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On the application of the multistage laplace adomian decomposition method with pade approximation to the rabinovich-fabrikant system

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ABSTRACT

In this paper, multi-staging (MLADM) and padé approximation are employed as tools to improve the performance of the Laplace Adomian Decomposition Method (LADM) scheme on the Rabinovich-Fabrikant system for the non-chaotic and chaotic case. We observe that the MLADM solutions were consistent with the fourth order Runge-Kutta (RK4) solutions and that padé approximation does not sufficiently improve the LADM or the MLADM solutions for the chaotic and non-chaotic Rabinovich-Fabrikant system.

Keywords: Chaos, Rabinovich-Fabrikant system, Laplace Adomian Decomposition method, Multistage, Padé approximation, Dynamical systems.

INTRODUCTION

Chaos theory is a nonlinear phenomenon that has applications in several disciplines including physics, geology, mathematics, biology, computer science, robotics, economics [1-2], engineering and meteorology. Chaotic systems are special nonlinear dynamic systems that have the special property of been sensitive to initial condition. This property of chaotic systems is generally known as the "butterfly effect" [3].

The Rabinovich–Fabrikant system is a set of equations developed by Mikhail Rabinovich and Anatoly Fabrikant in 1979 to model waves in non-equilibrium substances. This system is given by a set of three coupled ordinary differential as [4]:

$$\frac{dx}{dt} = y(z - 1 + x^2) + \gamma x$$
$$\frac{dy}{dt} = x(3z + 1 - x^2) + \gamma y \qquad (1)$$
$$\frac{dz}{dt} = -2z(\alpha + xy)$$

α, γ are constants that control the evolution of the system. For some values of α and γ, the system is chaotic, but for others it is periodic. Bifurcation studies have shown that for $\alpha = 1.1$, $\gamma = 0.87$ we have a chaotic system and $\alpha = 1.5$, $\gamma = 0.55$ correspond to a non-chaotic system [5].

The basic idea of the Laplace Adomian decomposition method (LADM) was first introduced by Khuri [6-7]. LADM is a promising method and has been applied in solving various differential equations [8-11]. However, LADM has some drawbacks like most semi-analytic schemes. By using the LADM, we obtain a truncated series solution which does not exhibit the real behaviours of the problem but gives a good approximation to the true solution in a very small region. Since the LADM has a very small convergence region, a multi-staging technique known as the Multistage Laplace Adomian Decomposition Method (MLADM) is proposed. Padé approximation is also used to improve the accuracy and convergence region of the truncated series. Padé approximation is a particular type of rational approximation that is known to be better than the Taylor series approximation.

The aim of this paper is to apply the LADM and MLADM to the Rabinovich–Fabrikant system for both chaotic and non-chaotic case and test the accuracy of the method with the well known fourth order Runge-Kutta method. The paper also investigates the effect of padé approximation on the convergence region of LADM and MLADM. The computations in this paper were carried out with Mathematica.

MATERIALS AND METHODS

LAPLACE ADOMIAN DECOMPOSITION METHOD (LADM)

In this section, we present a Laplace Adomian decomposition method for solving a differential equation written in operator form as:

$$L_t u + R(u) + N(u) = g \tag{2}$$

With initial condition

$$u(x,0) = f(x) \tag{3}$$

Where L_t is a first-order differential operator, R is a linear operator, N is a non-linear operator and g is the source term. We start by applying Laplace transform to both sides of equation (2) and then apply the initial condition (3).

$$\mathcal{L}[L_t u] + \mathcal{L}[R(u)] + \mathcal{L}[N(u)] = \mathcal{L}[g] \tag{4}$$

$$s\mathcal{L}[u] - f(x) = \mathcal{L}[g] - \mathcal{L}[R(u)] - \mathcal{L}[N(u)]$$
$$\mathcal{L}[u] = \frac{f(x)}{s} + \frac{\mathcal{L}[g]}{s} - \frac{\mathcal{L}[R(u)]}{s} - \frac{\mathcal{L}[N(u)]}{s} \tag{5}$$

The LADM defines the solution $u(x,t)$ as an infinite series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n \tag{6}$$

The non-linear term is expressed in terms of the Adomian polynomials given by [12]:

$$N(u) = \sum_{n=0}^{\infty} A_n \tag{7}$$

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\sum_{i=0}^{\infty}\lambda^i u_i\right]\right] \tag{8}$$

Substituting (6) and (7) into (4) gives:

$$\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] = \frac{f(x)}{s} + \frac{\mathcal{L}[g]}{s} - \frac{\mathcal{L}[R(u)]}{s} - \frac{\mathcal{L}[\sum_{n=0}^{\infty} A_n]}{s} \tag{9}$$

From (9), we can define the following recursive formula:

$$\mathcal{L}[u_0] = \frac{f(x)}{s} + \frac{\mathcal{L}[g]}{s} \tag{10}$$

$$\mathcal{L}[u_{n+1}] = -\frac{\mathcal{L}[R(u)]}{s} - \frac{\mathcal{L}[A_n]}{s} \tag{11}$$

Applying the inverse Laplace transform to both sides of (10) and (11) we obtain u_n $(n \geq 0)$ which is then substituted into (6).

MULTISTAGE LAPLACE ADOMIAN DECOMPOSITION METHOD (MLADM)

An efficient way of ensuring the validity of solutions to differential equations for large t $(t \gg 0)$ is by multi-staging the solution procedure to be employed. Let $[0, T]$ be the interval over which the solutions to the differential equation (1) is to be determined. The solution interval $[0, T]$ is divided into N subintervals $(n = 1, 2, \dots \dots \dots, N)$ of equal step size given by $h = T/N$ with the interval end points $t_n = nh$.

Initially, the LADM scheme is applied to obtain the approximate solutions of x, y and z of (1) over the interval $[0, t_1]$ by using the initial condition $x(0), y(0)$ and $z(0)$ respectively. For obtaining the approximate solution of (1) over the next interval $[t_1, t_2]$, we take $x(t_1), y(t_1)$ and $z(t_1)$ as the initial condition. Generally the scheme is repeated for any n with the right endpoints $x(t_{m-1}), y(t_{m-1})$ and $z(t_{m-1})$ at the previous interval being used as the initial condition for the interval $[t_{m-1}, t_m]$.

