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Optimizing Bicriteria in Flow Shop Production Schedule With String of Disjoint Job Block Criteria

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ABSTRACT

A bicriteria two stage flow shop scheduling is addressed to minimize the makespan and the rental cost of the machines under specified rental policy simultaneously. The processing times, independent setup times of the jobs are associated with their respective probabilities. Further the jobs are processed in a string of disjoint job-blocks. The study gives an optimal schedule rule to optimize the bicriteria through heuristic approach. The two criteria of minimizing the maximum utilization of the machines or rental cost and minimizing the maximum makespan are one of the combinations of our objective function reflecting the performance measure. A computer programme followed by a numerical illustration is given to substantiate the algorithm.

INTRODUCTION

In a general flowshop scheduling problem, n jobs are to be scheduled on m machines in order to optimize some measures of performance. All jobs have the same processing requirements so they need to be processed on all machines in the same order. Two-machine flowshop scheduling problem has been considered as a major problem due to its applications in real-life. Earlier, the algorithm which minimize one criteria does not take into consideration the effect of other criteria. In modern manufacturing and operations management, the minimization of makespan and rental/lease cost of machines are the significant factors as for the reason of upward stress of competition on the markets. Recently scheduling, so as to approximate more than one criteria received considerable attention. The bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. In this present work we introduced the processing of jobs in a string of disjoint job block. In one of the string the order of the jobs to be processed is fixed where as in other string disjoint from the first string, the jobs are in random order. The concept of job block has practical significance to create a balance between a cost of providing priority in service to the customer and cost of giving service with non-priority.

A flow shop scheduling problems has been one of the classical problems in the production scheduling since Johnson (1954) proposed the well known Johnson's rule in the two, three stage flow shop makespan scheduling problem. Smith (1956) whose work is one of the earliest considered minimization of mean flow time and maximum tardiness. Wassenhove and Gelders(1980) studied minimization of maximum tardiness and mean flow time explicitly as objective. Some of the noteworthy heuristic approaches are due to Maggu & Das (1977), Sen *et al*.(1983), Dileepan *et al*.(1988), Chandersekharan (1992), Bagga(1969), Bhambani(1997), Narain (2006), Chakarvrthy (1999), Singh T.P. *et al*. (2005) and Gupta *et al*.(2011). Gupta, Sharma, Seema & Shefali (2011) studied bicriteria in n x 2 flow shop scheduling under specified rental policy. The present work is an attempt to extend their study by introducing the concept of jobs in a string of disjoint job-blocks, thus making the problem wider and more practical in production / process industry.

Practical Situation

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology. Further the priority of one job over the other may be significant due to the relative importance of the jobs. It may be because of urgency or demand of that particular job. Hence, the job block criteria become important.

Notations

- S : Sequence of jobs 1,2,3,….,n
- S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -----
- M_j : Machine j, j= 1,2
- M : Minimum makespan
- a_{ij} : Processing time of i^{th} job on machine M_j
- p_{ij} : Probability associated to the processing time a_{ij}

s_{ij} : Set up time of i^{th} job on machine M_j
q_{ij} : Probability associated to the set up time s_{ij}
A_{ij} : Expected processing time of i^{th} job on machine M_j
S_{ij} : Expected set up time of i^{th} job on machine M_j
$L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M
t'_{ij} :Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $E_j(S_k)$
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Rental cost of i^{th} machine

Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$$t_{ij} = \max (t_{i-1,j}+ s_{(i-1)j} \times q_{(i-1)j} , t_{i,j-1}) + a_{ij} \times p_{ij} \quad \text{for } j \geq 2.$$

$$= \max (t_{i-1,j}+ S_{(i-1),j} , t_{i,j-1}) + A_{i,j}$$

where $A_{i,j}$= Expected processing time of i^{th} job on j^{th} machine
$S_{i,j}$= Expected setup time of i^{th} job on j^{th} machine.

