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On The Stability of a Four Species: A Prey-Predator-Host-Commensal-Syn Eco-System-VIII

(Host of the Predator Washed Out States)

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ABSTRACT

This paper deals with an investigation on a Four Species Syn-Ecological System (Host of the Predator washed out states). The System comprises of a Prey (S_1), a Predator (S_2) that survives upon S_1, two Hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are neutral. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of four of these sixteen equilibrium points: Host of the Predator washed out states only are established in this paper. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories are illustrated.

Keywords: Equillibrium point, Host, Prey, Predator, Trajectories, Unstable.

INTRODUCTION

Mathematical modeling of Eco-System was initiated in 1925 by Lotka [11] and in 1931 by Volterra[15]. The general concepts of modeling have been presented in the treatises of Meyer[12], Kushing[8], Kapur J.N. [6,7] and several others. The ecological interactions can be broadly classified as Prey-Predation, Commensalism, Competition, Neutralism, Mutualism and so on. N.C. Srinivas [14] studied competitive eco-systems of two species and three species with limited and unlimited resources. Later Lakshminarayan [9], Lakshminarayan and Pattabhi Ramacharyulu [10] studied Prey Preadtor ecological models with partial cover for the Prey and alternate food for the predator. Recently, Archana Reddy [1] and Bhaskara Rama Sharma [2] investigated diverse problems related to two species competitive systems with time delay, employing analytical and numerical techniques. Further Phani Kumar, Seshagiri Rao and

Pattabhi Ramacharyulu [13] studied the stability of a Host-A flourishing commensal species pair with limited resources. The present authors Hari Prasad and Pattabhi Ramacharyulu studied the stability of the Prey and Predator Washed out states [3], Prey washed out states [4] and the Predator Washed out states [5]. In continuation of this, the criteria for the stability of only the Host of the Predator washed out states of the system is presented in this paper.

A Schematic Sketch of the system under investigation is shown here under Fig.1.

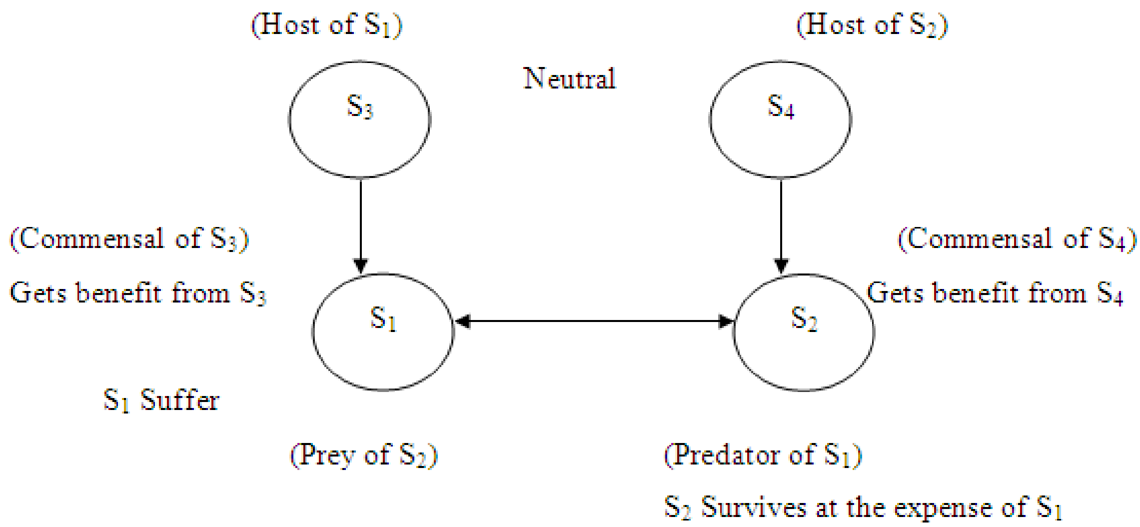

Fig. 1 Schematic Sketch of the Syn Eco - System

2. Basic equations of the model:

Notation Adopted:

S_1	:	Prey for S_2 and commensal for S_3.
S_2	:	Predator surviving upon S_1 and commonsal for S_4.
S_3	:	Host for the commonsal - Prey (S_1).
S_4	:	Host of the commonsal - Predator (S_2)
$N_1(t)$	:	The Population of the Prey (S_1)
$N_2(t)$	:	The Population of the Predator (S_2)
$N_3(t)$	:	The Population of the Host (S_3) of the Prey (S_1)
$N_4(t)$	:	The Population of the Host (S_4) of the Predator (S_2)
t	:	Time instant
a_1,a_2,a_3,a_4	:	Natural growth rates of S_1, S_2, S_3, S_4
$a_{11},a_{22},a_{33},a_{44}$	:	Self inhibition coefficients of S_1, S_2, S_3, S_4
	:	Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1
a_{13}	:	Coefficient for commensal for S_1 due to the Host S_3
a_{24}	:	Coefficient for commensal for S_2 due to the Host S_4
$K_i = \frac{a_i}{a_{ii}}$	:	Carrying capacities of S_i, i = 1,2,3,4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2 + a_{13} N_1 N_3 \qquad (2.1)$$

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{21} N_1 N_2 + a_{24} N_2 N_4 \qquad (2.2)$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2 \quad , \quad \frac{dN_4}{dt} = a_4 N_4 - a_{44} N_4^2 \qquad (2.3)$$

3. Equilibrium States

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \qquad (3.1)$$

as given in the following Table-1.

Table-1

S.No.	Equilibrium State	Equilibrium Point
1	E_1:Fully Washed out state	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$
2	E_2:Only the Host (S_4)of S_2 survives	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=K_4$
3	E_3:Only the Host (S_3)of S_1 survives	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=K_3, \overline{N_4}=0$
4	E_4:Only the Predator S_2 survives	$\overline{N_1}=0, \overline{N_2}=K_2, \overline{N_3}=0, \overline{N_4}=0$
5	E_5:Only the Prey S_1 survives	$\overline{N_1}=K_1, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$
6	E_6:Prey (S_1) and Predator (S_2) washed out	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=K_3, \overline{N_4}=K_4$
7	E_7:Prey (S_1) and Host (S_3) of S_1 washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2a_{44}+a_4a_{24}}{a_{22}a_{44}}, \overline{N_3}=0, \overline{N_4}=K_4$
8	E_8:Prey (S_1) and Host (S_4) of S_2 washed out	$\overline{N_1}=0, \overline{N_2}=K_2, \overline{N_3}=K_3, \overline{N_4}=0$
9	E_9:Predator (S_2) and Host (S_3) of S_1 washed out	$\overline{N_1}=K_1, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=K_4$
10	E_{10}:Predator (S_2) and Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{a_1a_{33}+a_3a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=K_3, \overline{N_4}=0$
11	E_{11}:Prey (S_1) and Predator (S_2)survives	$\overline{N_1}=\frac{a_1a_{22}-a_2a_{12}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_2}=\frac{a_1a_{21}+a_2a_{11}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_3}=0, \overline{N_4}=0$ This would exist only when $a_1a_{22} > a_2a_{12}$
12	E_{12}:Only the Prey (S_1) washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2a_{44}+a_4a_{24}}{a_{22}a_{44}}, \overline{N_3}=K_3, \overline{N_4}=K_4$
13	E_{13}:Only the predator (S_2) washed out	$\overline{N_1}=\frac{a_1a_{23}+a_3a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=K_3, \overline{N_4}=K_4$

