

On the effect of rotation on Soret driven double-diffusive stationary convection

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ABSTRACT

The effects of rotation and the mass flux induced by temperature gradient (Soret effect) on the double diffusion convection in a horizontal layer of fluid subjected to thermal and solutal gradients with cross diffusions are investigated analytically and shown graphically. Normal mode technique has been used for the linear stability analysis of the problem. The eigen value problem and the exact solution are obtained for stability investigations. The expression for stationary Rayleigh number is obtained as a function of the governing parameters. The analysis reveals that the Soret parameter and the Lewis number both have the destabilizing effect on the onset of stationary convection in the system while the stable solute gradient and rotation have stabilizing effect on the onset of stationary convection and introduces oscillatory modes in the system, which were non-existent in their absence. In the limiting cases some previous published results have been recovered.

Keywords: Rayleigh number, Double diffusive convection, Soret effect, Stationary convection, Cross diffusion, Rotation.

INTRODUCTION

Convection occurs in nature on a large scale in atmospheres, oceans, planetary mantles, and it provides the mechanism of heat transfer for a large numbers of processes. The earliest experiments to demonstrate the onset of thermal convection in the fluid are attributed to Bénard [3]. Inspired by the experimental works of Bénard, Lord Rayleigh [13] showed mathematically that if a quiescent fluid layer is heated uniformly from below, the adverse density gradient becomes unstable and the fluid motion ensues when a critical heating rate, measured in terms of Rayleigh number is exceeded. As a consequence of the works of Bénard and Rayleigh, the thermal instability or thermal convection problem is commonly known as Rayleigh-Bénard convection. Chandrasekhar [5] presented a comprehensive view of thermal convection problems under the varying assumptions of hydrodynamics and hydromagnetics in a treatise, Hydrodynamic and Hydromagnetic Stability.

Recently, the convection in two component systems (heat and mass diffusion) with different molecular diffusivities has received a considerable attention in the field of physical chemistry, oceanography, geophysics and astrophysics. A broader range of dynamical behavior is observed in the convective instabilities that may occur in a gravitational field containing two components of different diffusivities that effect the density; for example, temperature and solute. This phenomenon is known variously as thermohaline convection, double-diffusive convection, or thermosolutal convection. The presence of comparable magnitude of temperature and concentration gradients may play a significant role in the onset of double diffusive convection. In a binary mixture of fluids of different density

buoyancy may be created either by heat or concentration gradient, both of which are transported advectively and diffusively. The flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient are respectively known as Soret and Dufour effect (De Groot and Mazur [6] and Hurle & Jakeman [10]). Further, the Soret effect introduces a coupling between concentration transport and the local temperature gradient in the mixture, and this causes a concentration gradient to develop when a temperature gradient is imposed on the fluid layer. Therefore one cannot ignore the role of Soret effect chiefly in liquids.

Linear stability and weak nonlinear theories were used to investigate analytically the Coriolis Effect on three-dimensional gravity-driven convection in a layer rotating fluid with cross diffusion. A layer of such fluid heated from below under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics. From a geophysics point of view the effect of rotation acting on the convective present problem is of practical interest. Double diffusive convection is of importance in various fields; such as high quality crystal production, oceanography, production of pure medication, solidification of molten alloys, limnology and engineering. In view of these important applications in various fields, the problem has been examined by many researchers both theoretically and experimentally. Tewfik et al[19] were the first to study Soret-Dufour driven thermosolutal convection, followed by Sparrow et al[16]. Hurle and Jakeman [10] discuss Soret-driven thermosolutal convection and concluded that magnitude & sign of the Soret coefficient were changed by varying the composition of the mixture. McDougall[12] observed that the spatiotemporal properties of convection in binary mixture show quite different trends from those of the double-diffusive systems without these cross diffusions. Schechter et al[14] reported that in the study of two component thermosolutal problem the influence of Dufour effect is negligible (10^{-3}°C) in liquid mixtures and hence generally neglected. Dhiman and Goyal [8] recently studied the stability of Soret driven double-diffusive convection problem for the case of rigid, impervious and thermally perfectly conducting boundary conditions using Variational principle. Stommel and Fedorov [17] have observed that the length scales characteristic of double diffusing convecting layers in the ocean may be sufficiently large and hence the Earth rotation might be important in their formation. Veronis [21,22,23] studied the Bénard convection and the Bénard convection with rotating fluid with large amplitude disturbances. Sengupta and Gupta [15] extended the analysis of Veronis on thermohaline convection by including the effect of uniform rotation and found that for infinitesimal disturbances in the form of rolls, the marginal state is oscillatory and rotation parameter tends to stabilize the double-diffusive convection. They also studied thermohaline convection in a rotating fluid using finite amplitude disturbances. Antorang and Velarde [1] have analyzed the Soret driven convective instability with rotation. Further, the general theorems of Helmholtz and Kelvin relating to vorticity clearly established that a rotation introduces a number of new elements into hydrodynamical problem.