PADE APPROXIMATION

The padé approximation to $f(x)$ on $[a, b]$ is the quotient of two polynomials $P_N(x)$ and $Q_M(x)$ of degrees N and M respectively. This quotient denoted by $R_{N/M}(x)$ is the padé approximation to the function $f(x)$ and is given by [13] as:

$$R_{N/M}(x) = \frac{P_N(x)}{Q_M(x)} \tag{12}$$

Where $f(x)$ and its derivative must be continuous at $x = 0$. The polynomials $P_N(x)$ and $Q_M(x)$ are given by:

$$P_N(x) = \sum_{i=0}^{N} p_i x^i = p_0 + p_1 x + p_2 x^2 + \cdots + p_N x^N \tag{13}$$

$$Q_M(x) = \sum_{i=0}^{M} q_i x^i = q_0 + q_1 x + q_2 x^2 + \cdots + q_M x^M \tag{14}$$

To obtain the padé approximation $R_{N/M}(x)$, we set $q_0 = 1$. Hence, $Q_M(x)$ becomes

$$Q_M(x) = 1 + q_1 x + q_2 x^2 + \cdots + q_M x^M \tag{15}$$

The polynomials $P_N(x)$ and $Q_M(x)$ are such that the padé approximation $R_{N/M}(x)$ agrees with $f(x)$ at $x = 0$ and the derivatives up to the $(N + M)th$ derivative also agree at $x = 0$. Assuming that the padé approximation $R_{N/M}(x)$ is a series in the form

$$R_{N/M}(x) = \sum_{i=0}^{N+M} r_i x^i = r_0 + r_1 x + r_2 x^2 + \cdots + r_K x^K + \cdots \tag{16}$$

Then from equation (12) we can write,

$$R_{N/M}(x) Q_M(x) - P_N(x) = 0$$

$$\left[\sum_{i=0}^{N+M} r_i x^i\right]\left[\sum_{i=0}^{M} q_i x^i\right] - \left[\sum_{i=0}^{N} p_i x^i\right] = 0 \tag{17}$$

Collecting coefficients of the powers of x^i in equation (17) results in a set of $N+M+1$ linear equations that can be solved separately using Mathematica:

$$
\begin{aligned}
&r_0 - p_0 = 0\\
&r_0 q_1 + r_1 - p_1 = 0\\
&r_0 q_2 + r_1 q_1 + r_2 - p_2 = 0\\
&r_0 q_3 + r_1 q_2 + r_2 q_1 + r_3 - p_3 = 0\\
&r_{N-M} q_M + r_{N-M-1} q_{M-1} + r_N - p_N = 0
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&r_{N-M+1} q_M + r_{N-M+2} q_{M-1} + \cdots + r_N q_1 + r_{N+1} = 0\\
&r_{N-M+2} q_M + r_{N-M+3} q_{M-1} + \cdots + r_{N+1} q_1 + r_{N+2} = 0\\
&r_{N-M+3} q_M + r_{N-M+4} q_{M-1} + \cdots + r_{N+2} q_1 + r_{N+3} = 0\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&r_N q_M + r_{N+1} q_{M-1} + \cdots + r_{N+M-1} q_1 + r_{N+M} = 0
\end{aligned} \tag{19}
$$

The procedure is to first solve for the unknowns $q_1, q_2, \dots, q_M$ in equation (19), the values are then used to obtain the unknowns $p_1, p_2, \dots, p_N$ in equation (18).

If M is equal to N, the approximation is called a diagonal padé approximation of order N. The diagonal padé approximants are known to be the most effective [13-14].

APPLICATION

In this section, we apply the Laplace Adomian decomposition method to the Rabinovich-Fabrikant system in equation (1). The fundamental operation of Laplace Adomian decomposition method is applied to the Rabinovich-Fabrikant system is given below:

$$
\begin{aligned}
&\mathcal{L}\left[\frac{dx}{dt}\right] = \mathcal{L}[yz] - \mathcal{L}[y] + \mathcal{L}[yx^2] + \gamma\mathcal{L}[x]\\
&\mathcal{L}\left[\frac{dy}{dt}\right] = 3\mathcal{L}[xz] + \mathcal{L}[x] - \mathcal{L}[x^3] + \gamma\mathcal{L}[y]\\
&\mathcal{L}\left[\frac{dz}{dt}\right] = -2\alpha\mathcal{L}[z] - 2\mathcal{L}[xyz]
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
&s\mathcal{L}[x] - x(0) = \mathcal{L}[yz] - \mathcal{L}[y] + \mathcal{L}[yx^2] + \gamma\mathcal{L}[x]\\
&s\mathcal{L}[y] - y(0) = 3\mathcal{L}[xz] + \mathcal{L}[x] - \mathcal{L}[x^3] + \gamma\mathcal{L}[y]\\
&s\mathcal{L}[z] - z(0) = -2\alpha\mathcal{L}[z] - 2\mathcal{L}[xyz]
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{L}[x] = \frac{x(0)}{s} + \frac{1}{s}\mathcal{L}[yz] - \frac{1}{s}\mathcal{L}[y] + \frac{1}{s}\mathcal{L}[yx^2] + \frac{\gamma}{s}\mathcal{L}[x]\\
&\mathcal{L}[y] = \frac{y(0)}{s} + \frac{3}{s}\mathcal{L}[xz] + \frac{1}{s}\mathcal{L}[x] - \frac{1}{s}\mathcal{L}[x^3] + \frac{\gamma}{s}\mathcal{L}[y]\\
&\mathcal{L}[z] = \frac{z(0)}{s} - \frac{2\alpha}{s}\mathcal{L}[z] - \frac{2}{s}\mathcal{L}[xyz]
\end{aligned} \tag{21}
$$

The solution of the Rabinovich-Fabrikant system takes the form

$x(t) = \sum_n^\infty X_n \qquad y(t) = \sum_n^\infty Y_n \qquad z(t) = \sum_n^\infty Z_n$

The non-linear terms are expressed as:

