

Optimization of tilt angle for solar collectors in WA, Ghana

Felix A. Uba and Emmanuel A. Sarsah

Mechanical Engineering Department, Wa Polytechnic, Wa, Ghana

ABSTRACT

Tilt angle greatly influences the performance of solar collectors. Maximum output can be obtained by solar collectors if they are installed at an optimum tilt. In this study, a mathematical model is used to determine the solar radiation incident on an inclined surface and the optimum slope angles for each month, season and year are calculated for solar collectors at Wa, Upper West Region, Ghana. The calculations are based on the measured values of monthly mean daily global radiation and estimated diffuse solar radiation on a horizontal surface. It was found that the optimum tilt is different for each month of the year. A computer program in MATLAB software was developed for this purpose. The collected energy on the inclined collector is simulated as the tilt angle is varied. In this study the collector surface is assumed to be facing towards the equator. The yearly average fixed tilt angle is found to be 26.8° for the location (i.e. the latitude of Wa plus 17°). The maximum solar energy gains are 212.41MJ/m^2 and 211.461MJ/m^2 when using the yearly average fixed angle and seasonally adjusted tilt angle representing a loss of 1% and 0.6% respectively compared to the monthly optimum tilt.

Key words: Optimum tilt angle, solar collector, declination, latitude, solar radiation.

INTRODUCTION

Ghana has been facing energy crisis for the last decade [11]. Now, the demand exceeds supply and hence load-shedding is a common phenomenon through frequent power shutdowns. Due to increasing prices of fossil fuels in the country, it is difficult to concentrate on these conservative energy resources for production of electricity. Hence, the only way is to get benefit from renewable energy resources like solar, wind and biomass. The most plentiful power source we have in Ghana is solar.

To overcome the crisis of electric energy, people in Ghana are installing solar panels for electric power generation from solar energy. Solar energy can be utilized directly through a variety of devices such as solar collector or photovoltaic (PV) cell. Installing a collector properly can enhance its application benefit because the amount of radiation flux incident upon the collector is mainly affected by the azimuth and tilt angles, but the tilt angle varies with factors such as the geographic latitude, climate condition and utilization period of time [17]. The best way to collect maximum daily energy is to use tracking systems. A tracker is a mechanical device that follows the direction of the sun on its daily sweep across the sky. The trackers are expensive, need energy for their operation and are not always applicable [2]. To remove the cost component of a tracker, it is often practicable to orient the solar collector at fixed optimum tilt angle, β_{opt} , for the season/year round which will give a satisfactory output. To get the most from fixed-positioned (or seasonally adjusted) photovoltaic or solar collectors, they are positioned in the direction that captures the most solar radiation. But there are a number of variables in figuring out the best direction and this research is designed to find the best placement for solar collectors in Wa.

Researchers on solar energy often give the advice that the optimum tilt should be equal to the latitude [6, 19]. There are many papers in literature which make different recommendations for the optimum tilt, based only on the latitude (Iqbal, 1979; Yakup, 2001). In the past, many researchers have devoted to studying the optimal installation angle in the world. Gopinath [6] and Soulayman[19] showed the optimum tilt angle is almost equal to the latitude. It was found that the yearly optimum tilt angle in Basra, Iraq was higher than the latitude by about 8°[18].

Since solar radiation is not always available at the desired quantity and time, the challenge is how to maximize available solar energy. It is an established fact that the solar radiation intensity falling on a horizontal flat surface, at a given time and location, increases with the increase in surface tilt angle at a solar hemispherical inclination. Hence the aim of this work is to predict the optimum angles of inclination for maximum collection of solar energy for each month, season and year in Wa, Upper West region, Ghana.

In this study, the first aim was to analyse the mean daily radiation of horizontal surface observed in Wa, over a 2-year period from 2010-2011, conducted by the Department of Agricultural engineering. The second aim was to compute the beam and diffuse components of radiation received on a horizontal surface. A simple mathematical procedure was then used to transpose these quantities onto an inclined plane. This was followed by computing the optimum tilt angle by searching for the values for which the total radiation on the collector surface is a maximum for a particular day or a specific period. The optimum tilt angle obtained for each month of the year helps to obtain the maximum solar energy to be collected on a collector. The final aim was then to discuss the results obtained.