14	E_{14}:Only the Host (S_3) of S_1 washed out	$\overline{N_1}=\frac{\delta_2}{\delta_1}, \overline{N_2}=\frac{\delta_3}{\delta_1}, \overline{N_3}=0, \overline{N_4}=K_4$ $where\ \ \delta_1=a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$ $\delta_2=a_1a_{22}a_{44}-a_{12}(a_2a_{44}+a_4a_{24})$ $\delta_3=a_1a_{21}a_{44}-a_{11}(a_2a_{44}+a_4a_{24})$ This would exist only when $\delta_2,\delta_3>0$
15	E_{15}:Only the Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{\sigma_2}{\sigma_1}, \overline{N_2}=\frac{\sigma_3}{\sigma_1}, \overline{N_3}=K_3, \overline{N_4}=0$ $where$ $\sigma_1=a_{33}(a_{11}a_{22}+a_{12}a_{21})>0$ $\sigma_2=a_{22}(a_1a_{33}+a_3a_{13})-a_2a_{12}a_{33}$ $\sigma_3=a_{21}(a_1a_{33}+a_3a_{13})+a_2a_{11}a_{33}>0$ This would exist only when $\sigma_2>0$
16	E_{16}:The co-existent state (or) Normal steady state	$\overline{N_1}=\frac{a_{22}a_{44}\psi_1-a_{12}a_{33}\psi_2}{\psi_3}, \overline{N_2}=\frac{a_{21}a_{44}\psi_1+a_{11}a_{33}\psi_2}{\psi_3},$ $\overline{N_3}=K_3, \overline{N_4}=K_4$ $where$ $\psi_1=a_1a_{33}+a_3a_{13}>0,\ \psi_2=a_2a_{44}+a_4a_{24}>0$ $\psi_3=a_{33}a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$ This would exist only when $a_{22}a_{44}\psi_1>a_{12}a_{33}\psi_2$

The present paper deals with the host of the predator washed out states only (Sl.No.3,8,10,15). The stability of the other equilibrium states were already discussed and communicated to several International Journals.

4. Stability of the equilibrium states

Let $N=(N_1,N_2,N_3,N_4)=\bar{N}+U$ (4.1)

where $U=(u_1,u_2,u_3,u_4)^T$ is a small perturbation over the equilibrium state $\bar{N}=(\bar{N}_1,\bar{N}_2,\bar{N}_3,\bar{N}_4)$. The basic equations (2.1), (2.2), (2.3) are quasi linearized to obtain the equations for the perturbed state as

$$\frac{dU}{dt}=AU \tag{4.2}$$

with

$$\begin{bmatrix} a_1-2a_{11}\bar{N}_1-a_{12}\bar{N}_2+a_{13}\bar{N}_3 & -a_{12}\bar{N}_1 & a_{13}\bar{N}_1 & 0 \\ a_{21}\bar{N}_1 & a_2-2a_{22}\bar{N}_2+a_{21}\bar{N}_1+a_{24}\bar{N}_4 & 0 & a_{24}\bar{N}_2 \\ 0 & 0 & a_3-2a_{33}\bar{N}_3 & 0 \\ 0 & 0 & 0 & a_4-2a_{44}\bar{N}_4 \end{bmatrix} \tag{4.3}$$

The characteristic equation for the system is $\det[A-\lambda I]=0$ (4.4)

The equilibrium state is stable, if both the roots of the equation (4.4) are negative in case they are real or have negative real parts in case they are complex.

5. Stability of the Host (S_4) of the Predator (S_2) only is washed out states: (Sl. No's 3,8,10,15 in the above table)

The equilibrium points E_3, E_8, E_{10} were already discussed in the papers "On the Stability of a Four Species : A Prey Predator-Host-Commensal-Syn-Eco System – II, V, VI and published in "International eJournal of Mathematics and Engineering" (2010).

Now discussed about the Equilibrium point $E_{15}: \bar{N}_1 = \frac{\sigma_2}{\sigma_1},\ \bar{N}_2 = \frac{\sigma_3}{\sigma_1},\ \bar{N}_3 = K_3,\ \bar{N}_4 = 0$

where $\sigma_1 = a_{33}\ (a_{11}\ a_{22} + a_{12}\ a_{21}) > 0$ (5.1)

$\sigma_2 = a_{22}\ (a_1\ a_{33} + a_3\ a_{13}) - a_2\ a_{12}\ a_{33},\ \sigma_3 = a_{21}\ (a_1\ a_{33} + a_3\ a_{13}) + a_2\ a_{11}\ a_{33} > 0$ (5.2)

Let us consider small deviations from the steady state

i.e., $N_i(t) = \bar{N}_i + u_i\ (t),\ i = 1, 2, 3, 4$ (5.3)

where $u_i(t)$ is a small perturabations in the species S_i .