$yz = \sum_n^\infty A_n \qquad yx^2 = \sum_n^\infty B_n \qquad xz = \sum_n^\infty C_n \qquad x^3 = \sum_n^\infty D_n \qquad xyz = \sum_n^\infty E_n$

Then (21) can be written as a recursive formula in parameterized form as:

$$\mathcal{L}\left[\sum \lambda^n X_n\right] = \frac{x(0)}{s} + \frac{\lambda}{s}\mathcal{L}\left[\sum \lambda^n A_n\right] - \frac{\lambda}{s}\mathcal{L}\left[\sum \lambda^n Y_n\right] + \frac{\lambda}{s}\mathcal{L}\left[\sum \lambda^n B_n\right] + \frac{\lambda\gamma}{s}\mathcal{L}\left[\sum \lambda^n X_n\right]$$

$$\mathcal{L}\left[\sum \lambda^n Y_n\right] = \frac{y(0)}{s} + \frac{3\lambda}{s}\mathcal{L}\left[\sum \lambda^n C_n\right] + \frac{\lambda}{s}\mathcal{L}\left[\sum \lambda^n X_n\right] - \frac{\lambda}{s}\mathcal{L}\left[\sum \lambda^n D_n\right] + \frac{\lambda\gamma}{s}\mathcal{L}\left[\sum \lambda^n Y_n\right] \quad (22)$$
$$\mathcal{L}\left[\sum \lambda^n Z_n\right] = \frac{z(0)}{s} - \frac{2\alpha\lambda}{s}\mathcal{L}\left[\sum \lambda^n Z_n\right] - \frac{2\lambda}{s}\mathcal{L}\left[\sum \lambda^n E_n\right]$$

Comparing equal powers of λ in equation (22), we have:

$$\mathcal{L}[X_0] = \frac{x(0)}{s} \qquad \mathcal{L}[Y_0] = \frac{y(0)}{s} \qquad \mathcal{L}[Z_0] = \frac{z(0)}{s}$$
$$X_0 = \mathcal{L}^{-1}\left[\frac{x(0)}{s}\right] \qquad Y_0 = \mathcal{L}^{-1}\left[\frac{y(0)}{s}\right] \qquad Z_0 = \mathcal{L}^{-1}\left[\frac{z(0)}{s}\right] \quad (23)$$

$$\mathcal{L}[X_{n+1}] = \frac{1}{s}\mathcal{L}[A_n] - \frac{1}{s}\mathcal{L}[Y_n] + \frac{1}{s}\mathcal{L}[B_n] + \frac{\gamma}{s}\mathcal{L}[X_n]$$
$$\mathcal{L}[Y_{n+1}] = \frac{3}{s}\mathcal{L}[C_n] + \frac{1}{s}\mathcal{L}[X_n] - \frac{1}{s}\mathcal{L}[D_n] + \frac{\gamma}{s}\mathcal{L}[Y_n]$$
$$\mathcal{L}[Z_{n+1}] = -\frac{2\alpha}{s}\mathcal{L}[Z_n] - \frac{2}{s}\mathcal{L}[E_n]$$

$$X_{n+1} = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}[A_n] - \frac{1}{s}\mathcal{L}[Y_n] + \frac{1}{s}\mathcal{L}[B_n] + \frac{\gamma}{s}\mathcal{L}[X_n]\right]$$
$$Y_{n+1} = \mathcal{L}^{-1}\left[\frac{3}{s}\mathcal{L}[C_n] + \frac{1}{s}\mathcal{L}[X_n] - \frac{1}{s}\mathcal{L}[D_n] + \frac{\gamma}{s}\mathcal{L}[Y_n]\right] \quad (24)$$
$$Z_{n+1} = \mathcal{L}^{-1}\left[-\frac{2\alpha}{s}\mathcal{L}[Z_n] - \frac{2}{s}\mathcal{L}[E_n]\right]$$

The system is solved with the initial condition $x(0) = -1.0,\ y(0) = 0$ and $z(0) = 0.5$. For $\alpha = 1.1,\ \gamma = 0.87$ we have a chaotic system and $\alpha = 1.5,\ \gamma = 0.55$ correspond to a non-chaotic system. The recursive relations (23) and (24) are evaluated with the aid of Mathematica to obtain the solution up to a 11 terms approximation for the time range [0, 30] with a time step size 0.01. MLADM is implemented by dividing the solution interval $[0, 30]$ into 300 subintervals ($n = 1, 2, \ldots\ldots\ldots, 300$) of equal step size given by $h = 0.1$

RESULTS AND DISCUSSION

The Laplace Adomian decomposition method LADM and the Multistage Laplace Adomian decomposition method MLADM has been applied to the non-chaotic case of the Rabinovich-Fabrikant system. Padé approximation has also been applied to the LADM and MLADM results to improve the convergence region and hence the accuracy of the results. The 11 term approximate LADM solution and the corresponding diagonal padé approximants for the non-chaotic case were obtained and are presented below:

$$x = -1 - 0.55t - 0.52625t^2 + 0.432688t^3 - 1.19897t^4 + 0.972412t^5 + 0.181246t^6 - 0.0512276t^7 + 0.407736t^8 + 0.318605t^9 + 0.0619896t^{10}$$

$$y = -1.5t + 1.975t^2 + 0.0772917t^3 + 0.118073t^4 + 0.42832t^5 + 1.16637t^6 - 0.626623t^7 - 1.52433t^8 + 2.50421t^9 - 2.33947t^{10}$$

$$z = 0.5 - 1.5t + 1.5t^2 + 0.383333t^3 - 2.18146t^4 + 1.91417t^5 - 0.809985t^6 + 1.30261t^7 - 2.59824t^8 + 1.31422t^9 + 2.98515t^{10}$$

$$x_{[5/5]\,pade} = \frac{-1 - 0.245783t - 0.0396009t^2 + 1.16674t^3 - 0.976691t^4 + 1.24769t^5}{1 - 0.304217t - 0.319329t^2 - 0.398326t^3 + 0.0332179t^4 + 0.142647t^5}$$

$$y_{[5/5]\,pade} = \frac{-1.5t - 43.1127t^2 + 54.9589t^3 - 66.317t^4 + 139.266t^5}{1 + 30.0585t - 2.98928t^2 - 49.7748t^3 - 24.5014t^4 - 20.0994t^5}$$

$$z_{[5/5]\,pade} = \frac{0.5 - 0.163207t - 0.548661t^2 + 0.357611t^3 + 0.310687t^4 - 0.0700585t^5}{1 + 2.67359t - 3.92344t^2 - 3.69811t^3 + 2.25856t^4 + 0.369554t^5}$$

Table 1: Absolute differences between 11-term LADM and 11-term $\text{LADM}_{\text{padé}}$ with RK4 solutions ($\Delta t = 0.01$) for $\alpha = 1.5,\ \gamma = 0.55$.