MATERIALS AND METHODS

Given the monthly horizontal radiation, \overline{H} or monthly mean clearness index $\overline{K_T}$, the monthly diffuse radiation $\overline{H_d}$ can be calculated by [5]:

$$\frac{\overline{H_d}}{\overline{H}} = 1.391 - 3.560\overline{K_T} + 4.189\overline{K_T}^2 - 2.137\overline{K_T}^3 \quad (1)$$

for $\omega_s \leq 81.8^\circ$ and $0.3 \leq \overline{K_T} \leq 0.8$

$$\frac{\overline{H_d}}{\overline{H}} = 1.311 - 3.022\overline{K_T} + 3.427\overline{K_T}^2 - 1.821\overline{K_T}^3 \quad (2)$$

for $\omega_s > 81.4^\circ$ and $0.3 \leq \overline{K_T} \leq 0.8$

where $\overline{K_T} = \overline{H} / \overline{H_0}$ and $\overline{H_0}$ is the monthly mean daily extraterrestrial radiation on a horizontal surface, calculated by the following expression [4]:

$$\overline{H_0} = \frac{24 \times 3600}{\pi} \times G_{sc} \left(1 + 0.033 \cos \frac{360n}{365} \right) \times \left(\cos \phi \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \phi \sin \delta \right) \quad (3)$$

where \overline{n} is the monthly average daily hours of bright sunshine, G_{sc} is the solar constant (1367 W/m^2), ϕ is the latitude of the location, δ is declination angle and ω_s is the sunset hour angle in degrees. The declination, δ is given by the following expression:

$$\delta = 23.45 \sin \left(360 \times \frac{284 + \overline{n}}{365} \right) \quad (4)$$

and the hour angle, ω_s , is defined as

$$\omega_s = \cos^{-1} (-\tan \phi \tan \delta) \quad (5)$$

On obtaining $\overline{H_d}$, the mean daily beam radiation received on a tilted surface $\overline{H_b}$ can simply be expressed as

$$\overline{H_b} = \overline{H} - \overline{H_d} \quad (6)$$

Theoretically, the monthly collectible radiation on a tilted surface for any month of a year can be calculated by

$$\overline{H}_t = \overline{H}_b \overline{R}_b + \overline{H}_d \overline{R}_d + \rho \overline{H} \left(\frac{1 - \cos \beta}{2} \right) \quad (7)$$

where β is the tilt angle of the collector, ρ is the ground albedo and \overline{R}_b is the ratio of monthly mean daily beam radiation on a tilted surface to that on a horizontal surface. \overline{R}_b for fixed slope surfaces faced towards the equator in the northern hemisphere outlined by [13] is

$$\overline{R}_b = \frac{\cos(\phi - \beta) \cos \delta \sin \omega'_s + (\pi/180) \omega'_s \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta} \quad (8)$$

where ω'_s is the sunset hour angle for the tilted surface for the mean day of the month, which is given by

$$\omega'_s = \min \left[\cos^{-1}(-\tan \phi \tan \delta), \cos^{-1}(-\tan(\phi - \beta) \tan \delta) \right] \quad (9)$$

where 'min' means the smaller of the two terms in the bracket.

For surfaces in the southern hemisphere sloped towards the equator, the equations are [13]:

$$\overline{R}_b = \frac{\cos(\phi + \beta) \cos \delta \sin \omega'_s + (\pi/180) \omega'_s \sin(\phi + \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta} \quad (10)$$

and

$$\omega'_s = \min \left[\cos^{-1}(-\tan \phi \tan \delta), \cos^{-1}(-\tan(\phi + \beta) \tan \delta) \right] \quad (11)$$

\overline{R}_d in equation (7) is the ratio of the monthly diffuse radiation collected by unit area of a tilted surface to that by unit area of the surface when horizontal and is determined by the distribution of sky diffuse-radiation over the hemisphere. If the distribution of sky diffuse-radiation is isotropic, then \overline{R}_d is given as

$$\overline{R}_d = \left(\frac{1 + \cos \beta}{2} \right) \quad (12)$$

However, the actual distribution of the sky diffuse-radiation is not isotropic, as observed by (Hamilton, year) who found that 63% of sky diffuse-radiation comes from the southern part of the sky and recommended \overline{R}_d as

$$\overline{R}_d = \left(\frac{2 + \cos \beta}{3} \right) \quad (13)$$

Equation (13) was used in this present study.