Substituting (5.3) in (2.1), (2.2), (2.3) and neglecting products and higher powers of u_1, u_2, u_3, u_4.

we get

$$\frac{du_1}{dt} = \delta u_1 - \frac{a_{12}\ \sigma_2}{\sigma_1} u_2 + \frac{a_{13}\ \sigma_2}{\sigma_1} u_3 \quad (5.4)$$

$$\frac{du_2}{dt} = \frac{a_{21}\ \sigma_3}{\sigma_1} u_1 + \sigma\ u_2 + \frac{a_{24}\ \sigma_3}{\sigma_1} u_3 \quad (5.5)$$

$$\frac{du_3}{dt} = -\ a_3\ u_3\ \ ,\ \ \frac{du_4}{dt} = -\ a_4\ u_4 \quad (5.6)$$

where $\delta = a_1 - \frac{2a_{11}\ \sigma_2}{\sigma_1} - \frac{a_{12}\ \sigma_3}{\sigma_1} + \frac{a_3 a_{13}}{a_{33}},\ \sigma = a_2 - \frac{2a_{22}\ \sigma_3}{\sigma_1} + \frac{a_{21}\ \sigma_2}{\sigma_1}$ (5.7)

the characteristic equation for which is

$$\left[\lambda^2 - (\delta + \sigma)\lambda + \delta\,\sigma - \frac{a_{12}\ a_{21}\ \sigma_2\ \sigma_3}{\sigma_1^2}\right](\lambda + a_3)(\lambda - a_4) = 0 \quad (5.8)$$

One of the four roots a_4 is positive and $-a_3$ is negative. Hence the steady state is **unstable**.

Let λ_1, λ_2 be the zeros of the quadratic polynomial on the L.H.S of the above equation (5.8)

Case (A) : When the roots λ_1 and λ_2 have opposite signs.

The solutions of the equations (5.4) (5.5), (5.6) are

$$u_1 = \left[\frac{a_{12}\,\sigma_2\,(\beta - u_{20}) - \sigma_1\,(\alpha - u_{10})(\delta - \lambda_2)}{\sigma_1\,(\lambda_1 - \lambda_2)}\right] e^{\lambda_1 t}$$
$$+ \left[\frac{a_{12}\,\sigma_2\,(\beta - u_{20}) - \sigma_1\,(\alpha - u_{10})(\delta - \lambda_1)}{\sigma_1\,(\lambda_2 - \lambda_1)}\right] e^{\lambda_2 t} + A_2\,e^{-a_3 t} + B_2\,e^{a_4 t} \tag{5.9}$$
$$u_2 = \left[\frac{a_{12}\,\sigma_2\,(\beta - u_{20}) - \sigma_1\,(\alpha - u_{10})(\delta - \lambda_2)}{a_{12}\,\sigma_2\,(\lambda_1 - \lambda_2)}\right](\delta - \lambda_1)\,e^{\lambda_1 t}$$
$$+ \left[\frac{a_{12}\,\sigma_2\,(\beta - u_{20}) - \sigma_1\,(\alpha - u_{10})(\delta - \lambda_1)}{a_{12}\,\sigma_2\,(\lambda_2 - \lambda_1)}\right](\delta - \lambda_2)\,e^{\lambda_2 t}$$
$$+ \left[A_2\,(\delta + a_3) + \frac{a_{12}\,\sigma_2}{\sigma_1}\,u_{30}\right]\frac{\sigma_1}{a_{12}\,\sigma_2}\,e^{-a_3 t} + \frac{\sigma_1}{a_{12}\,\sigma_2}\,B_2\,(\delta - a_4)\,e^{a_4 t} \tag{5.10}$$
$$u_3 = u_{30}\,e^{-a_3 t}\ , \quad u_4 = u_{40}\,e^{a_4 t} \tag{5.11}$$

where $\alpha = A_2 + B_2$ (5.12)