	$\lvert \text{LADM}_{0.01} - \text{RK4}_{0.01} \rvert$			$\lvert \text{LADM}_{\text{padé}0.01} - \text{RK4}_{0.01} \rvert$		
t	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.192E-11	9.158E-12	3.598E-09	2.330E-10	1.887E-08	1.103E-09
0.45	8.647E-05	1.801E-05	6.452E-04	1.482E-04	8.081E-04	4.014E-06
1.05	1.066E+00	1.166E+00	4.594E+00	6.626E+00	1.883E+00	2.460E-03
2.10	5.340E+02	2.479E+03	5.249E+03	2.471E+01	8.953E+00	1.919E-01
4.05	1.972E+05	2.159E+06	3.763E+06	9.494E+00	7.687E+00	6.357E-01
7.10	3.741E+07	6.570E+08	1.016E+09	9.328E+00	7.099E+00	1.950E-01
11.30	3.169E+09	7.231E+10	1.046E+11	9.374E+00	7.417E+00	2.428E-01
16.05	9.462E+10	2.484E+12	3.468E+12	9.437E+00	7.881E+00	3.052E-01
22.10	2.147E+12	6.196E+13	8.447E+13	9.396E+00	7.837E+00	5.044E-01
25.65	9.249E+12	2.767E+14	3.738E+14	9.442E+00	8.129E+00	5.186E-01
30.00	4.314E+13	1.333E+15	1.787E+15	9.476E+00	8.107E+00	5.266E-01

Table 2: Absolute differences between 11-term MLADM and 11-term $\text{MLADM}_{\text{padé}}$ with RK4 solutions ($\Delta t = 0.01$) for $\alpha = 1.5,\ \gamma = 0.55$.

	$\lvert \text{MLADM}_{0.01} - \text{RK4}_{0.01} \rvert$			$\lvert \text{MLADM}_{\text{padé}0.01} - \text{RK4}_{0.01} \rvert$		
t	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.192E-11	9.158E-12	3.598E-09	2.330E-10	1.887E-08	1.103E-09
0.45	4.974E-11	2.527E-10	3.083E-10	4.967E-11	2.526E-10	3.083E-10
1.05	8.778E-11	6.365E-10	4.644E-11	8.732E-11	6.390E-10	4.644E-11
2.10	1.530E-10	2.237E-10	2.379E-10	1.559E-10	2.288E-10	2.451E-10
4.05	2.174E-11	1.906E-11	3.673E-11	2.174E-11	1.906E-11	3.673E-11
7.10	1.520E-12	4.360E-12	7.880E-13	1.530E-12	4.340E-12	7.880E-13
11.30	2.300E-13	7.440E-12	5.400E-13	2.400E-13	7.430E-12	5.390E-13
16.05	1.070E-12	9.350E-12	2.312E-12	1.070E-12	9.350E-12	2.312E-12
22.10	5.300E-12	2.061E-11	4.845E-12	5.298E-12	2.061E-11	4.851E-12
25.65	1.354E-12	1.161E-11	6.250E-13	1.354E-12	1.161E-11	6.250E-13
30.00	5.548E-12	6.353E-11	1.026E-11	4.643E-12	4.575E-11	7.484E-12

The accuracy of the LADM and MLADM and the effect of padé approximation is investigated by comparing their solutions to the RK4 solution for the parameters $\alpha = 1.5$, $\gamma = 0.55$ where the system is non-chaotic with the initial conditions $x(0) = -1.0$, $y(0) = 0$ and $z(0) = 0.5$. The RK4 with time step $\Delta t = 0.01$ with the number of significant digits set to 16 is used. Table 1 presents the absolute differences between the 11-term LADM solutions and the padé approximated LADM solutions for $\alpha = 1.5$, $\gamma = 0.55$ and the RK4 solutions. The absolute differences between the MLADM solutions and the padé approximated MLADM results and the RK4 solutions are presented in Table 2.

In Table 1, we can observe that LADM only gives valid result for $t \ll 1$ i.e. LADM does not give reliable results after $t = 0.45$. The absolute difference between the LADM and the RK4 solutions are as high as a million (1E+06) just after $t = 4.05$ and gave absolute difference of order 1E+15 after $t = 30$. Application of padé approximation to the truncated series used for the LADM solution improves considerably the performance of the LADM scheme but still does not produce desirable or acceptable absolute difference. The padé approximated LADM solution gave absolute difference of the order of 1E+01 at the solution interval.

From Table 2, we observe that the MLADM solution agree with the RK4 solution to at least 8 decimal places for the non-chaotic case. This shows that multi-staging technique applied to the LADM scheme is an effective method for solving the non-chaotic Rabinovich-Fabrikant system. The padé approximated MLADM solution gave comparable results with the ordinary MLADM scheme but without any noticeable improvements. Hence, MLADM scheme without padé approximation is sufficient for solving the non-chaotic Rabinovich-Fabrikant system.

The $x - y, x - z, y - z$ and $x - y - z$ phase portraits for the non-chaotic case obtained using the 11-term MLADM solutions are respectively shown in Figure 1 to Figure 4.