It can be seen from the above that the monthly mean radiation, \overline{H}_t , collected by unit area of a tilted surface will be influenced by the tilt angle β . The optimum tilt angle of a collector ensures that the collector receives maximum radiation during the given period.

MATERIALS AND METHODS

Available monthly mean daily global solar radiation and sunshine duration hours were taken from the site of the weather station of Wa Polytechnic, for the periods 2010 and 2011. The graphical location of Wa Polytechnic is latitude 10.01°N with an altitude of 322 m above sea level. Extraterrestrial radiation on a horizontal surface in monthly periods were calculated numerically using declination angle, latitude and sunset hour angle from the estimation methods. The mean clearness index, \overline{K}_T were then calculated from which the monthly diffuse radiation,

\bar{H}_d were obtained from the estimation methods. Table 1 shows the monthly averages over the two year period of data, processed in preparation for the estimation of the optimum tilt angle.

Table 1. Monthly averages over the two year period

Month	\bar{H}	\bar{H}_0	ω_s	\bar{K}_T	δ	\bar{H}_d	\bar{H}_b
Jan	19.65	31.99	86.13	0.61	-20.92	6.40	13.25
Feb	20.01	34.60	87.67	0.58	-12.95	7.15	12.86
Mar	21.95	36.91	89.57	0.59	-2.42	7.53	14.42
Apr	22.35	37.94	91.68	0.59	9.41	7.77	14.58
May	21.83	37.57	93.44	0.58	18.79	7.75	14.08
Jun	19.76	37.01	94.31	0.53	23.09	7.85	11.91
Jul	18.42	37.11	93.92	0.50	21.18	7.97	10.45
Aug	16.37	37.55	92.42	0.44	13.45	8.09	8.29
Sep	17.43	37.08	90.39	0.47	2.22	7.99	9.44
Oct	21.36	35.11	88.29	0.61	-9.60	7.07	14.29
Nov	17.95	32.50	86.53	0.55	-18.91	6.83	11.12
Dec	18.77	31.09	85.69	0.60	-23.05	6.29	12.48

Fig.1 shows the global, diffuse and beam monthly average radiation on a horizontal surface. It can be seen that the beam component is more dominant than the diffuse component all year round thus the main contribution of solar radiation comes from the beam component.