There are only few studies available on the effect of cross diffusion on double diffusion convection with rotation or magnetic fields because of the complexity in determining these coefficients and the problem under investigation has not given much attention. Motivated by the above discussions and keeping in mind the importance of the Soret effect and Coriolis force (which arises due to rotation) in convective instability, we in the present paper have studied the problem of Soret driven double-diffusion convection in a horizontal layer of a fluid in the presence of a uniform rotation subjected to thermal and solutal gradients with cross diffusion. The hydrodynamical stability of the configuration is investigated theoretically by means of a linear stability analysis for the case of dynamically free boundaries. The effects of, rotation, solute gradient, Soret parameter and that of Lewis number on the onset of double diffusive convection are investigated both analytically and numerically.

1. Physical Configuration And Basic Equations

Consider an infinite horizontal layer of two component viscous quasi-incompressible (Boussinesq) fluid is statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperature T_0 and T_1 ($T_0 > T_1$) and at uniform concentrations C_0 and C_1 ($C_0 > C_1$). This layer is acted upon by a uniform vertical rotation Ω (0, 0, Ω) and gravity field $g(0,0,-g)$. Both the boundaries are assumed to be dynamically free, pervious and perfectly heat conducting while the adjoining medium is assumed to be electrically non-conducting. The phenomenological equations relating the heat flux J_Q and the solute flux J_c to the thermal and solute gradients present in binary fluid mixture are given by (see for instance, De Groot and Mazur[6] as;

$$J_Q = -\kappa \frac{\partial T}{\partial x_j} - \rho T C \frac{\partial \mu}{\partial c} D' \frac{\partial C}{\partial x_j} \quad (1)$$

$$J_c = -\rho\kappa' \left[\frac{\partial C}{\partial x_j} + S_T N(1-N) \frac{\partial T}{\partial x_j} \right] \quad (2)$$

where, T is the temperature, C is the concentration, ρ is the density, κ is the thermal conductivity, $D' (= S_T \kappa')$ is the Dufour coefficient and μ is the chemical potential of the solute, κ' is the solutal diffusivity, S_T is the Soret coefficient. In the product $N(1-N)$; N and $1-N$ are respectively the mass fractions of two components. Bergeron et al [2] discussed that in general the Soret effect is small and it is assumed that the product $N(1-N)$ may be taken as constant and equal to its initial value $N_0(1-N_0)$ in the second term of the thermal diffusion flux given by equation [2]. La-Porta and Surko [11] found that the strength of the Soret forcing in mixtures is parameterized by the

stability ratio $\gamma = S_T N_0(1-N_0) \frac{\alpha'}{\alpha}$, (or Soret parameter) where α and α' are respectively thermal and concentration expansion coefficient, depending on the mixture, the Soret coefficient can be positive or negative, meaning thereby that solute can be driven toward the hotter, or the colder region. Hence γ can be taken positive or negative. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids.

Under these assumptions, the basic equations (i.e. the equations of continuity, motion, heat conduction, mass diffusion and the equation of state) in the presence of uniform rotation under Boussinesq [4] approximation that govern the present physical configuration are given by (cf. Chandrasekhar [5]);

$$\frac{\partial u_j}{\partial x_j} = 0 \quad (3)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial}{\partial x_i} \left[\frac{p}{\rho_0} - \frac{1}{2} |\vec{\Omega} \times \vec{r}|^2 \right] + \left[1 + \frac{\delta \rho_1}{\rho_0} + \frac{\delta \rho_2}{\rho_0} \right] X_i + \nu \nabla^2 u_i + 2 \epsilon_{ijk} u_j \Omega_k \quad (4)$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \kappa \nabla^2 T \quad (5)$$

$$\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = \kappa' \nabla^2 C + S_T N_0(1-N_0) \kappa' \nabla^2 T \quad (6)$$

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)] \quad (7)$$

In the above equations, $\Omega_k = (0, 0, \Omega)$ is the uniform *angular velocity* of fluid; \vec{r} is the position vector, ϵ_{ijk} is the *permutation tensor*; $X_i = (0, 0, -g)$ is the external force; g is gravity; $u_i = (u, v, w)$ are the components of velocity; p is the pressure, μ is the coefficient of viscosity and $\nu = \frac{\mu}{\rho_0}$ is the coefficient of kinematic viscosity.