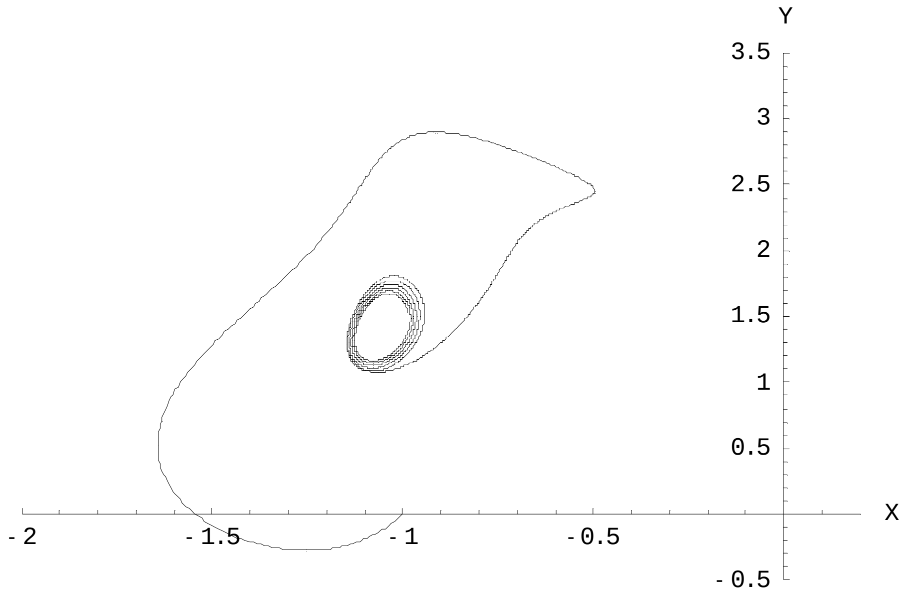

Figure 1: X-Y Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.5,\ \gamma = 0.55$.

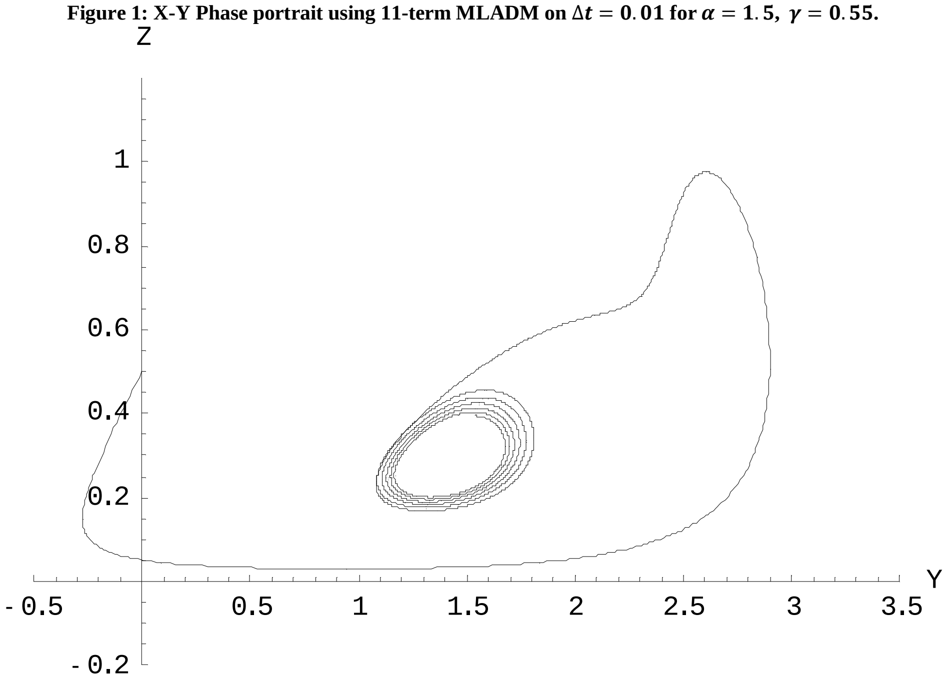

Figure 2: Y-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.5,\ \gamma = 0.55$.

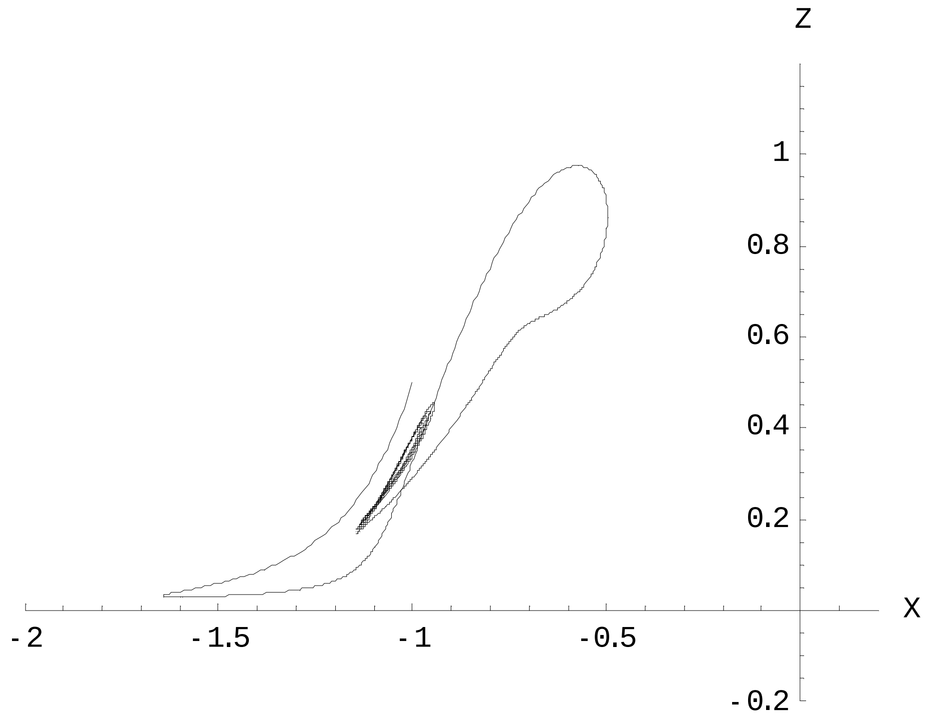

Figure 3: X-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.5,\ \gamma = 0.55$.

