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Advances in Applied Science Research, 2016, 7(4):190-196

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ISSN: 0976-8610
CODEN (USA): AASRFC

On Specially Structured Two Stage Flow Shop Scheduling Problem with Jobs In A String of Disjoint Job Blocks

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ABSTRACT

The present paper studies two stage specially structured flow shop scheduling problem with jobs in a string of disjoint job blocks in which processing times are associated with their respective probabilities, where the optimization criteria is the utilization time of machines. The objective is to minimize utilization time of machines for two the stage specially structured flow shop scheduling problem with jobs in a string of disjoint job blocks. The algorithm proposed in this paper is very simple and easy to understand and also provide an important tool for the decision maker. The algorithm is justified by a numerical illustration.

Keywords: Utilization Time, Specially Structured Flow Shop Scheduling, Processing time, Equivalent job, Jobs in a String, Disjoint Job Blocks, Elapsed Time.

INTRODUCTION

Scheduling is the determination of order of various jobs (tasks) for the set of machines (resources) such that certain performance measures are optimized. Scheduling involves time tabling as well as sequencing information of jobs (tasks). Scheduling is generally considered to be one of the most important issues in the planning and operation of a manufacturing system. Better scheduling system has significant impact on cost reduction, increased productivity, customer satisfaction and overall competitive advantage. Scheduling leads to increase in capacity utilization, improves efficiency and thereby reduces the time required to complete the jobs and consequently increases the profitability of an organization in present competitive environment.

In general flow shop scheduling problem, n-jobs are to be processed on m-machines in some particular order in which passing of jobs on machines is not permitted. Johnson [1] developed the heuristic algorithm for two and three stage production schedule for minimizing the makespan. Palmer, D.S., [2] gave a heuristic algorithm for sequencing jobs to minimize the total elapsed time. The general n×m problem was solved by Smith and Dudke [3]. Cambell et al. [4] proposed the generalization of Johnson's method by developing artificial two machine problems from the original m-machine problem and solved them using Johnson's algorithm. Gupta, J.N.D. [5] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Maggu, P. L. and Das, G. [6] gave the basic concept of equivalent job for job block in job sequencing. Anup [7] studied two machine flow shop problem with equivalent job for an ordered job block.

Anup and Maggu P.L. [8] gave an optimal schedule for n×2 flow shop problem with job blocks of jobs in which first job in each job block being the same. Heydari [9] studied flow shop scheduling problem with processing of jobs in a string of disjoint job blocks. Singh, T.P., Kumar, R. and Gupta, D. [10] studied the optimal three stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. Singh T.P., Kumar, V. and Gupta, D. [11] studied n×2 flow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. Gupta, D., Sharma,

S. and Gulati, N. [12] studied n×3 flow shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. Gupta, D., Sharma, S., and Bala, S. [13] studied specially structured two stage flow shop problem to minimize the rental cost of machines under pre-defined rental policy in which the probabilities have been associated with processing time. Gupta, D. et al. [14] studied two stage flow shop scheduling with job block criteria and unavailability of machines using branch and bound technique. Gupta, D. et al. [15] studied 3-stage specially structured flow shop scheduling to minimize the rental cost of machines including transportation time, weightage of jobs and job block criteria.

In this paper n×2 specially structured flow shop scheduling problem with jobs in a string of disjoint job block is considered. Two machine specially structured flow shop scheduling problem has been considered due to its applications in real life as there are cases when the processing time of jobs on machines does not take random values but follow some specific structural conditions. The string of disjoint job blocks consist of two disjoint job blocks such that in one job block the order of jobs is fixed and in second job block the order of jobs is arbitrary. The objective of the study is to obtain an optimal sequence of jobs to minimize the utilization time of machines in case of specially structured two stage flow shop scheduling problem with jobs in a string of disjoint job blocks and to develop a new heuristic algorithm, an alternative to the traditional algorithm proposed by Johnson's [1] to find the optimal sequence to minimize the utilization time of machines.

