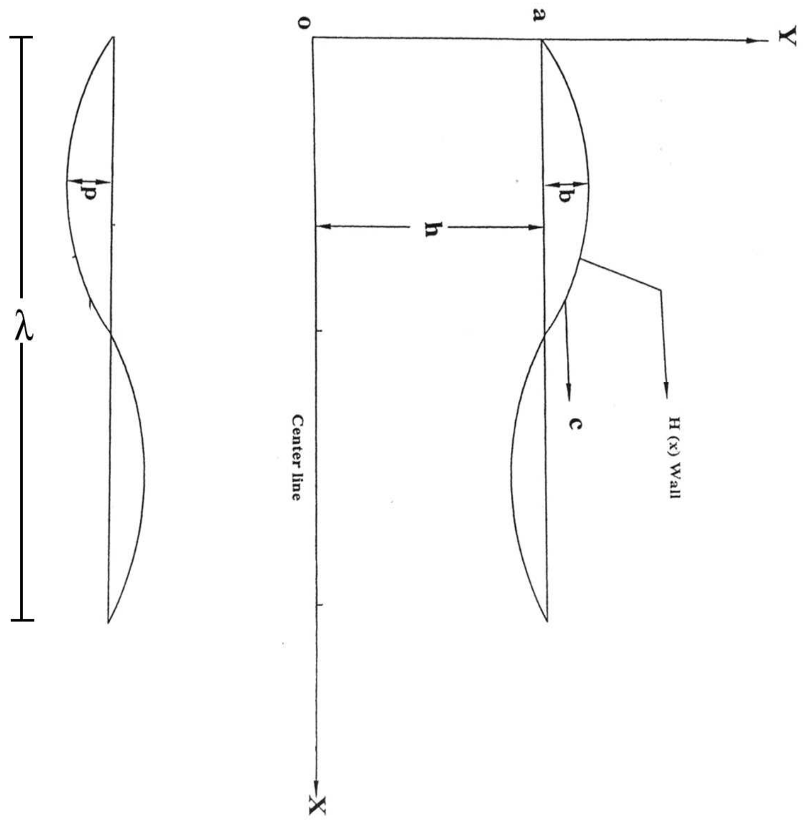


Figure 1 Physical Model

2.1. Equations of motion

Under the assumption that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame $(x,\ y)$ moving with velocity c away from the fixed (laboratory) frame $(X,\ Y)$. The transformation between these two frames is given by

$$x = X - ct;\ y = Y;\ u(x,y) = U(X - ct,\ Y) - c;\quad v(x,y) = V(X - ct,\ Y) \qquad (2)$$

Where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the flow is inertia-free. Further, we assume that the wavelength is infinite.

Using the non- dimensional quantities.

$$\bar{u} = \frac{u}{c};\ \bar{x} = \frac{x}{\lambda};\quad \bar{y} = \frac{y}{a};\quad \bar{p} = \frac{pa^2}{\lambda c\mu};\quad \bar{\Omega} = \frac{\Omega a}{c};\quad h = \frac{H}{a}$$

The non-dimensional form of equations governing the motion (dropping the bars) is

$$\frac{\partial^2 u}{\partial y^2} + N\frac{\partial \Omega}{\partial y} - (1-N)\frac{\partial p}{\partial x} + \eta = 0 \tag{3}$$

$$\frac{2-N}{m^2}\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial u}{\partial y} - 2\Omega = 0 \tag{4}$$

where $N = \frac{k}{\mu + k}$ is coupling number

Ω is the microrotation velocity

u is the velocity

μ is the viscosity of the fluid

k is the micropolar viscosity

m is the micropolar parameter

p is the fluid pressure

η is the gravity parameter, $\frac{a^2 g}{\gamma c}$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \tag{5}$$

$$\frac{\partial \Omega}{\partial y} = 0 \quad \text{at } y = 0 \tag{6}$$

$$u = -1 \quad \text{at } y = h(x) \tag{7}$$

$$\Omega = 0 \quad \text{at } y = h(x) \tag{8}$$

2.2. Solution

The general solution of (3) and (4) is given by

$$u = \frac{-N}{m}\left(C_2 \sinh my + C_3 \cosh my\right) + \left(\frac{(1-N)P - \eta}{(2-N)}\right)y^2 + \frac{2C_1}{m^2}y + C_4 \tag{9}$$