$$\beta = \frac{\sigma_1}{a_{12}\,\sigma_2}\left[A_2\,(\delta + a_3) + B_2\,(\delta - a_4)\right] + \frac{a_{13}}{a_{12}}\,u_{30} \tag{5.13}$$
$$A_2 = \frac{A_1}{a_3^2 + a_3\,(\delta + \sigma) + C_1}\,, \quad B_2 = \frac{B_1}{a_4^2 - a_4\,(\delta + \sigma) + C_1} \tag{5.14}$$
$$A_1 = -\frac{a_{12}\,\sigma_2}{\sigma_1}\,(a_2 + \sigma)\,u_{30} \tag{5.15}$$
$$B_1 = -\frac{a_{24}\,a_{12}\,\sigma_2\,\sigma_3}{\sigma_1^2}\,u_{40}, \qquad C_1 = \delta\sigma + \frac{a_{12}\,a_{21}\,\sigma_2\,\sigma_3}{\sigma_1^2} \tag{5.16}$$

and $u_{10}, u_{20}, u_{30}, u_{40}$ are the initial values of u_1, u_2, u_3, u_4 respectively.

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t). u_{20}(t), u_{30}(t), u_{40}(t)$ of the species S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations. And the solution curves are illustrated in the figures 2 to 5.

Case (i): If $u_{10} < u_{30} < u_{20} < u_{40}$ and $a_3 < a_1 < a_2 < a_4$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the Prey (S_1) till the time instant t_{13}^* and thereafter the dominance is reversed.

Case (ii): If $u_{20} < u_{10} < u_{30} < u_{40}$ and $a_4 < a_3 < a_1 < a_2$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the Predator (S_2), Prey (S_1) till the time instant t_{23}^*, t_{13}^* respectively and thereafter the dominance is reversed. Also the Host (S_4) of S_2 dominates over the Prey (S_1), Predator (S_2) till the time

instant t^*_{13}, t^*_{24} respectively and thereafter the dominance is reversed. Similarly the Prey (S_1) dominates over the Predator (S_2) till the time instant t^*_{21} and the dominance gets reversed thereafter.

Case(iii): If $u_{30} < u_{10} < u_{40} < u_{20}$ and $a_4 < a_1 < a_3 < a_2$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially the Host (S_4) of S_2 dominates over the Prey (S_1) till the time instant t^*_{14} and thereafter the dominance is reversed.

Case(iv): If $u_{40} < u_{10} < u_{30} < u_{20}$ and $a_1 < a_2 < a_4 < a_3$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the Prey (S_1), Host (S_4) of S_2 till the time instant t^*_{13}, t^*_{43} respectively and thereafter the dominance is reversed. Also the Prey (S_1), Predator (S_2) dominates over the Host (S_4) of S_2 till the time instant t^*_{41}, t^*_{42} respectively and the dominance gets reversed thereafter.

Case (B): When the roots λ_1 and λ_2 have same signs.
The solutions in this case are same as in case (A) and solution curves are illustrated in figures 6 to 9.

Case (i): If $u_{10} < u_{20} < u_{40} < u_{30}$ and $a_2 < a_1 < a_3 < a_4$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the Host (S_4) of S_2, Predator (S_2), Prey (S_1) till the time instant $t^*_{43}, t^*_{23}, t^*_{13}$ respectively and thereafter the dominance is reversed. Also the Predator (S_2) dominates over the Prey (S_1) till the time instant t^*_{12} and the dominance gets reversed thereafter.

Case (ii): If $u_{20} < u_{30} < u_{40} < u_{10}$ and $a_1 < a_3 < a_4 < a_2$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially the Prey (S_1), Host (S_4) of S_2, Host (S_3) of S_1 dominates over the Predator (S_2) till the time instant $t^*_{21}, t^*_{24}, t^*_{23}$ respectively and thereafter the dominance is reversed. Also the Prey (S_1) dominates over the Host (S_4) of S_2 till the time instant t^*_{41} and the dominance gets reversed thereafter.

Case (iii) If $u_{30} < u_{20} < u_{10} < u_{40}$ and $a_1 < a_4 < a_3 < a_2$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially the Host (S_4) of S_2, Prey (S_1) dominates over the Predator (S_2) till the time instant t^*_{24}, t^*_{21} respectively and thereafter the dominance is reversed.