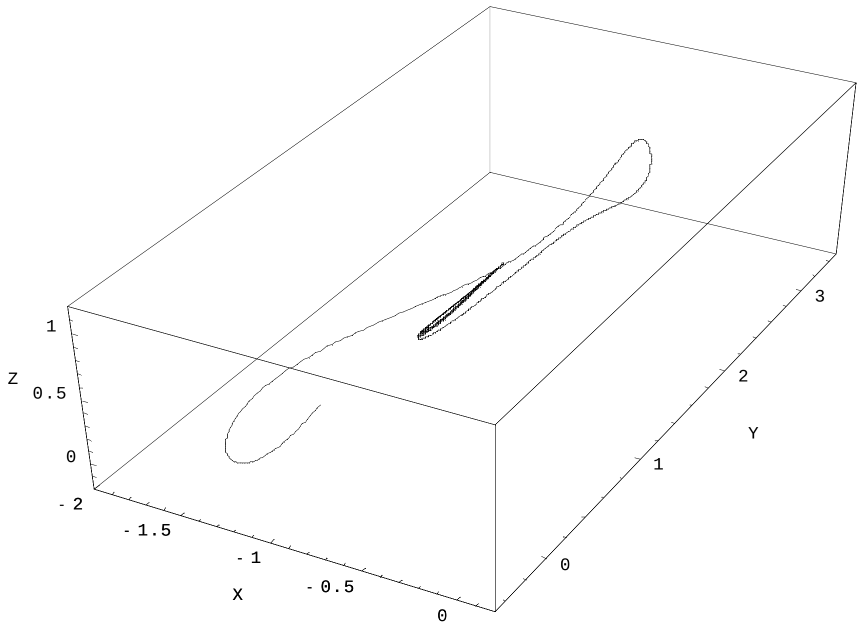

Figure 4: X-Y-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.5,\ \gamma = 0.55$.

The 11 term approximate LADM solution and the corresponding diagonal padé approximants for the chaotic Rabinovich-Fabrikant system (α = 1.1, γ = 0.87) were obtained as:

$$x = -1 - 0.87t - 0.75345t^2 - 0.336t^3 - 0.683627t^4 + 0.0837543t^5 + 1.02679t^6 + 1.69623t^7 + 2.11575t^8 + 2.15503t^9 + 1.73951t^{10} + \cdots$$

$$y = -1.5t + 1.215t^2 + 1.73183t^3 + 1.3385t^4 + 1.13646t^5 + 1.73087t^6 + 1.07902t^7 - 0.811878t^8 - 2.10469t^9 - 3.09986t^{10} + \cdots$$

$$z = 0.5 - 1.1t + 0.46t^2 + 0.732667t^3 - 0.238792t^4 - 0.409795t^5 - 0.171489t^6 + 0.825099t^7 + 0.0350825t^8 - 0.772747t^9 + 0.0268586t^{10} + \cdots$$

$$x_{[5/5]\,pade} = \frac{-1 + 1.04785t - 0.336679t^2 + 0.297421t^3 - 0.956598t^4 + 1.06331t^5}{1 - 1.91785t + 1.25176t^2 - 0.277448t^3 + 0.215612t^4 - 0.0675913t^5}$$

$$y_{[5/5]\,pade} = \frac{-1.5t + 3.68511t^2 - 2.12673t^3 + 1.75259t^4 - 1.64915t^5}{1 - 1.64674t + 1.23851t^2 - 1.17412t^3 + 0.866521t^4 + 0.357742t^5}$$

$$z_{[5/5]\,pade} = \frac{0.5 - 0.828568t + 0.174171t^2 + 0.359839t^3 + 0.132884t^4 - 0.0360036t^5}{1 + 0.542864t + 0.622643t^2 + 0.124724t^3 - 0.260564t^4 - 0.544664t^5}$$

The absolute differences between the 11-term LADM solutions and the padé approximated LADM solutions for $\alpha = 1.1$, $\gamma = 0.87$ and the RK4 solutions are presented in Table 3 while the absolute differences between the MLADM solutions and the padé approximated MLADM results and the RK4 solutions are presented in Table 4.

From Table 3, we also observe that LADM only gives valid result for $t \ll 1$ similar to what was obtained for the non-chaotic case. The absolute difference between the LADM and the RK4 solutions are of order 1E+15 after $t = 30$. Also, the padé approximated LADM scheme performed better than the ordinary LADM scheme for the chaotic Rabinovich-Fabrikant system but still gave an unacceptable absolute difference of the order 1E+01.

Table 3: Absolute differences between 11-term LADM and 11-term $\mathbf{LADM_{padé}}$ with RK4 solutions ($\Delta t = 0.01$) for $\alpha = 1.1,\ \gamma = 0.87$.

	$\lvert LADM_{0.01} - RK4_{0.01}\rvert$			$\lvert LADM_{padé0.01} - RK4_{0.01}\rvert$		
t	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.407E-10	4.552E-09	6.828E-10	5.124E-10	7.607E-10	1.626E-10
0.45	4.003E-04	1.048E-03	8.517E-05	4.337E-04	3.970E-04	5.843E-07
1.05	1.067E+01	2.777E+00	5.612E-01	2.671E+00	1.882E+00	3.342E-02
2.10	5.788E+03	6.718E+03	4.383E+02	2.059E+01	3.364E+00	1.417E+00
4.05	2.885E+06	4.327E+06	1.786E+05	4.412E+01	3.198E+00	4.485E-01
7.10	6.804E+08	1.110E+09	2.574E+07	2.183E+01	4.182E+00	3.616E-01
11.30	6.613E+10	1.117E+11	1.381E+09	1.840E+01	4.822E+00	7.061E-01
16.05	2.135E+12	3.668E+12	2.377E+10	1.674E+01	3.799E+00	2.961E-01
22.10	5.118E+13	8.884E+13	2.212E+11	1.675E+01	4.621E+00	8.347E-01
25.65	2.252E+14	3.924E+14	3.903E+11	1.656E+01	5.575E+00	6.167E-01
30.00	1.071E+15	1.872E+15	6.907E+11	1.549E+01	5.417E+00	3.458E-01

Table 4: Absolute differences between 11-term MLADM and 11-term $\mathbf{MLADM_{padé}}$ with RK4 solutions ($\Delta t = 0.01$) for $\alpha = 1.1,\ \gamma = 0.87$.