Practical Situation

Manufacturing units/industries play an important role in the economic development of a country. Productivity can be maximized if the available resources are utilized in an optimal manner. For optimal utilization of available resources there must be a proper scheduling system for the resources and this makes scheduling a highly important aspect of manufacturing systems. The practical situation of specially structured flow shop scheduling occurs in banking, offices, educational institutions, factories and industrial concern. In our day to day working in factories and industrial units different jobs are processed on various machines. In textile industry different types of fabric is produced using different types of yarn. Here, the time taken in dying of yarn on first machine is always less than the weaving of yarn on the second machine. In two machine problem the jobs are required to be processed on machines A, B in specified order. When certain ordering of the jobs to be processed is prescribed either by technological constraint or by externally imposed policy the concept of job block is significant. Thus, the job block represents the relative importance and group binding of jobs. Example of jobs in a string of disjoint job block occurs in steel manufacturing industries where certain jobs such as heating and molding must be carried out as a fixed job block in processing and other jobs such as cutting, grinding, chroming etc. can be processed in a block disjoint from the first block in an optimal order to minimize the makespan

Assumptions and Notations

The assumptions for the proposed algorithm are stated below:

a) Jobs are independent to each other. Let n jobs be processed thorough two machines M_1 and M_2 in order M_1 M_2.

b) Machine breakdown is not considered.

c) Pre-emption is not allowed. Once a job is started on a machine the process on that machine cannot be stopped unless the job is completed.

d) Expected processing times A_{i1} and A_{j2} for jobs i and j must satisfy the structural conditions viz. $A_{i1} \geq A_{j2}$ or $A_{i1} \leq A_{j2}$ for each i and j.

e) Each job has two operations and each job is processed through each of the machine once and only once.

f) Each machine can perform only one task at a time.

g) A job is not available to the next machine until and unless processing on the current machine is completed.

h) The independency of processing times of jobs on the schedule is maintained.

i) Only one machine of each type is available.

j) $\sum_{i=1}^{n} p_{i1} = 1, \sum_{i=1}^{n} p_{i2} = 1, 0 \leq p_{ij} \leq 1$

k) Jobs i_1, i_2, -----------------, i_h are to be processed as a job block (i_1, i_2, -----------------, i_h) showing priority of job i_1 over i_2 etc. in that order in case of a fixed order job block.

The following notations have been used throughout the paper:

σ: Sequence of n- jobs obtained by applying Johnson's algorithm.

σ_k: Sequence of jobs obtained by applying the proposed algorithm, k = 1, 2, 3, ------.

M_j: Machine j, here j= 1, 2.

a_{ij}: Processing time of i^{th} job on machine M_j.

p_{ij}: Probability associated to the processing time a_{ij}.

A_{ij}: Expected processing time of i^{th} job on machine M_j.

t_{ij} (σ_k): Completion time of i^{th} job of sequence σ_k on machine M_j, where,
$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + A_{ij}$; $j \geq 2$.
T (σ_k): Total elapsed time for jobs 1, 2, --------, n for sequence σ_k.
I_{ij} (σ_k): Idle time of machine M_j for job i in the sequence σ_k.
U_j (σ_k): Utilization time for which machine M_j is required for sequence σ_k.
A_{ij} (σ_k): Expected processing time of i^{th} job on machine M_j for sequence σ_k.
α: Fix order job block.
β: Job block with arbitrary order.
β_k: Job block with jobs in an optimal order.
S: String of job blocks α and β i.e. S = (α, β)
S′: Optimal string of job blocks α and β_k.