Case(iv): If $u_{40} < u_{10} < u_{30} < u_{20}$ and $a_2 < a_4 < a_3 < a_1$
In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the Host (S_4) of S_2, Prey (S_1) till the time instant t^*_{43}, t^*_{13} respectively and thereafter the

dominance is reversed. Also the Predator (S_2) dominates over the Prey (S_1), Host (S_4) of S_2 till the time instant t^*_{12}, t^*_{42} respectively and the dominance gets reversed thereafter.

6. Trajectories of Perturbations

The trajectories in the $u_1 - u_3,\ u_1 - u_4,\ u_2 - u_3,\ u_2 - u_4,\ u_3 - u_4$ planes are

$$x_1 = A y_1^{\frac{-\lambda_1}{a_3}} + B y_1^{\frac{-\lambda_2}{a_3}} + C y_1 + D y_1^{\frac{-a_4}{a_3}}, \quad x_1 = A y_2^{\frac{\lambda_1}{a_4}} + B y_2^{\frac{\lambda_2}{a_4}} + C y_2^{\frac{-a_3}{a_4}} + D y_2 \tag{6.1}$$

$$x_2 = \bar{A} y_1^{\frac{-\lambda_1}{a_3}} + \bar{B} y_1^{\frac{-\lambda_2}{a_3}} + \bar{C} y_1 + \bar{D} y_1^{\frac{-a_4}{a_3}}, \quad x_2 = \bar{A} y_2^{\frac{\lambda_1}{a_4}} + \bar{B} y_2^{\frac{\lambda_2}{a_4}} + \bar{C} y_2^{\frac{-a_3}{a_4}} + \bar{D} y_2 \tag{6.2}$$

$$y_1^{a_4} = y_2^{-a_3} \qquad \text{respectively} \tag{6.3}$$

Where

$$A = \frac{a_{12}\sigma_2(\beta - u_{20}) - \sigma_1(\alpha - u_{10})(\delta - \lambda_2)}{\sigma_1(\lambda_1 - \lambda_2)u_{10}} \tag{6.4}$$

$$B = \frac{a_{12}\sigma_2(\beta - u_{20}) - \sigma_1(\alpha - u_{10})(\delta - \lambda_1)}{\sigma_1(\lambda_2 - \lambda_1)u_{10}}, \ C = \frac{A_2}{u_{10}}, D = \frac{B_2}{u_{10}} \tag{6.5}$$

$$\bar{A} = \left[\frac{a_{12}\sigma_2(\beta - u_{20}) - \sigma_1(\alpha - u_{10})(\delta - \lambda_2)}{a_{12}\sigma_2(\lambda_1 - \lambda_2)u_{20}}\right](\delta - \lambda_1) \tag{6.6}$$

$$\bar{B} = \left[\frac{a_{12}\sigma_2(\beta - u_{20}) - \sigma_1(\alpha - u_{10})(\delta - \lambda_1)}{a_{12}\sigma_2(\lambda_2 - \lambda_1)u_{20}}\right](\delta - \lambda_2) \tag{6.7}$$

$$\bar{C} = \frac{\sigma_1}{a_{12}\sigma_2 u_{20}}\left[A_2(\delta + a_3) + \frac{a_{13}\sigma_2}{\sigma_1}u_{30}\right], \ \bar{D} = \frac{B_2\sigma_1}{a_{22}\sigma_2 u_{20}}(\delta - a_4) \tag{6.8}$$

$$x_1 = \frac{u_1}{u_{10}},\ x_2 = \frac{u_2}{u_{20}},\ y_1 = \frac{u_3}{u_{30}},\ y_2 = \frac{u_4}{u_{40}} \tag{6.9}$$