2. Characteristic Value Problem

Following the usual steps of the linear stability analysis [1,7,10], we obtain the following system of non-dimensional linearized perturbation equations;

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = Ra^2\theta - R'a^2\phi + TD\zeta \quad (8)$$

$$\left(D^2 - a^2 - p\right)\theta = -w \quad (9)$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi = -\frac{w}{\tau} - S\left(D^2 - a^2\right)\theta \quad (10)$$

$$\left(D^2 - a^2 - \frac{p}{\sigma}\right)\zeta = -Dw \quad (11)$$

together with the following dynamically free, pervious and perfectly heat conducting boundary conditions,

$$w = 0 = \theta = \phi = D^2w = D\zeta \quad \text{at } z = 0, \quad \text{and } z = 1 \quad (12)$$

In the above equations; $D \equiv d/dz$ represents the derivative with respect to the vertical co-ordinate z ($0 \leq z \leq 1$); w , θ and ϕ respectively denote the perturbed velocity, temperature and concentration and are complex valued function of z only, and $R = \frac{g\alpha\beta d^4}{\kappa\nu}$ is the thermal Rayleigh number; $R' = \frac{g\alpha'\beta' d^4}{\kappa\nu}$ is the solutal Rayleigh number; $T = \frac{4\Omega^2 d^4}{\nu^2}$ is the Taylor number; ζ is z-component of vorticity; $S = \frac{\beta}{\beta'} S_T N_0 (1 - N_0)$ is the Soret number associated with Soret effect, where $\beta' = -\beta S_T N_0 (1 - N_0)$

The above definitions yield that $S = -1$ and $\mathcal{R} = -R'$ (Takashima Masaki[18]) The system of equations (8)-(11) together with boundary conditions (12) constitutes an characteristic value problem for p for the prescribed values of other parameters namely; $R, R', a^2, \sigma, \tau, S$ and T .

Remarks:

(i) A given state of the system is stable, neutral or unstable according as; $p_r < 0, p_r = 0$ or $p_r > 0$. Further, if $p_r = 0 \Rightarrow p_i = 0$ for all wave numbers a^2 (where p_r and p_i are the real and imaginary parts of p) then we have $p = 0$. This situation in hydrodynamic stability is termed as the validity of principle of exchange of stabilities (PES), otherwise we have over stability at least when instability sets in as certain modes.

(ii) The system of equations (8)-(11) together with the boundary conditions (12) when $S = 0$ yields the non dimensional linear perturbation equations governing Veronis Type rotary thermohaline convection (Gupta et al[9])

(iii) Further, when $S = -1$ and $\mathcal{R} = -R'$ the system of equations (8)-(11) together with the boundary conditions (12) yields the non-dimensional linearized perturbation equations governing rotatry double diffusive convection with Soret effect in terms of stability ratio γ .

3. Mathematical Analysis

a. An exact solution of the problem

We shall now obtain an exact solution of the characteristic value problem described by equations (8)–(11) together with the boundary conditions (12). We proceed as follows;

Upon utilizing boundary conditions (12) in equations (8)-(11), we have

$$D^2\theta = D^2\phi = D^4w = 0 \quad \text{at } z = 0 \text{ and } z = 1 \quad (13)$$

Differentiating equation (11) once and (8)-(10) twice with respect to z , we have

$$D^3\zeta = 0 = D^6w = D^4\theta = D^4\phi \quad \text{at } z=0 \quad \text{and } z=1 \quad (14)$$

Proceeding likewise, it follows that

$$D^{2m}w = 0 \quad (m=1, 2, 3, \dots), \quad \text{at } z=0 \quad \text{and } z=1 \quad (15)$$

Hence, the solution that satisfies conditions (15) can be considered of the form;

$$w = A \sin n\pi z \quad (16)$$

where, A is constant.

Substituting this value of w in equations (9) and (11), solving the resulting differential equations analytically, we obtain the following solutions

$$\theta = \frac{A \sin n\pi z}{n^2\pi^2 + a^2 + p} \quad (17)$$

and

$$\zeta = \frac{n\pi A \cos n\pi z}{n^2\pi^2 + a^2 + \frac{p}{\sigma}} \quad (18)$$

Substituting the values of w and θ from solutions (16) and (17) in equation (10) and solving the resulting equation for ϕ , we get

$$\phi = \frac{-A}{\tau \left(n^2\pi^2 + a^2 + \frac{p}{\tau} \right)} \left[-1 + \tau \mathcal{S} - \frac{\tau \mathcal{S} p}{n^2\pi^2 + a^2 + p} \right] \sin n\pi z. \quad (19)$$