Problem Formulation

Let n-jobs (i = 1, 2, -----------, n) be processed on two machines M_j (j =1, 2) in the order M_1M_2. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probability p_{ij} such that $0 \leq p_{ij} \leq 1$ and $\sum_{i=1}^{n} p_{ij} = 1$. Take two job blocks α and β such that the job block α consist of s jobs with fixed order of jobs and β consist of r jobs in which order of jobs is arbitrary such that s + r = n and α ∩ β =∅ i.e. the two job blocks α and β are disjoint in the sense that the two blocks have no job in common. Let S = (α, β). The mathematical model of the problem in matrix form can be stated as:

Table -1

Jobs	Machine M_1		Machine M_2	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	a_{11}	p_{11}	a_{12}	p_{12}
2	a_{21}	p_{21}	a_{22}	p_{22}
3	a_{31}	p_{31}	a_{32}	p_{32}
-	-	-	-	-
n	a_{n1}	p_{n1}	a_{n2}	p_{n2}

Our aim is to find job block β_k with jobs in optimal order and an optimal string S′ of job blocks α and β_k i.e. to find a sequence σ_k of jobs which minimizes the total elapsed time and hence minimizes the utilization times of machines given that S = (α, β).

Mathematically, the problem is stated as:

Minimize T (σ_k) and hence
Minimize U_2 (σ_k).

Proposed Algorithm

Step 1: Calculate the expected processing times A_{ij} given by $A_{ij} = a_{ij} \times p_{ij}$.

Step 2: Take equivalent job α for the job block (r, m) and calculate the processing time $A_{\alpha 1}$ and $A_{\alpha 2}$ on the guidelines of Maggu and Das [6] as follows:

$$A_{\alpha 1} = A_{r1} + A_{m1} - \min(A_{m1}, A_{r2}).$$
$$A_{\alpha 2} = A_{r2} + A_{m2} - \min(A_{m1}, A_{r2}).$$

If a job block has three or more than three jobs then to find the expected flow times we use the property that the equivalent job for job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$.

Step 3: Obtain the new job block β_k from the job block β (disjoint from job block α) by the proposed algorithm using step 6 by treating job block β as sub-flow shop scheduling problem of the main problem. Obtain the processing times $A_{\beta_k 1}$ and $A_{\beta_k 2}$ as defined in step 2.

Step 4: Now, reduce the given problem to a new problem by replacing s-jobs by job block α with the processing times $A_{\alpha 1}$ and $A_{\alpha 2}$ and remaining r (= n−s)-jobs by a disjoint job block β_k with processing times $A_{\beta_k 1}$ and $A_{\beta_k 2}$ as defined in step 2. The new reduced problem can be represented as:

Table: 2

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
α	$A_{\alpha 1}$	$A_{\alpha 2}$
β_k	$A_{\beta_k 1}$	$A_{\beta_k 2}$

Step 5: Check the structural conditions that $A_{i1} \geq A_{j2}$ or $A_{i1} \leq A_{j2}$ for each i and j. If the structural conditions hold good go to Step 6 to obtain β_k and follow step 7 to find S′ otherwise modify the problem.

Step 6: Obtain the new job block β_k having jobs in an optimal order from the job block β (disjoint from job block α) by treating job block β as sub-flow shop scheduling problem of the main problem. For finding β_k follow the following steps:

(A): Obtain the job J_1 (say) having maximum processing time on 1st machine and job J_r (say) having minimum processing time on 2nd machine. If $J_1 \neq J_r$ then put J_1 on the first position and J_r at the last position and go to 6 (D) otherwise go to 6 (B).

(B): Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on machine M_1. Call this difference as G_1. Also take the difference of processing time of job J_r on machine M_2 from job J_{r-1} (say) having next minimum processing time on M_2. Call this difference as G_2. Now follow step 6(C).

(C): If $G_1 \leq G_2$ then put J_r on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{r-1} on the last position. Now go to step 6(D).

(D): Arrange the remaining (r-2) jobs, if any between 1st job J_1or J_2 & last job J_r or J_{r-1} in any order; thereby due to structural conditions we get the job blocks $\beta_1, \beta_2 \ldots \beta_m$ of jobs each having same elapsed time when treated as sub-flow shop scheduling problem of the main problem. Let $\beta_k = \beta_1$ (say).