where $P = \frac{\partial p}{\partial x}$

$$\Omega = C_2 \cosh my + C_3 \sinh my - \left(\frac{(1-N)p - \eta}{(2-N)}\right)y - \frac{C_1}{m^2} \tag{10}$$

using the boundary conditions (5) to (8) in (9) and (10), we obtain the velocity of the fluid and microrotation velocity as

$$u = \frac{(1-N)p - \eta}{(2-N)}\left[D_4(\sinh my - my) - D_5 \cosh my + y^2 + D_6\right] - 1 \tag{11}$$

where $D_1 = \frac{(mh - \sinh mh)}{(2\cosh mh - N)}$

$$D_2 = [\sinh mh - mh]$$

$$D_3 = [\cosh mh - \frac{m^2h^2}{N}]$$

$$D_4 = \frac{-2N D_1}{m^2}$$

$$D_5 = \frac{N}{m^2}$$

$$D_6 = D_5D_3 - D_4D_2$$

$$\Omega = C_2 \cosh my + C_3 \sinh my - \frac{(1-N)P-\eta}{(2-N)} y - \frac{C_1}{m^2} \qquad (12)$$

where
$$C_1 = \frac{mN\left[(1-N)P-\eta\right][mh-\sinh mh]}{(2-N)(2\cosh mh - N)}$$

$$C_2 = \frac{2\left[(1-N)P-\eta\right][mh-\sinh mh]}{m\,(2-N)(2\cosh mh - N)}$$

$$C_3 = \frac{(1-N)P-\eta}{(2-N)m}$$

$$C_4 = \frac{2N((1-N)P-\eta))(mh-\sinh mh)}{(2-N)\,(2\cosh mh - N)}\left[\frac{\sinh mh}{m^2} - \frac{h}{m}\right] + \frac{(1-N)P-\eta}{(2-N)}\left[\frac{N\cosh mh}{m^2} - h^2\right] - 1$$

Integrating the equation (11) and using the condition $\psi = 0$ at $y = 0$, we get the stream function as

$$\psi = \frac{(1-N)p-\eta}{(2-N)}\left[D_4\left[\frac{\cosh my}{m} - \frac{my^2}{2}\right] - D_5\frac{\sinh my}{m} + \frac{y^3}{3} + D_6 y\right] - y \qquad (13)$$

The volume flux q through each cross-section in the wave frame is given by

$$q = \int_0^h u\, dy$$

$$q = \frac{(1-N)p-\eta}{(2-N)}\left[D_4\left[\frac{\cosh mh}{m} - \frac{mh^2}{2}\right] - D_5\frac{\sinh mh}{m} + \frac{h^3}{3} + D_6 h\right] - h \qquad (14)$$

where $D_1 = \dfrac{(mh-\sinh mh)}{(2\cosh mh - N)}$

$$D_2 = [\sinh mh - mh]$$

$$D_3 = [\cosh mh - \frac{m^2h^2}{N}]$$

$$D_4 = \frac{-2N D_1}{m^2}$$

$$D_5 = \frac{N}{m^2}$$

$$D_6 = D_5 D_3 - D_4 D_2$$

The pressure gradient is obtained from equation (14)

$$\frac{dp}{dx} = \frac{(q+h)(2-N)}{(1-N)}\left[\frac{1}{D_4\left[\frac{\cosh mh}{m} - \frac{mh^2}{2}\right] - D_5\frac{\sinh mh}{m} + \frac{h^3}{3} + D_6 h}\right] + \eta \qquad (15)$$

The time averaged flow rate is

$$\bar{Q} = q + 1 \qquad (16)$$

2.3. The pumping characteristics

Integrating the equation (15) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \left[\frac{(\bar{Q}-1+h)(2-N)}{(1-N)}\left(\frac{1}{D_4\left[\frac{\cosh mh}{m} - \frac{mh^2}{2}\right] - D_5\frac{\sinh mh}{m} + \frac{h^3}{3} + D_6 h}\right) + \eta\right] dx \qquad (17)$$

