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Original Article

Optimal Two Stage Open Shop Specially Structured Scheduling To Minimize The Rental Cost

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A R T I C L E I N F O

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A B S T R A C T

The present paper is an attempt to develop a heuristic algorithm for two machines specially structured open shop scheduling in which order of processing is not given ($A \rightarrow B$ or $B \rightarrow A$). Further the processing time of jobs are associated with their probabilities under some well defined structural relationship to one another. The objective of the paper is to minimize the rental cost of machines under a specified rental policy. A numerical illustration is given to support the algorithm.

INTRODUCTION

A feasible combination of machine and job orders is called a schedule. A job passing through the machines following a certain order is known as the processing route.If the processing route is not given in advance, but have to be chosen, processing system is called open shop. Open shop scheduling problem is to determine the feasibly combination of machine order and job order. application of open shop scheduling problem are in automobile repair quality control centre semi-conductor manufacturing etc. The research into flow shop problems has drawn a great attention in the last decades with the aim to decrease the cost and to increase the effectiveness of industrial production. Johnson [1] gave a procedure to obtain the optimal sequence for n-jobs, two – three machines flow shop scheduling problem with an objective to minimize the makespan. The work was develop by Ignall and Scharge [2], Bagga [3], Sahni,et.al.[5] Gupta, J.N.D[4], Maggu and Das [7], Yoshida and Hitomi[8], Singh, T.P. [12], Chander sekharan [13], V.A. Strusevich[15], Anup [19], Gupta Deepak [22], etc. by considering the various parameters.

Singh T. P. And Gupta Deepak,[20] made an attempt to study the optimal two stage open shop scheduling in which processing time is associated with their respective probabilities including job block criteria. In the present paper we have develop a new heuristic algorithm for a specially structured two stage open shop scheduling.

PRACTICAL SITUATION

Various practical situations occur in real life when one has got the assignments. Many applied and experimental situations occur in our day to day working in factories and industrial concern where we have to restrict the processing of some jobs. The practical situation may be taken in a production industry; manufacturing industry etc, where some jobs has to give priority over other. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography, Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up-gradation to new technology.

NOTATIONS

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------ r.
M_j : Machine j= 1,2
p_{ij} : Probability associated to the processing time a_{ij}.
a_{ij} : Processing time of i^{th} job on machine M_j
A_{ij} : Expected processing time.
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$T_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k.
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Renal cost of i^{th} machine.
$CT(S_k)$: Total completion time of the jobs for sequence S_k

DEFINITION

Completion time of ith job on machine Mj is denoted by tij and is defined as:

tij = max (ti-1, j , ti , j-1) + aij $\times$ pij for $j \geq 2$.

= max (ti-1,j , ti,j-1) + Ai,.j , where Ai,,j= Expected processing time of ith job on jth machine.

RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

PROBLEM FORMULATION

Let n jobs 1,2,……..,n processed on two machines M_1 and M_2 under the specified rental policy p. Let a_{ij} be the processing time of i^{th} job (i = 1, 2, ... n) on machine a and p_{ij} be the probability associated with a_{ij} such that $\sum p_{ij} = 1$ & $o \leq p_{ij} \leq 1$.
A_{ij} be the expected processing time of i^{th} job on machine such that either $A_{i1} \geq A_{i2}$ or $A_{i1} \leq A_{i2}$ for all values of i & j. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.
The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M1		Machine M2	
I	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	a_{11}	p_{11}	a_{12}	p_{12}
2	a_{21}	p_{21}	a_{22}	p_{22}
3	a_{31}	p_{31}	a_{32}	p_{32}
4	a_{41}	p_{41}	a_{42}	p_{42}
-	-	-	-	-
N	a_{n1}	p_{n1}	a_{n2}	p_{n2}

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P)
Our objective is to minimize rental cost of machines while minimizing the utilization time.

ASSUMPTIONS

1. Two jobs cannot be processed on a single machine at a time.
2. Jobs are independent to each other.
3. Per-emption is not allowed i.e. once a job started on a machine, the process on that machine cannot be stopped unless the job is completed.
4. Either the expected processing time of the i^{th} job of machine M_1 is longer than the expected processing time of j^{th} job on machine M_2 or the processing time i^{th} job on machine M_1 is shorter than the expected processing time of j^{th} job on machine M_2 for all i,j.
   i.e. either $A_{i1} \geq A_{j2}$
   or $A_{i1} \leq A_{j2}$ for all i,j.
5. $\sum p_{ij} = 1$, j=1, 2.
6. Let n jobs be processed through two machines A, B in order AB and in order BA.
7. Machine break down is not considered.

ALGORITHM

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij} \forall i, j.$
Step 2: Obtain the job J1 (say) having maximum processing time on 1st machine obtain the job Jn (say) having minimum processing time on 2nd machine.

Step 3: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 11, Otherwise go to step 9.
Step 4: Take the difference of processing time of job J1 on M1 from job J2 (say) having next maximum processing time on M1. Call this difference as G1.Also, Take the difference of processing time of job Jn on M2 from job Jn-1(say) having next minimum processing time on M2. Call the difference as G2.
Step 5: If G1 ≤ G2 put Jn on the last position and J2 on the first position otherwise put J1 on 1st position and Jn-1 on the last position.
Step 6: Arrange the remaining (n-2) jobs between 1^{st} job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.
Step 7: Compute the total completion time $CT(S_k)$ k=1, 2…r. by computing in – out table for sequences S_k (K= 1, 2, ...r.) in.
Step 8: Calculate utilization time U'_2 of 2^{nd} $(M_1 \rightarrow M_2)$.

$$U_2 = CT(S_k) - A_{11}(S_k);\ k=1,2,\ldots r.$$

Step 9: Calculate utilization time U_2 of 2^{nd} $(M_2 \rightarrow M_1)$.

$$U_2 = CT(S_k) - A_{12}(S_k);\ k=1,2,\ldots r.$$

Step 10: Find rental cost $R(S_i) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U_2 \times C_2$, where C_1 & C_2 are the rental cost per unit time of M_1 & M_2 respectively $(M_1 \rightarrow M_2)$.