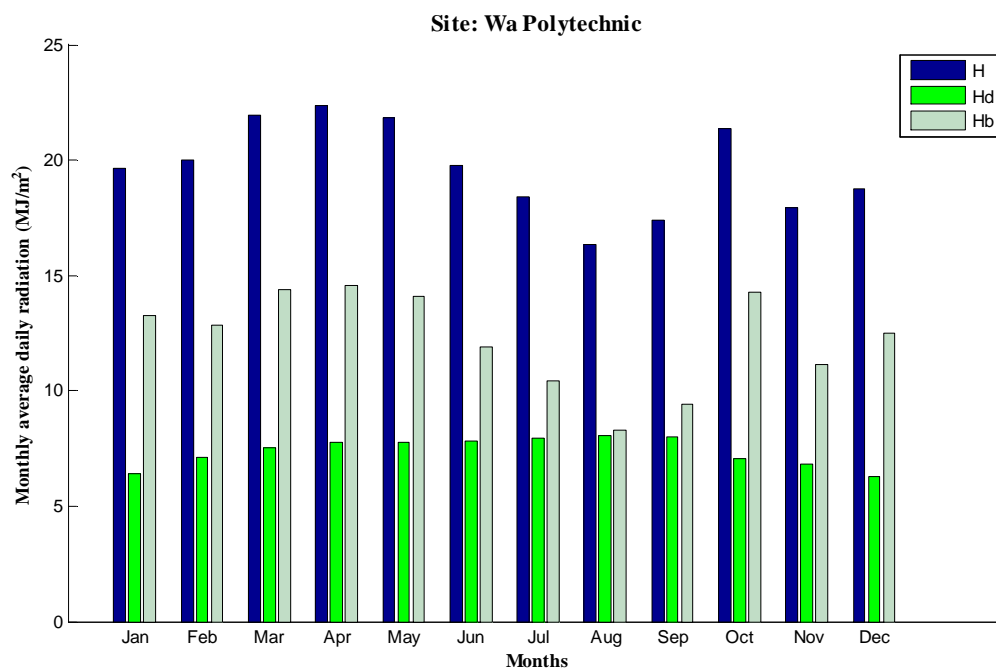


Fig.1. Monthly average daily global radiation (\bar{H}), diffuse radiation (\bar{H}_d) and beam radiation (\bar{H}_b) on a horizontal surface in Wa Polytechnic

The total solar radiation falling on tilted surface was computed for different tilt angles (0° until 90°) for each month of the year for Wa Polytechnic site. As no information is available on ground albedo, ρ values are assumed to be 0.22[2]. Graphs were plotted between the total radiation on tilted surface and tilt angle for each month, using Matlab computer software package (as shown in Fig.2).

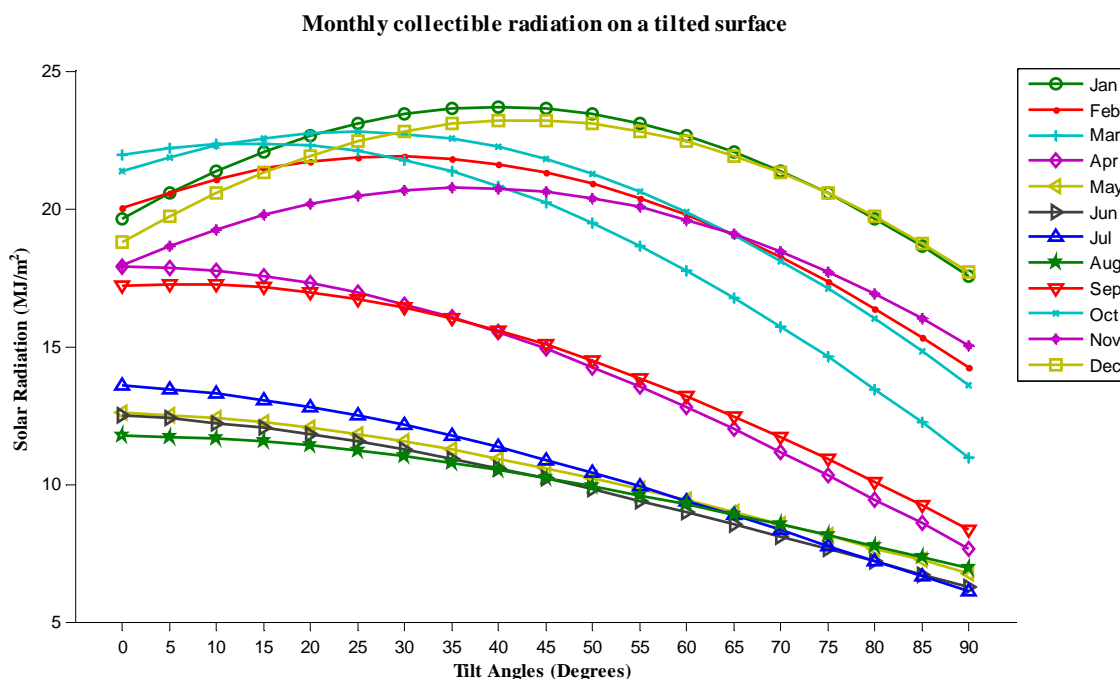


Fig.2. Monthly collectible radiation on a tilted surface in Wa Polytechnic over the year for different tilt angles varying from 0° to 90°

Second order polynomial equations were developed to fit the curves. The resulting polynomial equations were then differentiated with respect to tilt angle and equated to zero. The optimum tilt angle was then obtained for each month.