The pressure rise required to produce zero average flow rate is denoted by ΔP_0 . Hence ΔP_0 is given by

$$\Delta p_0 = \int_0^1 \left[\frac{(h-1)(2-N)}{(1-N)}\left(\frac{1}{D_4\left[\frac{\cosh mh}{m} - \frac{mh^2}{2}\right] - D_5\frac{\sinh mh}{m} + \frac{h^3}{3} + D_6 h}\right) + \eta\right] dx \qquad (18)$$

The dimensionless frictional force F at the wall across one wavelength in the inclined channel is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right) dx$$

$$= \int_0^1 -h\left[\frac{(\bar{Q}-1+h)(2-N)}{(1-N)}\left(\frac{1}{D_4\left[\frac{\cosh mh}{m} - \frac{mh^2}{2}\right] - D_5\frac{\sinh mh}{m} + \frac{h^3}{3} + D_6 h}\right) + \eta\right] dx \qquad (19)$$

RESULT AND DISCUSSION

From equation (17), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of coupling number N with $a=1, \; m=4, b=0.6$ and $\eta=2$ and is shown in Figure (2). It is observed that for chosen parameters the pumping curves intersect at a point in the first quadrant closure to $\bar{Q}\approx 0.5$. For $\bar{Q}<0.5$ we observed that the pressure rise increases with the coupling number N. The behavior is otherwise when $\bar{Q}$ > 0.5. For free pumping the $\bar{Q}$ decreases with the increasing N.

The variation of pressure rise with time averaged flow rate is calculated from equation (17) for different values of micropolar parameter 'm', and is shown in Figures (3) for fixed $a=1, b=0.6 \quad n=0.8, \; \eta=2$. It is observed that the pumping curves that the pumping curves meet at a point between $\bar{Q}=0.4$ and $\bar{Q}=0.6$. This value is estimated as $\bar{Q}=0.44$. When $\bar{Q}<0.44$ the pressure rise decreases with increasing m. The opposite behavior is noticed for $\bar{Q}>0.44$.

From equation (17), we have calculated pressure rise as a function of $\bar{Q}$ for different values of η and is shown in Figure (4) for fixed $n=0.2, \quad m=2$ and $a=1, b=0.6$. We observed that with a given $\bar{Q}$, the value Δp is increases with an increasing in the parameter η.

The variation of frictional force with time averaged flow rate is calculated from equation (19) for different values of N, m, η for a fixed a = 0.1, b = 0.6 and is shown in Figures (5) to (7). It is observed that the frictional force F has the opposite behavior compared to pressure rise.