Definition

Completion time of i^{th} job on machine M_j starts processing jobs at time L_j is denoted by t'_{ij} and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j}$$

Also $t'_{i,j} = \max(t'_{i,j-1}, t'_{i-1,j} + S_{i-1,j}) + A_{i,j}$.

Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine.

Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let w_i be the weight of i^{th} job. Let A_{ij} be the expected processing time and $S_{i,j}$ be the expected setup time of i^{th} job on j^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines while minimizing total elapsed time. Let $\alpha =(i_k, i_m)$ be an equivalent job for job block in which job i_k is given priority over job i_m Take two job blocks α and β such that block α consists of m jobs out of n jobs in which the order of jobs is fixed and β consists of r jobs out of n in which order of jobs is arbitrary such that $m + r = n$. let $\alpha \cap \beta = \emptyset$ i.e. the two job blocks α & β form a disjoint set in the sense that the two blocks have no job in common. A string S of job blocks α and β is defined as $S = (\alpha, \beta)$.

The mathematical model of the problem in matrix form can be stated as:

Table 1: Jobs with processing and setup time

Jobs	Machine M_1				Machine M_2			
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}
4	a_{41}	p_{41}	s_{41}	q_{41}	a_{42}	p_{42}	s_{42}	q_{42}
5	a_{51}	p_{51}	s_{51}	q_{51}	a_{52}	p_{52}	s_{52}	q_{52}

Mathematically, the problem is stated as:

Minimize $U_j\left(S_k\right)$ and

Minimize $R(S_k) = t_{n,1}(S_k)\times C_1 + U_2(S_k)\times C_2.\ \forall i$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

THEOREM

The processing of jobs on M_2 at time $L_2 = \sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof. Let $t'_{i,2}$ be the completion time of i th job on machine M_2 when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let P(n) : $t'_{n,2} = t_{n,2}$

Basic step: For n = 1, j =2;

$$t'_{1,2} = L_2 + \sum_{k=1}^{1} A_{k,2} + \sum_{k=1}^{1-1} S_{k,2} = \sum_{k=1}^{1} I_{k,2} + \sum_{k=1}^{1} A_{k,2} + \sum_{k=1}^{1-1} S_{k,2}$$

$$= \sum_{k=1}^{1} I_{k,2} + A_{1,2} = I_{1,2} + A_{1,2} = A_{1,1} + A_{1,2} \quad = t_{1,2}\ ,$$

$\therefore$ P(1) is true.

Induction Step: Let P(m) be true, i.e., $t'_{m,2} =$ $t_{m,2}$

Now we shall show that P(m+1) is also true, i.e., $t'_{m+1,2} = t_{m+1,2}$

Since $t'_{m+1,2} = \max(t_{m+1,1}, t'_{m,2} + S_{m,2}) + A_{m+1,2}$

$$= \max\left(t_{m+1,1}, L_2 + \sum_{i=1}^{m} A_{i,2} + \sum_{i=1}^{m-1} S_{i,2} + S_{m,2}\right) + A_{m+1,2}$$

$$= \max\left(t_{m+1,1}, \sum_{i=1}^{m+1} I_{i,2} + \sum_{i=1}^{m} A_{i,2} + \sum_{i=1}^{m-1} S_{i,2} + S_{m,2}\right) + A_{m+1,2}$$

$$= \max\left(t_{m+1,1}, \left(\sum_{i=1}^{m} I_{i,2} + \sum_{i=1}^{m} A_{i,2} + \sum_{i=1}^{m-1} S_{i,2}\right) + I_{m+1} + S_{m,2}\right) + A_{m+1,2}$$

$$= \max\left(t_{m+1,1}, t'_{m,2} + I_{m+1} + S_{m,2}\right) + A_{m+1,2}$$

$$= \max\left(t_{m+1,1}, t'_{m,2} + \max\left(\left(t_{m+1,1} - t_{m,2}\right), 0\right) + S_{m,2}\right) + A_{m+1,2}$$

$$= \max\left(t_{m+1,1}, t_{m,2} + S_{m,2}\right) + A_{m+1,2}$$

$$= t_{m+1,2}$$

Therefore, *P(m+1)* is true whenever *P(m)* is true.