RESULTS AND DISCUSSION

The mean yearly optimum tilt angles for 2010 and 2011 were seen to be 25.9° and 27.8° respectively. The mean optimum tilt angle for the two years was therefore 26.8° approximately. From yearly analysis of the collected data, it was clearly seen that a unique β_{opt} exists for each month of the year for which the solar radiation is at a peak for the given month. Fig. 3 shows the tilt angles for each month of the year when the collector panel is tilted at the optimum angle at Wa Polytechnic site.

The optimum tilt angle of a flat-plate collector in January is 40°. The optimum tilt angle then decreased in February and March to 28.7° and 10.6° respectively. These values are so to be expected because during this period, the sun is imagined to be moving towards the latitude of Wa from the southern hemisphere (i.e. having a decreasing declination). A positive sign indicates that the collector is directed towards the south. This also means that collectors used during this period must have an upward facing component to the direction of the sun.

The negative values of the optimum tilt angles from April to August determine the orientation of the solar collector. This means that collectors used during this period in Wa Polytechnic site must have a downward facing component, i.e. facing the north. The values of the optimum tilt angles for this period also show an increasing trend to June. This is because the sun has just crossed the latitude of the observer and moving towards the north until it has reached its maximum declination in June (i.e. $\delta = 23.09^\circ$).

On the latitude of the observer (i.e. when the sun is directly overhead the observer), the incident angle is zero and this occurs on a particular day in March and September. There was however a decrease in values from July to September because the sun is now imagined to be moving from its maximum declination in June towards the equator from the north. The values of the optimum tilt angles show an increasing trend from September to December. This is because the sun has once again crossed the latitude of the observer and moving towards the south until it has reached its maximum declination in December ($\delta = -23.05^\circ$).

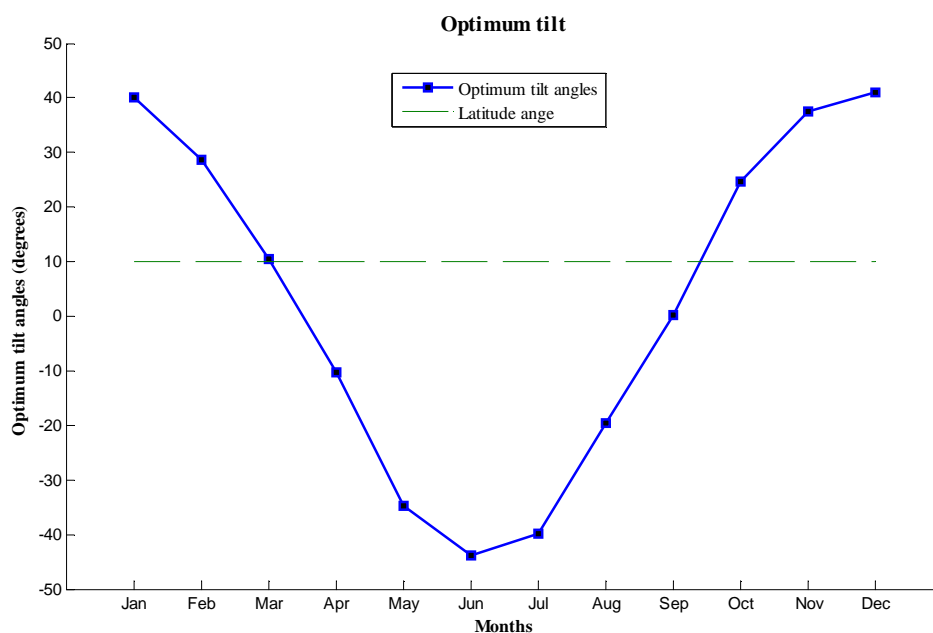


Fig.3. Optimum average tilt angle for each month of the year at Wa Polytechnic site

The average value of the optimum tilt angle for three periods, January – March, April – August and September – December, were found. Fig.3 requires that collector tilt be changed three times a year: January – March (26.4°), April – August (29.7°) and September – December (25.9°). These values clearly indicate that the optimum tilt angles for the periods January – March and September – December should be the latitude of the location plus 16° . The optimum tilt angle for the period April – August should be the latitude of the location plus 20° .