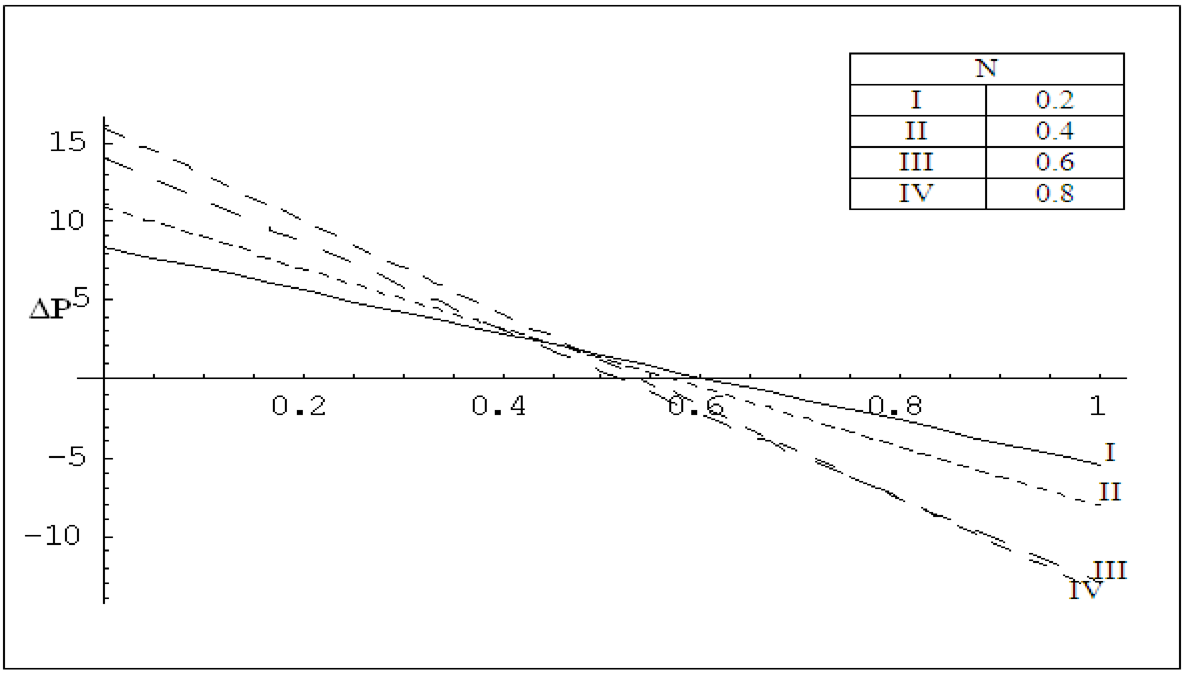


Figure 2: The variation of ΔP with $\bar{Q}$ for different values of N with a=1, b=0.6, m=4 and η=2

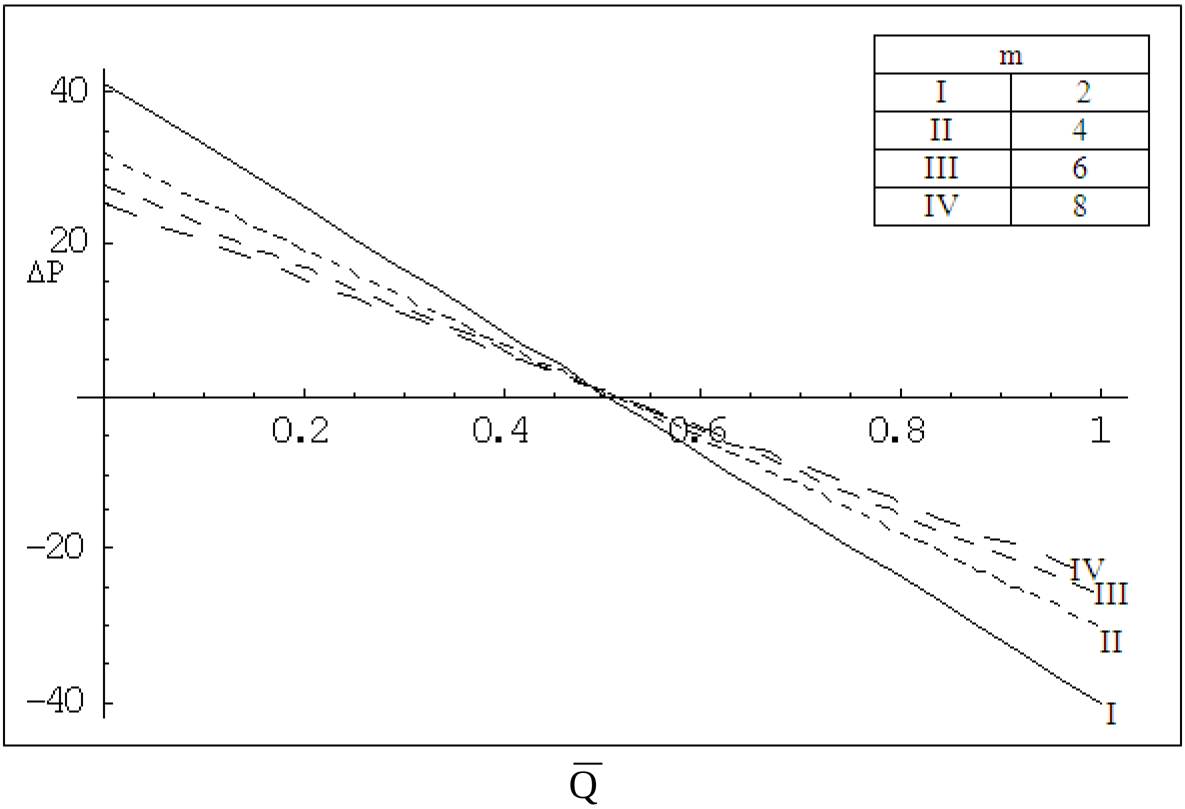


Figure 3: The variation of ΔP with $\bar{Q}$ for different values of m with a=1, b=0.6, n=0.8 and η=2