Step 11: Find rental cost $R(S_i) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U'_2 \times C_1$.

NUMERICAL ILLUSTRATION

Consider 6 jobs, 2 machines problem to minimize the rental cost. The processing times associated with their probabilities are given. The rental cost per unit time for machines M_1 and M_2 are 10 units and 5 units respectively. Our aim is to obtain optimal schedule to minimize the utilization time and hence the rental cost of machines under the rental policy P.

Jobs	Machine M_1		Machine M_2	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	25	.1	30	.3
2	20	.2	50	.2
3	15	.3	35	.2
4	30	.2	72	.1
5	40	.1	75	.1
6	35	.1	80	.1

SOLUTION: As per step 1: The expected processing time for machines M_1 and M_2 as follow

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	2.5	9.0
2	4.0	10.0
3	4.5	7.0
4	6.0	7.2
5	4.0	7.5
6	3.5	8.0

For flow shop $M_1 \rightarrow M_2$.
each $A_{i1} \leq A_{j2}$ for all i,j
i.e. $\max A_{i1} \leq \min A_{j2}$ for all i,j
also $\max A_{i1} = 6.0$ which is for job 4
$J_1 = 4$.
and $\min A_{j2} = 7.0$ which is for job 3
$J_n = 3$.
Since $J_1 \neq J_n$ so put $J_1 = 4$ on the first position and $J_n = 3$ on the last position. Arrange remaining 4 jobs between and J_n in any order we get 24 sequences in all.
$S_1 : 4-1-2-5-6-3$
$S_2 : 4-2-1-5-6-3$

$S_{22} : 4-5-6-2-1-3$
Since all the sequences will have the same elapsed time and utilization time & hence rental cost. So we shall find in – out table from any one of these say from
$S_1 : 4-1-2-5-6-3$

In – out table for the sequence $S_1 : 4-1-2-5-6-3$ is

Jobs	Machine M_1	Machine M_2
I	In-Out	In-Out
4	0.0 – 6.0	6.0 – 13.2
1	6.0– 8.5	13.2 – 22.2
2	8.5 – 12.5	22.2 – 32.2
5	12.5 – 16.5	32.2 – 39.7
6	16.5 – 20.0	39.7 – 47. 7
3	20.2 – 24.5	47.7 – 54.7

Total elapsed time = CT (S_1) = 54.7 units
Utilization time for M2 = U_2 (S_1) = 54.7 – 6.0 = 48.7 units
Also $\sum A_{i1} = 24.5$
Total rental cost $= 24.5 \times 10 + 48.7 \times 5$
$= 245 + 243.5$
$= 488.5$ units

For flow shop $M_2 \rightarrow M_1$.
each $A_{i2} \geq A_{j1}$ for all i,j
i.e. $\max A_{i2} \geq \min A_{j1}$ for all i,j
also $\max A_{i2} = 10.0$ which is for job 2
$J_2 = 2$.
and $\min A_{j2} = 2.5$ which is for job 1
$J_n = 1$.
Since $J_1 \neq J_n$ so put job 2 on the first position and job 1 on the last position. Arranging remaining 4 jobs between job 2 and job 1 in any order we get 24 sequences in all.
$S_1 : 2-4-3-5-6-1$
$S_2 : 2-3-4-5-6-1$
$S_{22} : 2-5-6-4-3-1$
Since due to our rental sequences all the sequences have the same elapsed time and utilization time & hence rental cost. Thus find in – out for any of the sequences say of
$S_1 : 2-4-3-5-6-1$ is
In – out table for the sequence $S_1 : 2-4-3-5-6-1$ is

Jobs	Machine M_1	Machine M_2
I	In-Out	In-Out
2	0 – 10.0	10.0 – 14.0
4	10.0 – 17.2	17.2 – 23.2
3	17.2 – 24.2	24.2 – 29.7
5	24.2 – 31.7	31.7- 35.7
6	31.7 – 39.7	39.7 – 43.2
1	39.7 – 48.7	48.7 – 51.2

Total elapsed time = CT (S_1) = 51.2 units
Utilization time for $M1_1$ = U'_2 (S_1) = 51.2 – 10.0 = 41.2 units
Also $\sum A_{i2} = 48.7$
Total rental cost $= 41.2 \times 10 + 48.7 \times 5$
$= 412 + 243.5$

= 655.5 units

Total rental cost when the flow is from $M_1 \rightarrow M_2$ for the sequence (4 – 1 – 2 – 5 – 6 – 3) is 488.5 units. And for the sequence (2 – 4 – 3 – 5 – 6 – 1) is 655.5 units when flow is $M_2 \rightarrow M_1$.

CONCLUSION

Hence the optimal sequence of all the jobs which minimize the rental cost of machines is (4 – 1 – 2 – 5 – 6 – 3) for flow shop $M_2 \rightarrow M_1$.

REMARKS

The study on n × 2 open shop scheduling may be further extented by including various parameters such as weightage of jobs, job block criteria etc.
The study may further be extended for n jobs 3 machine open shop problem.

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