Hence by Principle of Mathematical Induction P(n) is true for all n i.e. $t'_{n,2} = t_{n,2}$ for all n.

Remark: If M_2 starts processing the job at $L_2 = t_{n,2} - \sum_{i=1}^{n} A_{i,2} - \sum_{i=1}^{n-1} S_{i,2}$, then total time elapsed $t_{n,2}$ is not altered and M_2 is engaged for minimum time. If M_2 starts processing the jobs at time L_2 then it can be easily shown that. $t_{n,2} = L_2 + \sum_{i=1}^{n} A_{i,2} + \sum_{i=1}^{n-1} S_{i,2}$.

Algorithm

Step 1: Calculate the expected processing times and expected set up times as follows

$$A_{ij} = a_{ij} \times p_{ij} \text{ and } S_{ij} = s_{ij} \times q_{ij} \quad \forall i, j$$

Step 2: Calculate the expected flow time for the two machines M_1 and M_2 as follows

$$A'_{i1} = A_{i1} - S_{i2} \text{ and } A'_{i2} = A_{i2} - S_{i1} \; \forall i$$

Step 3: Take equivalent job block $\alpha = (i_k, i_m)$ and calculate the processing times G_α and H_α on the guide lines of Maggu and Das (1977) as follows

$$G_\alpha = A'_{k1} + A'_{m1} - \min\left(A'_{m1}, A'_{k2}\right) \text{ and}$$

$$H_\alpha = A'_{k2} + A'_{m2} - \min\left(A'_{m1}, A'_{k2}\right)$$

Step 4: Obtain the order of jobs in the job block β in an optimal manner using Johnson's (1954) technique by treating job block β as sub flow shop scheduling problem of the main problem. Let β' be the new job block. Define its processing times $G_{\beta'}$ & $H_{\beta'}$ as defined in step 3.

Now, the given problem reduce into new problem replacing m jobs by job block α with processing times G_α & H_α as defined in step 3 and *r* jobs of job block β by β' with processing times $G_{\beta'}$ & $H_{\beta'}$ as defined in step 4.

The new problem can be represented as –

Jobs *(i)*	Machine M_1	Machine M_2
α	G_α	H_α
β'	$G_{\beta'}$	$H_{\beta'}$

Step 5: Let S_1 denote set of all the processing time G_i when $G_i \leq Hi$ and let S_2 denote the set of processing times which are not covered in set S_1.

Step 6: Let S'_1 denote a suboptimal sequence of jobs corresponding to non decreasing times in set S_1 & let S'_2 denote a suboptimal sequence of jobs corresponding to non-decreasing times in set S_2.

Step7: The augmented ordered sequence $S=(S'_1, S'_2)$ gives optimal sequence for processing the jobs for the original problem.

Step 8: Compute total elapsed time $t_{n2}(S)$ by preparing in-out tables for sequence S.

Step 9: Compute $L_2(S)$ for each sequence S as follows

$$L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$$

Step 10: Find utilization time of 2nd machine $U_2(S)$ and minimum rental cost $R(S)$ for the sequence S as follows

$$U_2(S) = t_{n2}(S) - L_2(S) \text{ and}$$

$$R(S) = t_{n,1}(S) \times C_1 + U_2(S) \times C_2.$$

Numerical Illustration

Consider 5 jobs, 2 machine flow shop problem with processing time and setup time associated with their respective probabilities as given in the following table and jobs 2, 5 are to be processed as a group job (2,5). The rental cost per unit time for machines M_1 and M_2 are 5 units and 7 units respectively. Our objective is to obtain optimal schedule to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines, under the rental policy P.