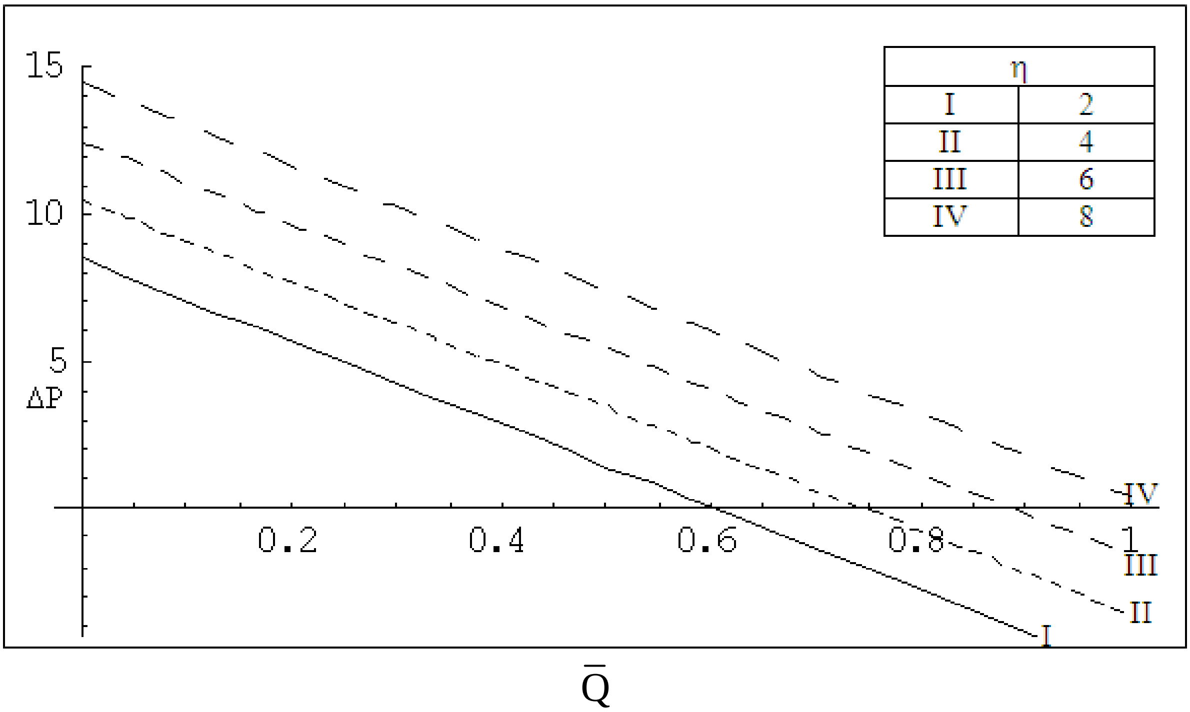
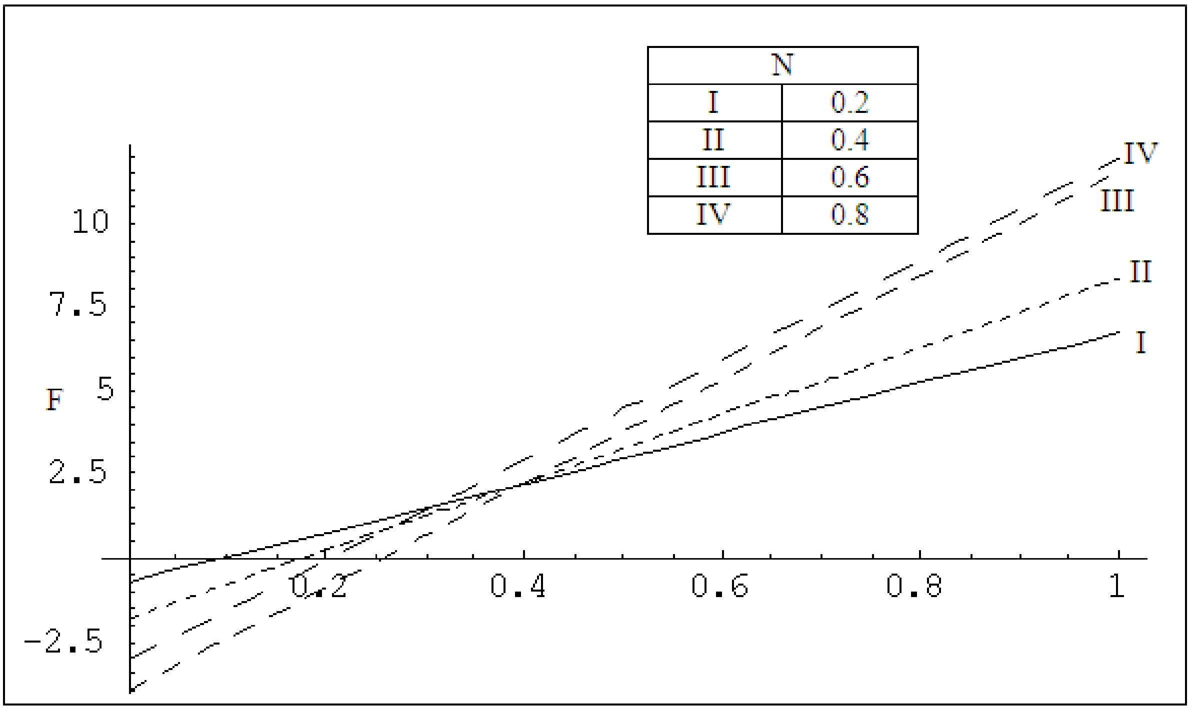