Obtain the processing times $A_{\beta_k 1}$ and $A_{\beta_k 2}$ for the job block β_k as defined in step 2.

Step 7: For finding optimal string S′ follow the following steps:

(α) Obtain the job I_1 (say) having maximum processing time on 1st machine and job I'_1 (say) having minimum processing time on 2nd machine. If $I_1 \neq I'_1$ then put I_1 at the first position and I'_1 at the second position to obtain S′ otherwise go to step 7 (b).

(b): Take the difference of processing time of job I_1 on M_1 from job I_2 (say) having next maximum processing time on machine M_1. Call this difference as H_1. Also take the difference of processing time of job I'_1 on machine M_2 from job I'_2 (say) having next minimum processing time on M_2. Call this difference as H_2. If $H_1 \leq H_2$ then put I'_1 on the second position and I_2 on the first position otherwise put I_1 on 1st position and I'_2 on the second position to obtain the optimal string S′.

Step 8: Compute the in - out table for sequence σ_k of jobs in the optimal string S′.

Step 9: Compute the total elapsed time T (σ_k)

Step 10: Calculate the utilization time U_2 of 2nd machine for σ_k, given by
$U_2(\sigma_k) = T(\sigma_k) - A_{11}(\sigma_k)$.

Numerical Illustration
Consider 5 jobs to be processed in a string S of disjoint job blocks on 2 machines as job block α = (3, 5) with fixed order of jobs and job block β = (1, 2, 4) with arbitrary order of jobs such that $\alpha \cap \beta = \emptyset$. The processing times with respective probabilities are given in the following table:

Table: 3

Jobs	Machine M_1		Machine M_2	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	18	0.2	9	0.1
2	26	0.1	8	0.3
3	12	0.2	7	0.2
4	20	0.2	8	0.2
5	17	0.3	3	0.2

Solution: As per step 1: The expected processing time for machines M_1 and M_2 are as follow:

Table: 4

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	3.6	0.9
2	2.6	2.4
3	2.4	1.4
4	4.0	1.6
5	5.1	0.6

As per step 2: Processing time $A_{\alpha 1}$ and $A_{\alpha 2}$ for the equivalent job block (3, 5) are calculated as:

$$A_{\alpha 1} = A_{r1} + A_{m1} - \min(A_{m1}, A_{r2}) \qquad \text{(Here r=3 \& m=5)}$$
$$= 2.4 + 5.1 - \min(5.1, 1.4)$$
$$= 7.5 - 1.4 = 6.1$$
$$A_{\alpha 2} = A_{r2} + A_{m2} - \min(A_{m1}, A_{r2})$$
$$= 1.4 + 0.6 - \min(5.1, 1.4)$$
$$= 2.0 - 1.4 = 0.6$$

As per step 3, 5 and 6: Since $A_{i1} \geq A_{j2}$ for each i and j for jobs in block β and so using step 6 we calculate β_k. We have, $\beta_k = (4, 2, 1)$.

Now, we know that the equivalent job for job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$ and so we have,

$$\beta_k = (4, 2, 1) = ((4, 2), 1) = (\alpha_1, 1), \text{ where } \alpha_1 = (4, 2)$$

Therefore,

$$A_{\alpha_1 1} = 4.0 + 2.6 - \min(2.6, 1.6) = 6.6 - 1.6 = 5.0.$$
$$A_{\alpha_1 2} = 1.6 + 2.4 - \min(2.6, 1.6) = 4.0 - 1.6 = 2.4.$$
$$A_{\beta_k 1} = 5.0 + 3.6 - \min(3.6, 2.4) = 8.6 - 2.4 = 6.2$$
$$A_{\beta_k 2} = 2.4 + 0.9 - \min(3.6, 2.4) = 3.3 - 2.4 = 0.9$$

As per step 4: The reduced problem is defined below:

Table: 5

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
α	6.1	0.6
β_k	6.2	0.9

Here $A_{i1} \geq A_{j2}$ for each i and j and thus the structural relations hold good.

