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Advances in Applied Science Research, 2011, 2 (3): 536-543

ISSN: 0976-8610
CODEN (USA): AASRFC

On Some Topological Properties of Multigranular Rough Sets

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ABSTRACT

Rough set theory was introduced by Pawlak [5] as a model to capture impreciseness in data and since then it has been established to be a very efficient tool for this purpose. The definition of basic rough sets depends upon a single equivalence relation defined on the universe or several equivalence relations taken one each taken at a time. There have been several extensions of the basic rough sets introduced since then in the literature. Rough set model based on tolerance relation ([1]) one of several such extensions. In the view of granular computing, classical rough set theory is researched by a single granulation. The basic rough set model has been extended to rough set model based on multi-granulations (MGRS) in [10], where the set approximations are defined by using multi-equivalences on the universe and their properties were investigated. Topological properties of rough sets introduced by Pawlak in terms of their types was recently studied by Tripathy and Mitra [15] to find the types of the union and intersection of such sets and also complement of one such set. In this paper we extend these results to the multi granulation context. The rough set model based on tolerance relations was also extended to the multi granulation context in [10, 11, 12] by introducing incomplete rough set model based on multi-granulations. Since the basic properties of both types of rough sets based on multi granulation are identical, our findings are also true for both complete and incomplete rough set models based upon multi granulation.

Keywords: Rough sets, equivalence relations, tolerance relations, type of rough sets, multi granular rough sets

INTRODUCTION

The observation that most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character, which restricts their applicability in real life situations, led to the extension of the concept of crisp sets so as to model imprecise data and enhance their modeling power. One such approach to capture impreciseness is due to Pawlak [5, 6], who introduced the notion of Rough Sets, which is an excellent tool to capture impreciseness in data. The basic assumption of rough set theory is that human knowledge about a universe depends upon their capability to classify its objects. Classifications of a universe and equivalence relations defined on it are known to be interchangeable notions. So, for mathematical reasons equivalence relations were considered by Pawlak to define rough sets. A rough set is represented by a pair of crisp sets, called the lower approximation comprises of elements, which belong to it definitely and upper approximation, which comprises of elements, which are possibly in the set with respect to the available information.

To improve the modeling capability of basic rough sets several extensions have been made in different directions. One such extension is the rough sets based upon tolerance relations instead of equivalence relations. These rough sets are sometimes called incomplete rough set models. In the view of granular computing, classical rough set theory is researched by a single granulation. The basic rough set model has been extended to rough set model based on multi-granulations (MGRS) in [10], where the set approximations are defined by using multi-equivalences on the universe. Using similar concepts, that is taking multiple tolerance relations instead of multiple equivalence relations, incomplete rough set model based on multi-granulations was introduced in [11]. Several fundamental properties of these types of rough sets have been studied [10, 11, 12]. Employing the notions of lower and upper approximations of rough sets, an interesting characterization of rough sets has been made by Pawlak in [6], where he introduced the types (originally called kinds) of rough sets. There are two different ways of characterising rough sets; the accuracy coefficient and the topological characterisation introduced through the notion of types. As mentioned by Pawlak himself [6],in practical applications of rough sets we combine both types of information about the borderline region, that is, of the accuracy of measure as well as the information about the topological classification of the set under consideration. Keeping this in mind, Tripathy and Mitra[15] have studied the types of rough sets by finding out the types of union and intersection of rough sets of different types. In this paper, we extend these results to the multi granular context, which remain the same for both the basic and incomplete cases.