Now, to obtain the characteristic equation, operating on equation (8) on both sides by an operator

$$\begin{aligned} & (D^2 - a^2 - p) [\tau(D^2 - a^2) - p] \left[D^2 - a^2 - \frac{p}{\sigma} \right], \text{ we have} \\ & (D^2 - a^2 - p) [\tau(D^2 - a^2) - p] (D^2 - a^2) \left[D^2 - a^2 - \frac{p}{\sigma} \right]^2 w \\ & = Ra^2 (D^2 - a^2 - p) [\tau(D^2 - a^2) - p] \left[D^2 - a^2 - \frac{p}{\sigma} \right] \theta \\ & - Ra^2 (D^2 - a^2 - p) [\tau(D^2 - a^2) - p] \left[D^2 - a^2 - \frac{p}{\sigma} \right] \phi + TD [D^2 - a^2 - p] [\tau(D^2 - a^2) - p] \left(D^2 - a^2 - \frac{p}{\sigma} \right) \zeta \quad (20) \end{aligned}$$

Equation (20) together with equations (9)-(11) yields, after a little simplification, the following equation

$$\begin{aligned} & (D^2 - a^2 - p) [\tau(D^2 - a^2) - p] \left[\left(D^2 - a^2 \right) \left(D^2 - a^2 - \frac{p}{\sigma} \right)^2 + TD^2 \right] w \\ & + \left(D^2 - a^2 - \frac{p}{\sigma} \right) [Ra^2 \{ \tau(D^2 - a^2) - p \} - Ra^2 \{ (D^2 - a^2 - p) - \tau \mathcal{S} (D^2 - a^2) \}] w = 0 \quad (21) \end{aligned}$$

Substituting the value of w from equation (16) in equation (21), we have

$$\begin{aligned} & (\pi^2 + a^2 + p) \left[\tau(\pi^2 + a^2) + p \right] \left[(\pi^2 + a^2) \left(\pi^2 + a^2 + \frac{p}{\sigma} \right)^2 + T\pi^2 \right] - \\ & \left(\pi^2 + a^2 + \frac{p}{\sigma} \right) \left[Ra^2 \{ \tau(\pi^2 + a^2) + p \} - R'a^2 \{ (\pi^2 + a^2 + p) - \tau(\pi^2 + a^2) \} \right] = 0 \end{aligned} \quad (22)$$

Equation (22) is required characteristic equation belonging to the lowest mode ($n = 1$) studying the effect of Soret, rotation and other parameters on the system.

In view of Remarks (ii) and (iii) above, substituting $S = 0$ in equation (22), we obtain the characteristic equation for Veronis Type rotatry thermohaline convection, and for $S = -1$, $\gamma R = -R'$. equation (22) yields the characteristic equation for rotatry double diffusive convection with Soret effect in terms of stability ratio γ .

b. Mode of Instability

The system of equations (8)-(12) upon using the linear transformations

$$\left. \begin{aligned} \hat{w} &= w \\ \hat{\theta} &= \theta \\ c &= (1-\tau)\phi + \tau\theta \\ \hat{\zeta} &= \zeta \end{aligned} \right\}$$

and dropping the caps for convenience in writing are transformed into equations

$$\left(D^2 - a^2 \right) \left(D^2 - a^2 - \frac{p}{\sigma} \right) w = R_T a^2 \theta - R_s a^2 \phi + T D \zeta \quad (23)$$

$$(D^2 - a^2 - p) \theta = -w \quad (24)$$

$$\left(D^2 - a^2 - \frac{p}{\tau} \right) \phi = -\frac{w}{\tau} \quad (25)$$

$$\left(D^2 - a^2 - \frac{p}{\sigma} \right) \zeta = -Dw, \quad (26)$$

together with the boundary conditions

$$w = 0 = D^2 w = \theta = \phi = D\zeta \quad \text{at } z = 0 \text{ and } z=1 \quad (27)$$

$$\text{where } R_T = R + \frac{\tau R'}{1-\tau} \text{ and } R_s = \frac{R'}{1-\tau}$$

To examine the existence of oscillatory or non-oscillatory, multiplying equation (23) by w^* and integrating the resulting equation over the vertical range of z , we get

$$\int w^* \left(D^2 - a^2 \right) \left(D^2 - a^2 - \frac{p}{\sigma} \right) w \, dz = R_T a^2 \int w^* \theta \, dz - R_s a^2 \int w^* \phi \, dz + T \int w^* D \zeta \, dz \quad (28)$$