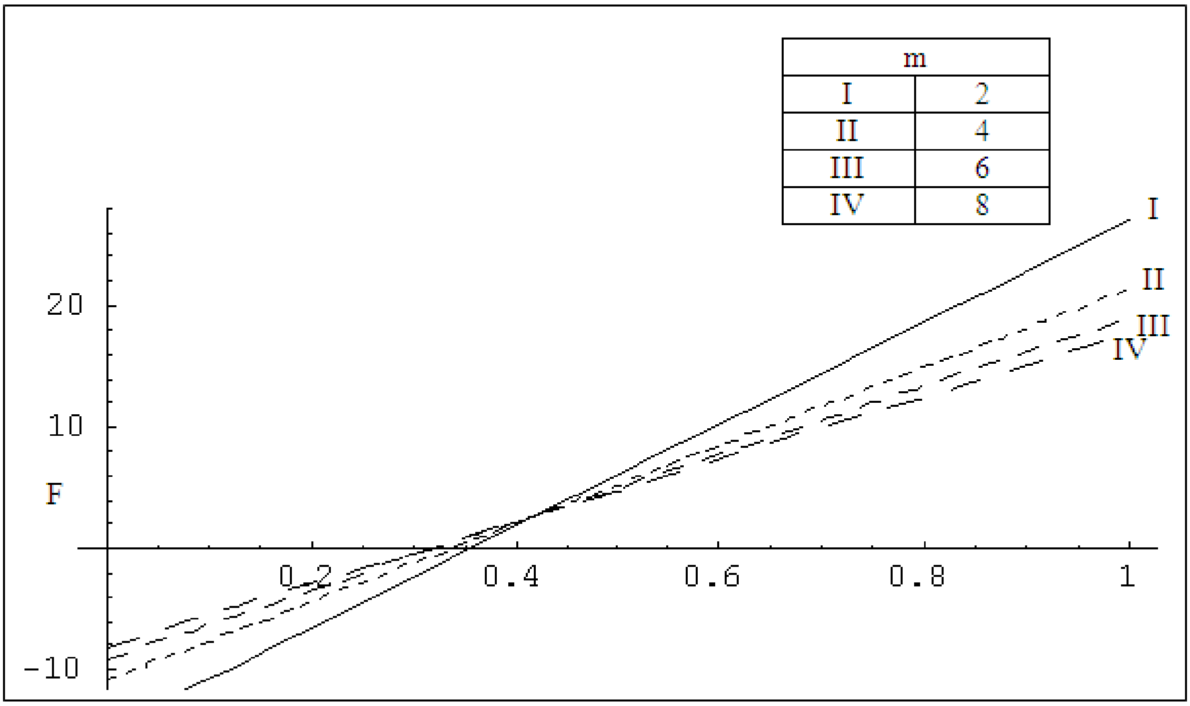