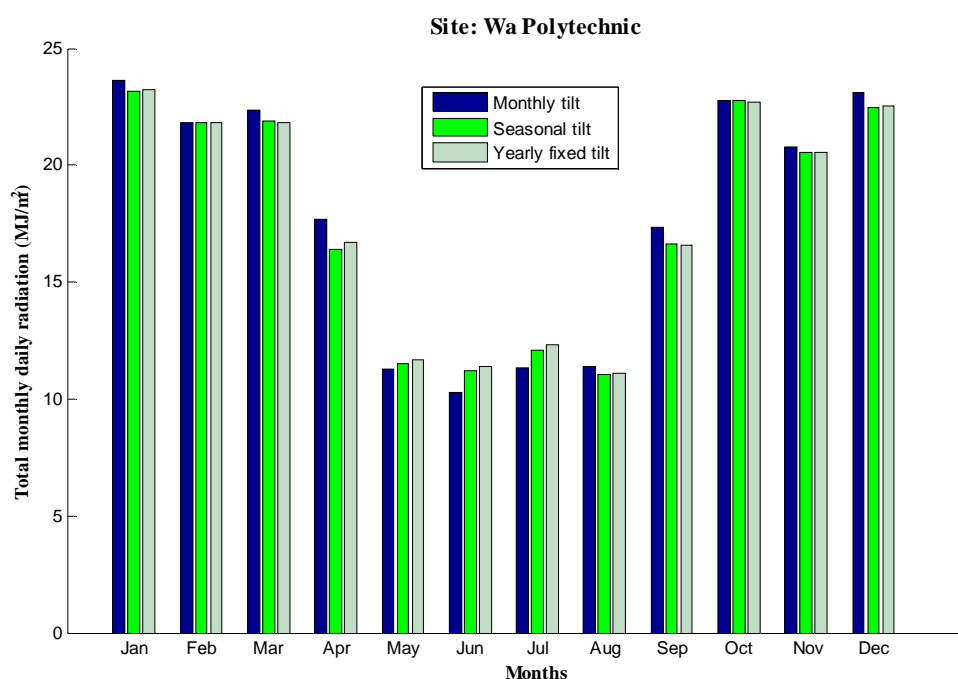


Fig.4. Total monthly radiation for optimum, seasonally and yealy fixed tilt angles

The yearly average tilt was calculated by finding the average value of the tilt angle for all months of the year. This value was found to be 26.7° which is a fixed tilt throughout the year required to give a greater absorption of solar radiation. Fig. 4 shows the total monthly daily solar radiation collected for the optimum tilt angles, the seasonal average tilt angles and for the yearly average fixed tilt angles. When the monthly optimum tilt angle was used, the yearly collected solar energy was 213.74 MJ/m^2 . With the seasonally adjusted angle, the yearly collected solar energy was 211.46 MJ/m^2 . With the yearly average tilt angle, the yearly collected solar energy was 212.41 MJ/m^2 . Hence the loss of energy when using both the yearly average fixed angle and the seasonal average tilt angle is around 1%.

CONCLUSION

The objective of this study was to estimate the optimum tilt angle of a solar collector based on monthly horizontal radiation. It was found out that the optimum tilt is different for each month of the year. We have also found out that the yearly average optimum tilt is equal to the latitude of the site plus 17° .

The results show that the optimum tilt angles for the periods January-March and September-December is the latitude of the location plus 16° while the optimum tilt for the period April-August is the latitude of the location plus 20° .

The loss of energy when using both the yearly average fixed angle and the seasonal average tilt angle is around 1%. It can, however, be concluded that a yearly average fixed tilt can be used in many applications (e.g. Domestic heating) in order to keep the manufacturing and installation costs of collectors low. The loss of 1% also shows that the models used in this present study give satisfactory results.

Acknowledgment

The authors would like to thank the staff of Agricultural Engineering Department, Wa Polytechnic, for their initiative for setting up the weather station for academic work.

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