Using equations (24), (25) in equation (28), we have

$$\int w^* (D^2 - a^2)^2 w dz - \frac{P}{\sigma} \int w^* (D^2 - a^2) w dz = -R_T a^2 \int \theta (D^2 - a^2 - p^*) \theta^* dz$$

$$+ R_S a^2 \tau \int \phi \left(D^2 - a^2 - \frac{P}{\tau} \right) \phi^* dz + T \int w^* D \zeta dz. \quad (29)$$

Integrating the last term on the right hand side of equation (29) by parts once over the vertical range of z and using boundary conditions (27), we have

$$\int w^* (D^2 - a^2)^2 w dz - \frac{P}{\sigma} \int w^* (D^2 - a^2) w dz = -R_T a^2 \int \theta (D^2 - a^2 - p^*) \theta^* dz$$

$$+ R_S a^2 \tau \int \phi \left(D^2 - a^2 - \frac{P}{\tau} \right) \phi^* dz + T \int \zeta \left(D^2 - a^2 - \frac{P}{\sigma} \right) \zeta^* dz. \quad (30)$$

Integrating by parts equation (30) for an appropriate number of times by using the boundary conditions (27), we have

$$\int \left[|D^2 w|^2 + a^4 |w|^2 + 2a^2 |Dw|^2 \right] + \frac{P}{\sigma} \int \left[|Dw|^2 + a^2 |w|^2 \right]$$

$$- R_T a^2 \int \left[|D\theta|^2 + a^2 |\theta|^2 + p^* |\theta|^2 \right] dz$$

$$+ R_S a^2 \tau \int \left[|D\phi|^2 + a^2 |\phi|^2 + \frac{P}{\tau} |\phi|^2 \right] + T \int \left[|D^2 \zeta|^2 + a^2 |\zeta|^2 + \frac{P}{\tau} |\zeta|^2 \right] = 0. \quad (31)$$

Putting $p = ip_i$ in equation (31) where p_i is real and then equating the imaginary parts, we get

$$p_i \left[\frac{1}{\sigma} \int \left\{ |Dw|^2 + a^2 |w|^2 \right\} dz - R_S a^2 \int |\phi|^2 dz + R_T a^2 \int |\theta|^2 dz - \frac{T}{\sigma} \int |\zeta|^2 \right] = 0 \quad (32)$$

Equation (32) clearly implies that for given positive values of R_S , R_T , σ and T , we have either $p_i = 0$ or $p_i \neq 0$. These situations in the terms of hydrodynamic stability means that either the instability occurs through non oscillatory modes or through oscillations. In equation (32), the bracket is positive definite if $R_S = 0$ and $T = 0$, which implies that rotation and solutal gradient introduce oscillatory modes in the system which were non-existent in their absence.

c. Stationary Convection

For the case of stationary convection, putting $p = 0$ in the characteristic equation (22) we get

$$R = \frac{(\pi^2 + a^2)^3}{a^2} + \frac{T\pi^2}{a^2} + \frac{R'(1 - \tau S)}{\tau}$$

or equivalently for $S = -1$, we have

$$R = \frac{(\pi^2 + a^2)^3}{a^2} + \frac{T\pi^2}{a^2} + \frac{R'(1 + \tau)}{\tau}$$

And in terms of stability ratio it can be written as

$$R = \frac{(\pi^2 + a^2)^3}{a^2} + T\pi^2 \left(\frac{\tau}{\tau + \gamma(1 + \tau)} \right) \quad (33)$$

$$\text{Let, } x = \frac{a^2}{\pi^2}, R_1 = \frac{R}{\pi^4}, T_1 = \frac{T}{\pi^4}, R'_1 = \frac{R'}{\pi^4}$$

Equation (33) can be written as

$$R_1 = \frac{(1+x)^3 + T_1}{x} + \frac{R'_1(1+\tau)}{\tau} \quad (34)$$

Further, equation (34) in terms of stability ratio reduces to

$$R_1 = \frac{(1+x)^3 + T_1}{x} \left(\frac{\tau}{\tau + \gamma(1+\tau)} \right) \quad (35)$$

5. Numerical Results and Discussion

We shall now investigate analytically the effects of rotation, stability ratio and solute gradient on the onset of stationary convection. From equation (34), we can have

$$\frac{\partial R_1}{\partial R'_1} = \frac{1+\tau}{\tau}$$

which is positive for wave number $x (x \neq 0)$ and given positive value of τ (the Lewis number). This implies that the solute gradient has stabilizing effect on the onset of stationary convection in the system.

Further the equation (34) yields that

$$\frac{\partial R_1}{\partial T_1} = \frac{1}{x}, \quad (36)$$

which is positive for all wave number $x (x \neq 0)$. This yields that the value of the Rayleigh number increases for increasing values of Taylor number which implies that the rotation has stabilizing effect on the system.