7. Perturbation Graphs

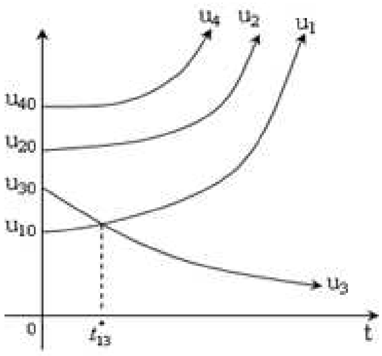

Fig. 2

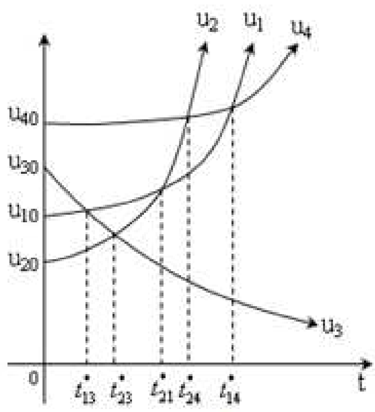

Fig. 3

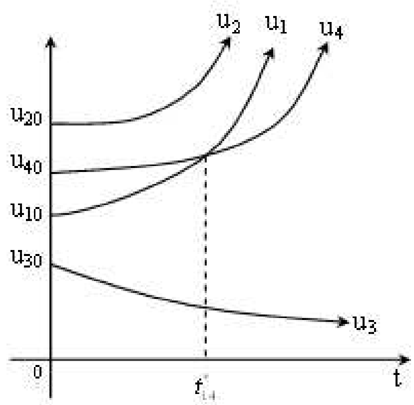

Fig. 4

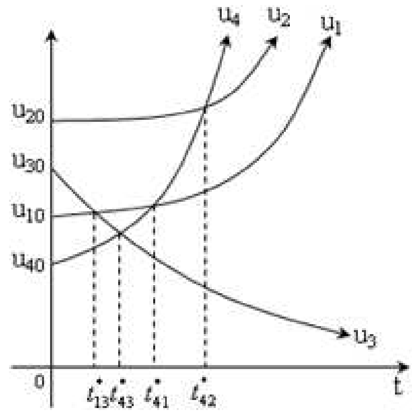

Fig. 5

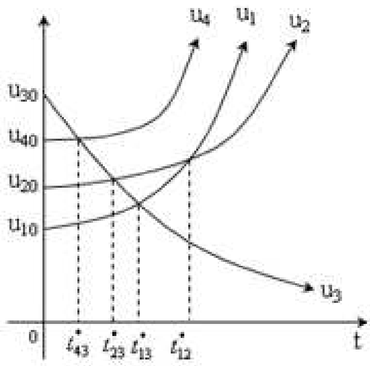

Fig. 6

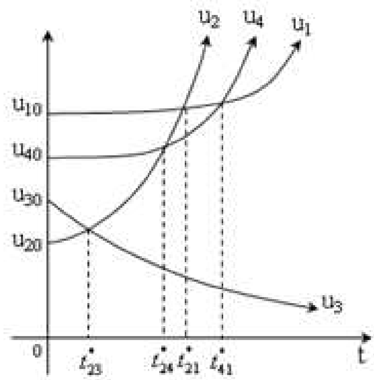

Fig. 7

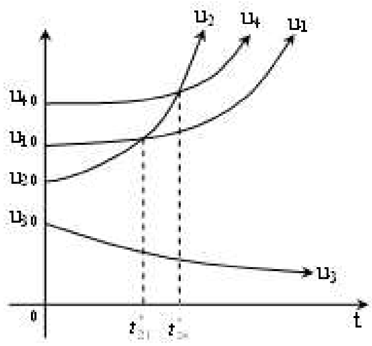

Fig. 8

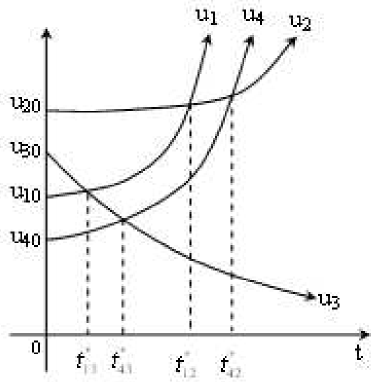

Fig. 9

CONCLUSION

A Four Species Syn-Eco System consisting of a Prey(S_1), a Predator (S_2) that survives upon S_1, two hosts S_3 and S_4 for which S_1, S_2 are Commensal respectively. Further S_3 and S_4 are neutral. In the above system it is observe that the host of the Predator Washed out state is unstable. The stability of the other equilibrium states were already investigated and communicated to several International Journals.

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