	$\lvert MLADM_{0.01} - RK4_{0.01}\rvert$			$\lvert MLADM_{padé0.01} - RK4_{0.01}\rvert$		
t	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.407E-10	4.552E-09	6.828E-10	5.124E-10	7.607E-10	1.626E-10
0.45	4.064E-10	3.177E-10	4.378E-10	3.990E-10	3.175E-10	4.378E-10
1.05	7.022E-11	5.363E-10	3.718E-10	7.578E-11	5.345E-10	3.718E-10
2.10	2.435E-10	4.909E-11	4.626E-10	2.708E-10	1.571E-11	4.508E-10
4.05	2.417E-11	1.541E-11	1.839E-11	2.417E-11	1.541E-11	1.839E-11
7.10	1.224E-09	1.553E-09	9.840E-10	5.618E-10	2.487E-09	9.744E-10
11.30	4.385E-10	1.947E-11	1.583E-10	4.660E-10	1.983E-10	6.004E-11
16.05	8.462E-10	7.770E-10	6.206E-10	8.443E-10	7.770E-10	6.206E-10
22.10	3.171E-11	7.643E-11	3.571E-11	3.235E-11	8.278E-11	4.526E-11
25.65	6.414E-12	9.170E-12	3.612E-11	6.414E-12	9.170E-12	3.612E-11
30.00	1.762E-06	8.250E-06	1.779E-08	1.463E-06	1.330E-07	2.200E-08

From Table 4, we observe that the MLADM solution agree with the RK4 solution to at least 5 decimal places for the chaotic case. The padé approximated MLADM solution also does not improve on the ordinary MLADM scheme and is therefore unnecessary. The $x-y, x-z, y-z$ and $x-\ y-z$ phase portraits for the chaotic Rabinovich-Fabrikant system obtained using the 11-term MLADM solutions are presented in Figure 5 to Figure 8 respectively.

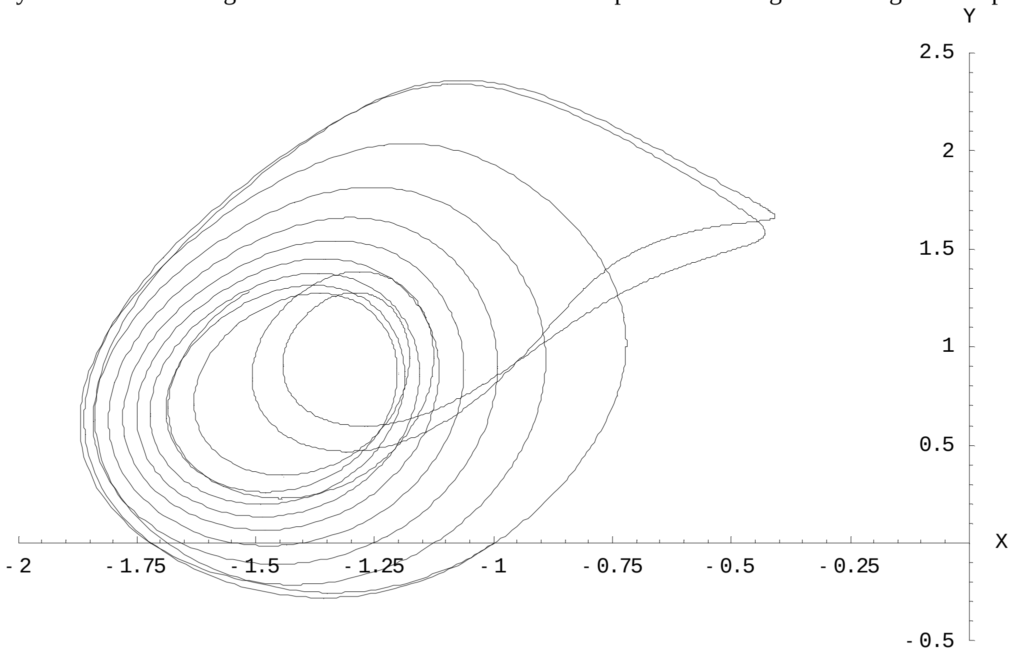

Figure 5: X-Y Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.1,\ \gamma = 0.87$.

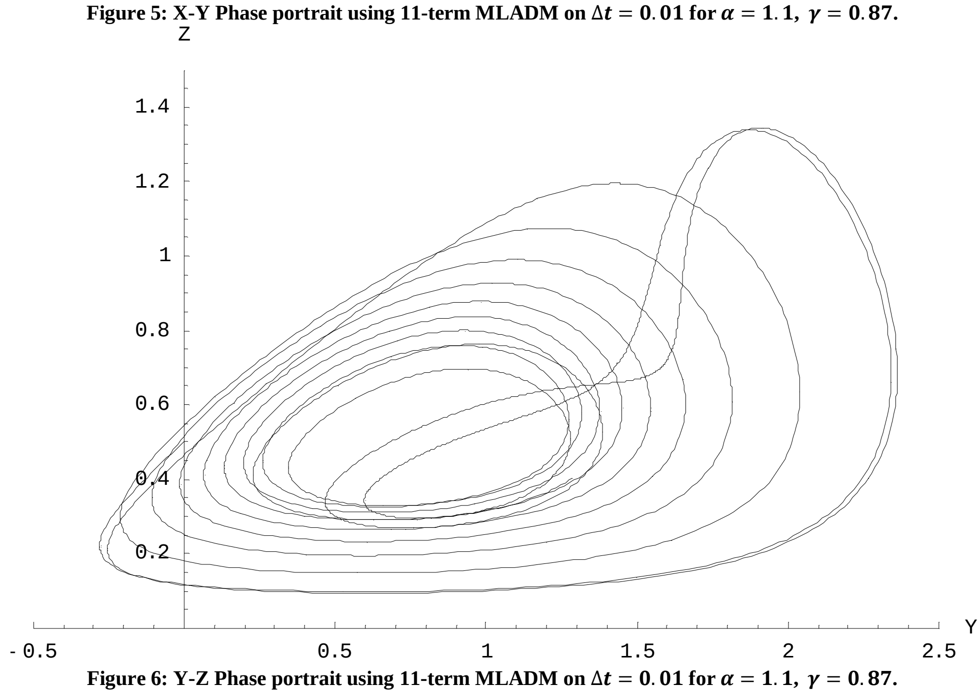

Figure 6: Y-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.1,\ \gamma = 0.87$.