Also, from equation (35), we have

$$\frac{\partial R_1}{\partial \gamma} = -\frac{(1+x)^3 + T_1}{x} \left[\frac{\tau(1+\tau)}{(\tau + \gamma(1+\tau))^2} \right] \quad (37)$$

which is negative for wave number $x (x \neq 0)$ and given positive values of τ, T_1 . This yields that the value of the Rayleigh number decreases for increasing values of γ which implies that the stability ratio (the Soret parameter) has destabilizing effect on the onset of stationary convection.

Further, we can have from equation (34) that

$$\frac{\partial R_1}{\partial \tau} = -\frac{R'_1}{\tau^2} \quad (38)$$

which yields that for all wave number $x (x \neq 0)$ and given positive values of R'_1 . This means that the value of the Rayleigh number decreases for increasing τ which implies that the Lewis number has destabilizing effect on the onset of stationary convection in the system. It is found that the stationary convection in double diffusive convection with a rotating fluid depends on the Soret parameter. In the absence of Soret effect, the results thus obtained for stationary convection are in good agreement with the results which were obtained by Sengupta and Gupta[18].

To have a better insight of the physical problem, we have presented the variations of Rayleigh number with wave numbers x ($x \neq 0$); representing the effects of solute gradient, Taylor number (rotation), Soret parameter and that of Lewis number on the stationary convection in the double diffusive system. The values of Rayleigh number are calculated numerically from the expression (34) for fixed values of the governing parameters except the one varying parameter. These numerical values are shown graphically to assess the effect of each varying parameter on the onset of convection in double diffusive system. The convection curves (stability curves) for these parameters in (R_1-x) plane for different values of one of the parameter are shown in the following figures (1)-(4).

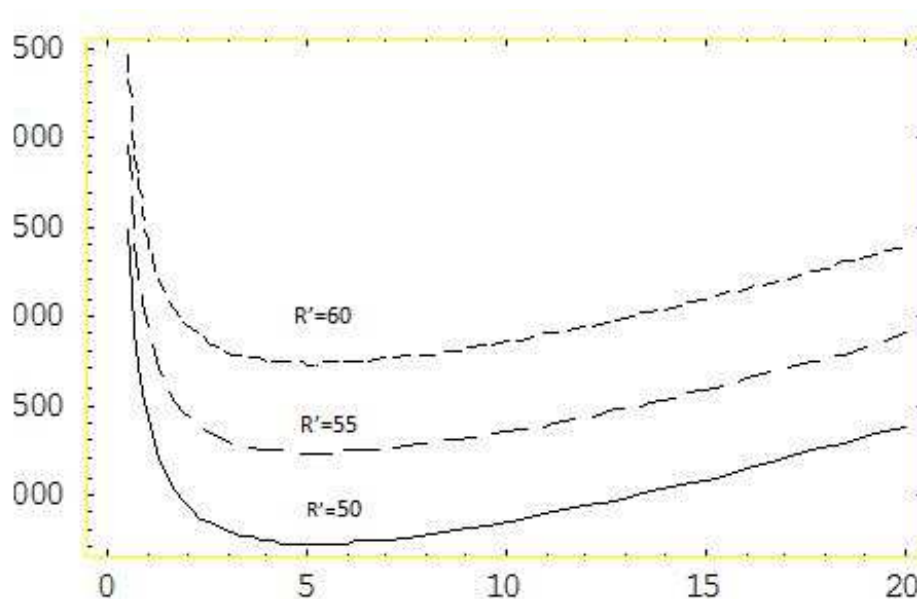


Figure 1. Variation of Rayleigh number R_1 with wave number x for fixed values of $\tau = 0.01$, $T = 10$ and for different values of Solutal Number R'_1