Figure 4: The variation of ΔP with $\bar{Q}$ for different values of η with a=1, b=0.6, n=0.2 and m=2



$\bar{Q}$

Figure 5: The variation of F with $\bar{Q}$ for different values of N with a=1, b=0.6, m=4 and η=2



$\bar{Q}$

Figure 6: The variation of F with $\bar{Q}$ for different values of m with a=1, b=0.6, N=0.8 and η=2

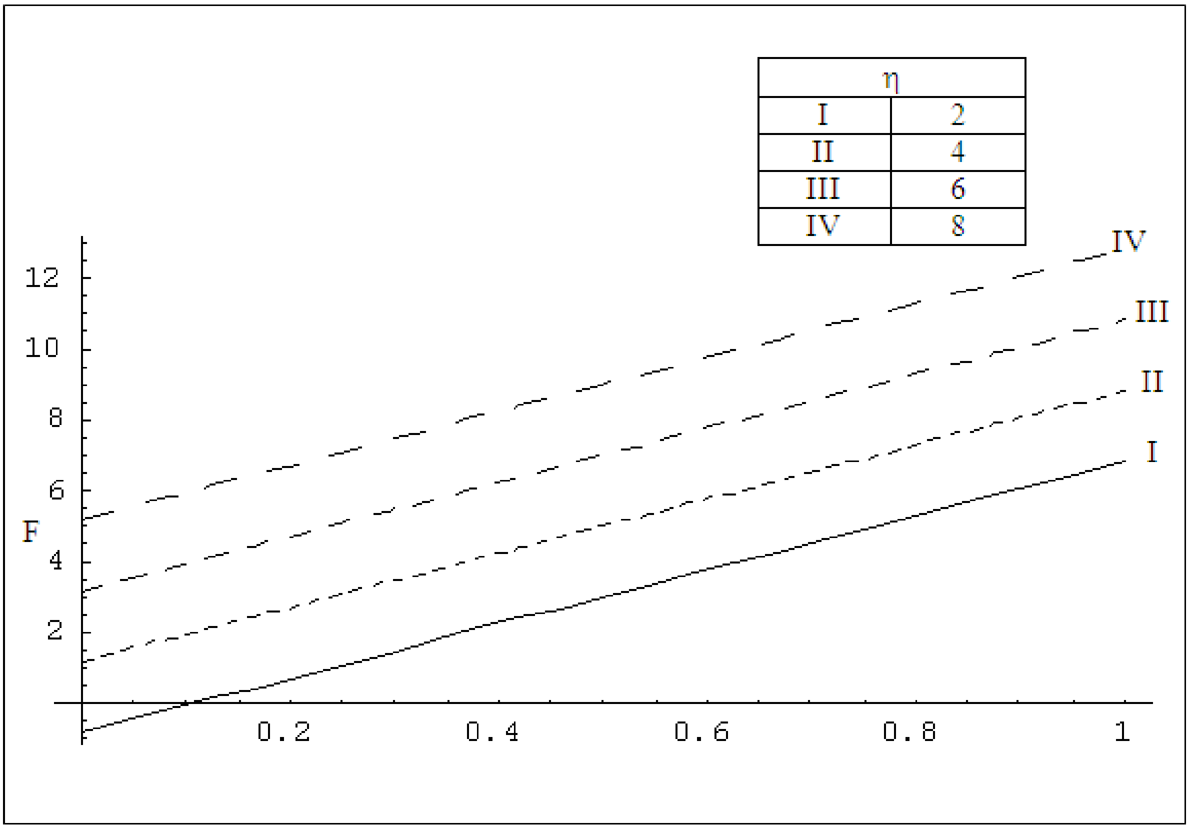


Figure 7: The variation of F with $\bar{Q}$ for different values of η with a=1, b=0.6, m=2 and N=0.2

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