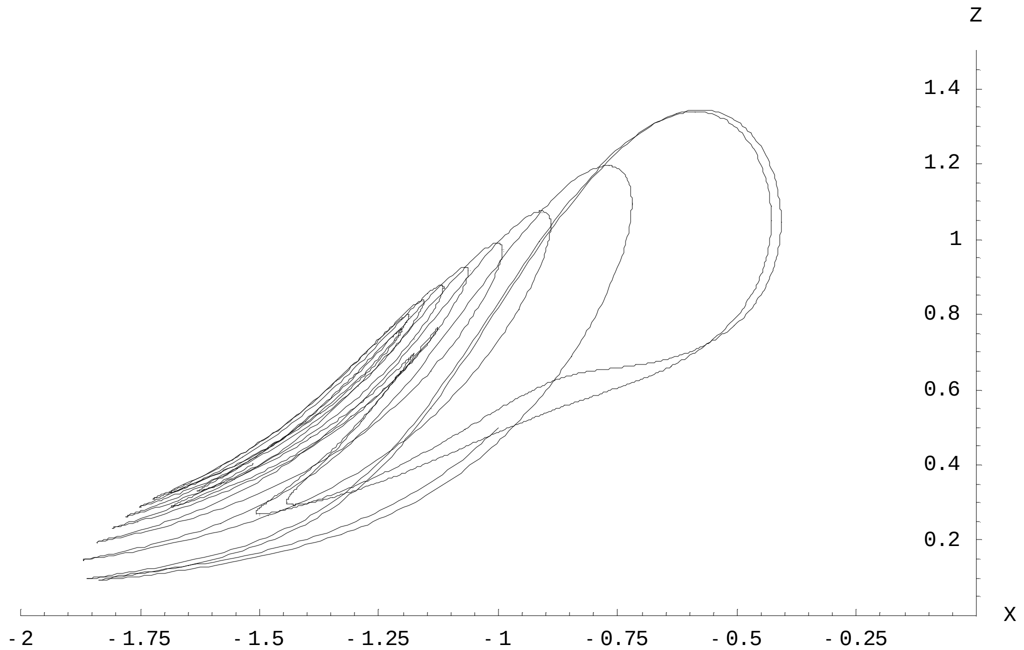

Figure 7: X-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.1,\ \gamma = 0.87$.

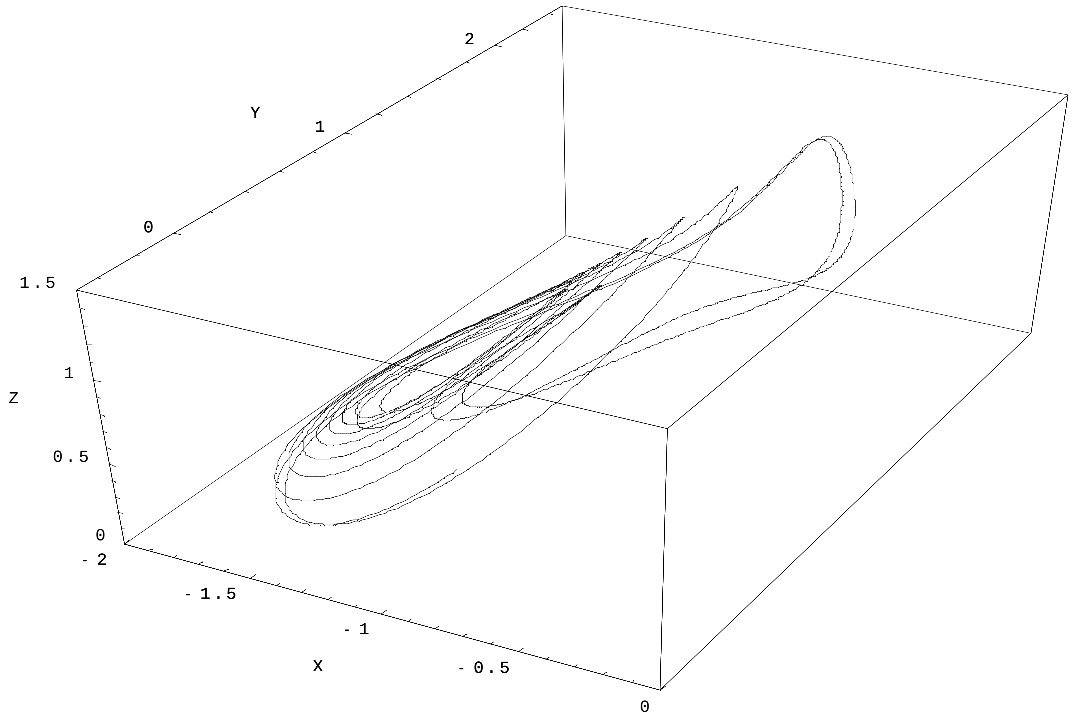

Figure 8: X-Y-Z Phase portrait using 11-term MLADM on $\Delta t = 0.01$ for $\alpha = 1.1,\ \gamma = 0.87$.

CONCLUSION

In this work, multi-staging (MLADM) and padé approximation are employed as tools to improve the performance of the Laplace Adomian Decomposition Method (LADM) scheme on the Rabinovich-Fabrikant system. Comparisons were made between these methods and the fourth-order Runge-Kutta (RK4) method. For the chaotic and non-chaotic case, we observe that the MLADM solutions were consistent with the RK4 solutions and that padé approximation does not sufficiently improve the LADM or the MLADM solutions. Conclusively, multi-staging employed with the LADM scheme in MLADM is a simple and accurate method of solving the Rabinovich-Fabrikant system and by extension other similar nonlinear systems.

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