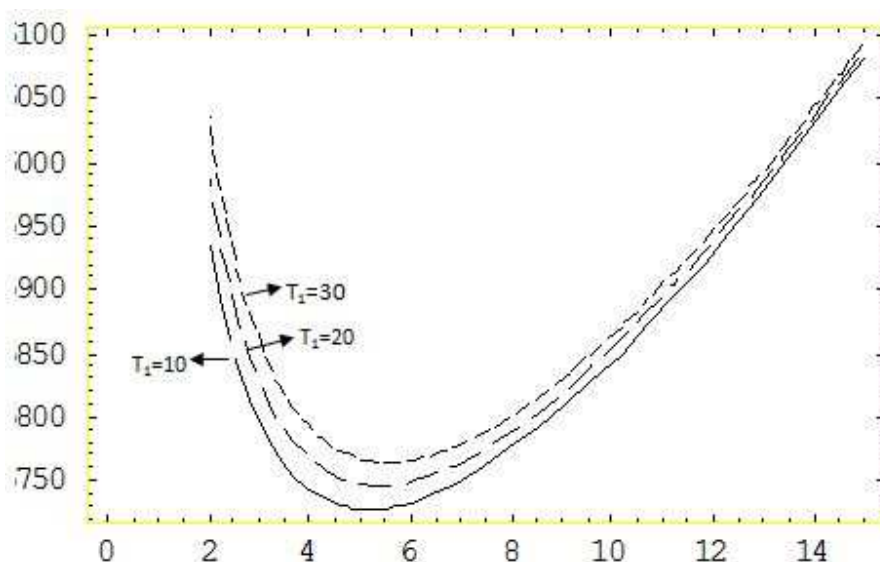


Figure 2. Variation of Rayleigh number R_1 with wave number x for fixed values of $\tau = 0.01$, $R'_1 = 50$ and for different values of Taylor Number T

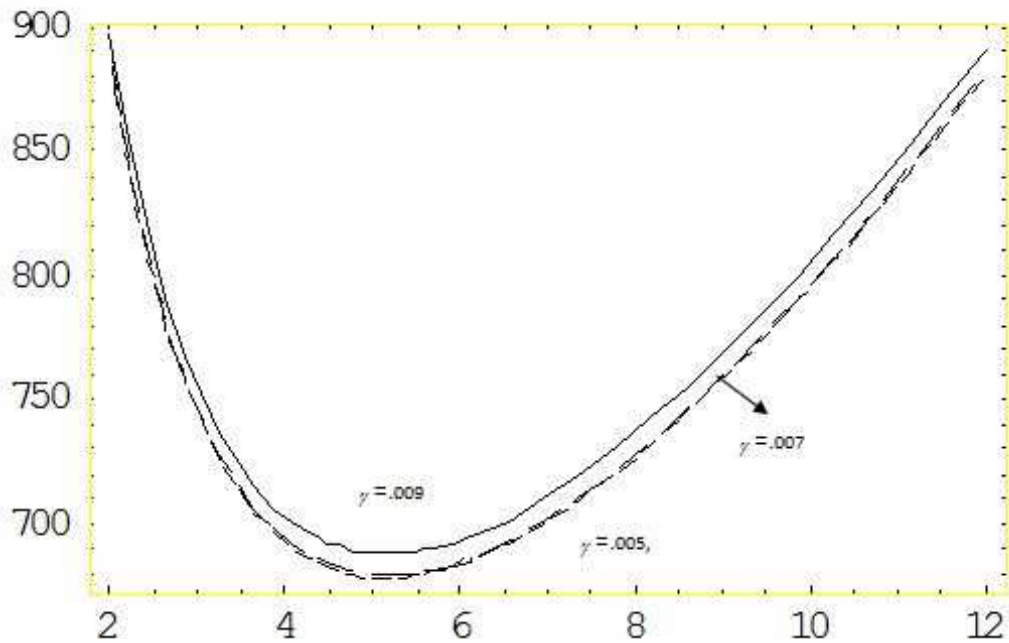


Figure 3. Variation of Rayleigh number R_1 with wave number x for fixed values of $R'_1=50$, $T=10$ and for different values of Stability Ratio γ