Table 2: Jobs with processing and setup time

Job	Machine M_1				Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	26	0.2	3	0.3	5	0.3	3	0.2
2	30	0.2	3	0.1	20	0.2	4	0.2
3	40	0.1	2	0.3	20	0.2	3	0.3
4	25	0.2	4	0.1	25	0.1	4	0.1
5	20	0.3	2	0.2	7	0.2	5	0.2

Solution: As per step 1: Expected processing and setup times for machines M_1 and M_2 are as shown in table 3.

Table 3: Processing and setup times for machines M_1 and M_2

Job	Machine M_1		Machine M_2	
i	A_{i1}	S_{i1}	A_{i2}	S_{i2}
1	5.2	0.9	1.5	0.6
2	6.0	0.3	4.0	0.8
3	4.0	0.6	4.0	0.9
4	5.0	0.4	2.5	0.4
5	6.0	0.4	1.4	1.0

As per step 2: The expected flow times for the machines M_1 and M_2 are as shown in table 4.

Table 4: Expected flow times for the machines M_1 and M_2

Job	Machine M_1	Machine M_2
i	A'_{i1}	A'_{i2}
1	4.6	0.6
2	5.2	3.7
3	3.1	3.4
4	4.6	2.1
5	5.0	1.0

As per step 3: Here $\alpha = (2, 5)$

Therefore, $G_\alpha = 5.2 + 5.0 - 3.7 = 6.5$ and $H_\alpha = 3.7 + 1.0 - 3.7 = 1.0$

As per step 4: Here $\beta = (1, 3, 4)$

Job	Machine M_1	Machine M_2
1	4.6	0.6
3	3.1	3.4
4	4.6	2.1

Now, using Johnson[1] technique by treating job block β as sub flow shop scheduling problem of the main problem. Let β' be the new job block. Here we get $\beta' = (3, 4, 1)$

Also $\beta' = (3,4, 1) = ((3, 4), 1) = (\alpha', 1)$, where $\alpha' = (3, 4)$

$G_{\beta'} = 4.3 + 4.6 - 2.1 = 6.8$ and $H_{\beta'} = 2.1 + 0.6 - 2.1 = 0.6$

Now problem is

Jobs(i)	G_i	H_i
α	6.5	1.0
β'	6.8	0.6

As per step 5: $S_1 = \varphi$, $S_2 = [6.5, 6.8]$

As per step 6: $S'_1 = \varphi$, $S'_2 = (\alpha, \beta')$

As per step 7: Optimal sequence is S= (2, 5, 3, 4, 1)

As per step 8: The In-Out table for the sequence S is as shown in table 5.

Table 5: In-Out table for the sequence S

Jobs	Machine M_1	Machine M_2
i	In – Out	In – Out
2	0 – 6.0	6.0 – 10.0
5	6.3 – 12.3	12.3 – 13.7
3	12.7 – 16.7	16.7 – 20.7
4	17.3 – 22.3	22.3 – 24.8
1	22.7 – 27.9	27.9 – 29.4

Total elapsed time $t_{n,2}(S) = 29.4$

As per step 9: The latest time at which machine M_2 is taken on rent

$$L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$$

$$= 29.4 - 13.4 - 3.1 = 12.9 \text{ units}$$

As per step 10: The utilization time of machine M_2 is

$$U_2(S) = t_{n,2}(S) - L_2(S) = 29.4 - 12.9 = 16.5 \text{ units}$$

Bi-objective In – Out table is as follows

Jobs	Machine M_1	Machine M_2
i	In – Out	In – Out
2	0 – 6.0	12.9 – 16.9
5	6.3 – 12.3	17.7 – 19.1
3	12.7 – 16.7	20.1 – 24.1
4	17.3 – 22.3	25.0 – 27.5
1	22.7 – 27.9	27.9 – 29.4

Total Minimum Rental Cost = $R(S) = t_{n,1}(S) \times C_1 + U_2(S) \times C_2$

$$= 27.9 \times 5 + 16.5 \times 7$$

$$= 255 \text{ units}$$

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