As per step 7 the max $A_{i1} = 6.2$ is for job block β_k i.e. $I_1 = \beta_k$ and min $A_{i2} = 0.6$ is for job block α i.e. $I'_1 = \alpha$. Since $I_1 \neq I'_1$, so we put $I_1 = \beta_k$ on the first position and $I'_1 = \alpha$ on the last position. Therefore, the optimal string S′ as per step 7 is given by $S' = (\beta_k, \alpha)$.

Hence, the optimal sequence σ_k of jobs as per string S′ is $\sigma_k = 4 - 2 - 1 - 3 - 5$.
The in-out table for optimal sequence σ_k is:

Table: 6

Jobs	Machine M_1	Machine M_2
i	In-Out	In-Out
4	0.0 − 4.0	4.0 − 5.6
2	4.0 − 6.6	6.6 − 9.0
1	6.6 − 10.2	10.2 − 11.1
3	10.2 − 12.6	12.6 − 14.0
5	12.6 − 17.7	17.7 − 18.3

Therefore, the total elapsed time = T (σ_k) = 18.3 units.

Utilization time of machine M_2 = U_2 (S′) = (18.3 – 4.0) units.
= 14.3 units.

Remarks

If we solve the same problem by Johnson's [1] method by treating job block β (disjoint from job block α) as sub flow shop scheduling problem of the main problem we get the new job block β′ as:

β′ = (2, 4, 1)

The processing time $A_{\beta'1}$ and $A_{\beta'2}$ for the job block β′ on the guidelines of Maggu and Das [6] are calculated below.

We have, β′ = (2, 4, 1).

Now, β′ = (2, 4, 1) = ((2, 4), 1) = (α′, 1), where α′ = (2, 4).

$A_{\alpha'1}$ = 2.6 + 4.0 – min (4.0, 2.4) = 6.6 – 2.4 = 4.2.
$A_{\alpha'2}$ = 2.4 + 1.6 – min (4.0, 2.4) = 4.0 – 2.4 = 1.6.
$A_{\beta'1}$ = 4.2 + 3.6 – min (3.6, 1.6) = 7.8 – 1.6 = 6.2.
$A_{\beta'2}$ = 1.6 + 0.9 – min (3.6, 1.6) = 2.5 – 1.6 = 0.9.

The reduced problem is defined below:

Table: 7

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
α	6.1	0.6
β′	6.2	0.9

By Johnson's [1] algorithm the optimal string S′ is given by S′ = (β′, α).

Therefore, the optimal sequence σ for the original problem corresponding to optimal string S′ is given by σ = 2 – 4 – 1 – 3 – 5.

The in – out flow table for the optimal sequence σ is:

Table: 8

Jobs	**Machine M_1**	**Machine M_2**
i	In - Out	In - Out
2	0.0 − 2.6	2.6 − 5.0
4	2.6 − 6.6	6.6 − 8.2
1	6.6 − 10.2	10.2 − 11.1
3	10.2 − 12.6	12.6 − 14.0
5	12.6 − 17.7	17.7 − 18.3

Therefore, the total elapsed time = T (σ) = 18.3 units.

Utilization time of machine M_2 = U_2 (σ) = (18.3−2.6) units.
= 15.7 units.

CONCLUSION

The algorithm proposed here to minimize the utilization time for specially structured two stage flow shop scheduling problem with processing time associated with probabilities including jobs in a string of disjoint job blocks is more efficient as compared to the algorithm proposed by Johnson [1] for optimization of utilization time of machines. From table 8 we see that the utilization time of machine M_2 is U_2 (σ) = 15.7 units with makespan of 18.3 units. However, if the proposed algorithm is applied the utilization time of machine M_2 as per table 6 is U_2 (σ_k) = 14.3 with the same makespan of 18.3 units. Hence, the proposed algorithm is more efficient as it optimizes both the makespan and utilization time simultaneously.

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