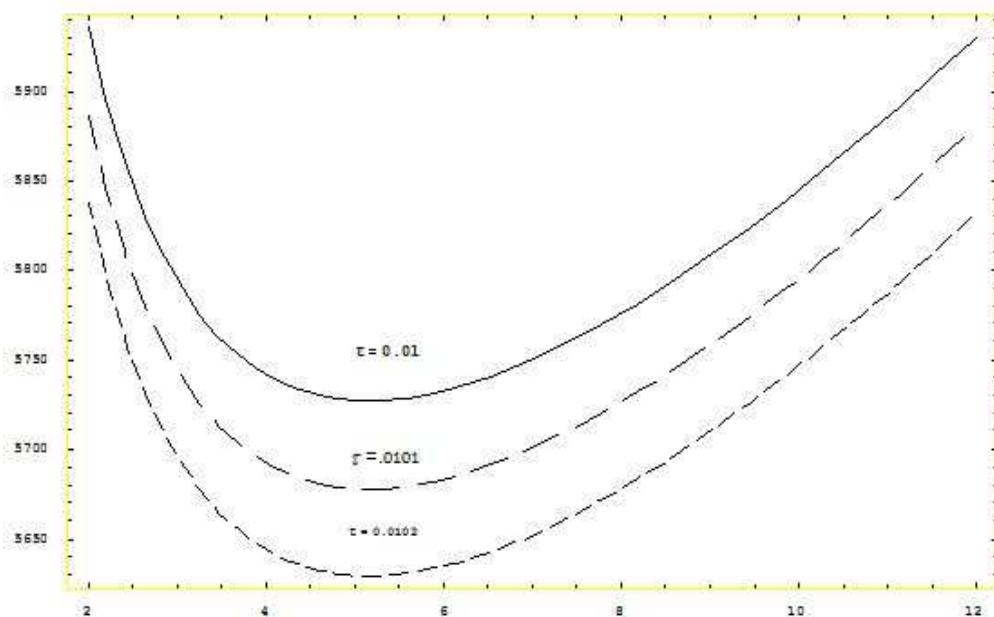


Figure 4. Variation of Rayleigh number R_1 with wave number x for fixed values of $R'_1=50$, $T=10$ and for different values of Lewis Number τ

CONCLUSION

In the present paper, the effects of rotation and the mass flux induced by temperature gradient (Soret effect) on the double diffusion convection in a horizontal layer of fluid subjected to thermal and solutal gradients with cross diffusions has been investigated analytically using linear stability analysis. The eigenvalue problem and the exact solution are obtained for stability investigations. The expressions for stationary Rayleigh number is obtained as a

function of the governing parameters, which characterize the stability of the system. The effects of various physical parameters on the onset of stationary convection in the system are studied both analytically and graphically. Our investigation leads to the following conclusions:

- (1) From the analysis, we found that that solute gradient and rotation has stabilizing effect on the onset of stationary convection in the system. Figures (1)-(2) support the analytical results graphically.
- (2) It is found that the Soret parameter and the Lewis number both has destabilizing effect on the onset of stationary convection in the system. Figures (3)-(4) depict these effects graphically.
- (3) It is shown the onset of instability may either be through non oscillatory or oscillatory modes depending upon the values of the parameters. From equation (32), it is found that rotation and stable solutal gradient introduce oscillatory modes in the systems which were non-existent in their absence.

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REFERENCES

- [1] Antoranz J.C and Velarde M.G, *Physics Letters*, **1978**, 65a, 5, 6.
- [2] Bergeron A. and Tuckerman L.S, *J. Fluid Mech.* **1998**, 375 143
- [3] Bénard H., *Revue generale des sciences pures et appliquéés*, **1900**, 11, 1261.
- [4] Boussinesq J.E., *Gauthier Villars*, **1998**, 172.
- [5] Chandrasekhar S., *Hydrodynamic and Hydromagnetic Stability*, Dover Publication New York. **1981**.
- [6] De Groot S.R and Mazur P., 'Non-equilibrium Thermodynamics', Amsterdam, 273.
- [7] Dhiman J.S and Goyal M.R, *Int. J. Appl. Maths and Mech.* **2**, **2013**, 11
- [8] Dhiman J.S, *Proc. Indian Acad. Sci. (Math. Sci.)* 110(3), **2000**, 335.
- [9] Gupta J.R, Dhiman J.S and Thakur J, *J. Math. Anal. Appl.* **2001**, 287, 398.
- [10] Hurler D.T.J and Jakeman E., *J Fluid Mech.*, **1971**, 47, 667.
- [11] La Porta A and Surko C.M, *Physical Review Letter*, **1998**, 80, 17, 3759
- [12] McDougall T.J, *J. Fluid Mech.*, **1983**, 126, 379
- [13] Rayleigh L, *Phil. Mag.*, **1916**, 32, 529
- [14] Schechter R.S, I. Prigogine I and Hamm J.R, *Phys. of Fluids*, **1972**, 15, 379
- [15] Sengupta, S. and Gupta, A.S, *J. Appl. Maths. Phys. (ZAMP)*, **1971**, 22, 906
- [16] Sparrow, E.M., Minkowycz, W.J., *ASME J. Heat Trans.*, **1964**, 86, 4, 508
- [17] Stommel H and Frodov K.N, *Tellus*, **1967**, 19, 306.
- [18] Takashima M, *J. Phy. Soc. Japan*, **1976**, 41, 4
- [19] Tewfik, O.E. and Yang, J.W, *Int. J. Heat Mass Trans*, **1963**, 6, 10, 915
- [20] Veronis, G., *J. Mar Res.*, **1965**, 23, 1
- [21] Veronis, G., *J. Fluid Mech.*, **1966a** 24, 545.
- [22] Veronis, G., *J. Fluid Mech.*, **1966b**, 26, 49
- [23] Veronis, G., *J. Fluid Mech.*, **1968**, 31, 113