Definitions and Notations

Let *U* be a universe of discourse and *R* be an equivalence relation over *U*. By *U/R* we denote the family of all equivalence class of *R*, referred to as categories or concepts of *R* and the equivalence class of an element $x \in U$, is denoted by $[x]_R$. By a knowledge base, we understand a relational system $K = (U, P)$, where *U* is as above and P is a family of equivalence relations over *U*. For any subset Q $(\neq \phi) \subseteq$ P, the intersection of all equivalence relations in Q is denoted by IND(Q) and is called the indiscernibility relation over Q. Given any $X \subseteq U$ and $R \in IND(K)$, we associate two subsets, $\underline{R}X = \bigcup\{Y \in U/R : Y \subseteq X\}$ and $\overline{R}X = \bigcup\{Y \in U/R : Y \cap X \neq \phi\}$, called the R-lower and R-upper approximations of X respectively. The R-boundary of *X* is denoted by $BN_R(X)$ and is given by $BN_R(X) = \overline{R}X - \underline{R}X$. The elements of $\underline{R}X$ are those elements of U, which can certainly be classified as elements of X, and the elements of $\overline{R}X$ are those elements of U, which can possibly be classified as elements of X, employing knowledge of *R*. We say that X is rough with respect to R if and only if $\underline{R}X \neq \overline{R}X$, equivalently $BN_R(X) \neq \phi$. X is said to be R-definable if and only if $\underline{R}X = \overline{R}X$, or $BN_R(X) = \phi$.

In the view of granular computing (proposed by L. A. Zadeh), an equivalence relation on the universe can be regarded as a granulation, and a partition on the universe can be regarded as a granulation space [2, 3]. For an incomplete information system, similarly, a tolerance relation on the universe can be regard as a granulation, and a cover induced by the relation can be regarded as a granulation space. Several measures in knowledge base closely associated with granular computing, such as knowledge granulation, granulation measure, information entropy and rough entropy, were discussed in [2-4]. On research of rough set method based on multi-granulations, Y. H. Qian and J. Y. Liang brought forward a rough set model based on multi-granulations [10], which is established by using multi equivalence relations. In [11] an extension of MGRS, rough set model based on multi tolerance relations in incomplete information systems.

Definition 2.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X \subseteq U$ and $P, Q \in$ **R.** We define the lower approximation and upper approximation of X in U as

(2.1) $\underline{P+Q}X = \cup\{x/[x]_P \subseteq X \ or\ [x]_P \subseteq Y\}$

and

(2.2) $\overline{P+Q}X = (\underline{P+Q}(X^C))^C$

Property 2.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X \subseteq U$ and $P, Q \in$ **R.** The following properties hold true.

(2.3) $\underline{P+Q}X \subseteq X \subseteq \overline{P+Q}X$

(2.4) $\underline{P+Q}\phi = \overline{P+Q}\phi = \phi$

(2.5) $\underline{P+Q}U = \overline{P+Q}U = U$

(2.6) $\underline{P+Q}(X^C) = (\overline{P+Q}X)^C$

(2.7) $\underline{P+Q}X = \underline{P}X \cup \underline{Q}X$

(2.8) $\overline{P+Q}X = \overline{P}X \cap \overline{Q}X$

Property 2.2: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X, Y \subseteq U$ and $P, Q \in$ **R.** The following properties hold true.

(2.9) $\underline{P+Q}(X \cap Y) \subseteq \underline{P+Q}(X) \cap \underline{P+Q}(Y)$

(2.10) $\underline{P+Q}(X \cup Y) \supseteq \underline{P+Q}(X) \cup \underline{P+Q}(Y)$

(2.11) $\overline{P+Q}(X \cap Y) \subseteq \overline{P+Q}(X) \cap \overline{P+Q}(Y)$

(2.12) $\overline{P+Q}(X \cup Y) \supseteq \overline{P+Q}(X) \cup \overline{P+Q}(Y)$

Next, we define MGRS in incomplete information systems.

Definition 2.2: An information system is a pair S = (U, A), where U is a non-empty finite set of objects, A is a non-empty finite set of attributes. For every $a \in A$, there is a mapping $a: U \to V_a$, where V_a is called the value set of a.

If V_a contains a null value for at least one attribute $a \in A$, then S is called an incomplete information system. Otherwise, it is complete.

Definition 2.3: Let S = $((U, A)$ be an incomplete information system, P $\subseteq$ A an attribute set. We define a binary relation on U as follows

(2.13) SIM(P) = {(u,v) $\in$ U X U | $\forall$ a $\in$ P, a(u) = a(v) or a(u) = * or a(v) = *}.

In fact, SIM(P) is a tolerance relation on U, the concept of a tolerance relation has a wide variety of applications in classifications [2-4].

It can be shown that SIM(P) = $\bigcap_{a\in P}$ SIM({A}).
Let S_p(u) denote the set {v $\in$ U| (u,v) $\in$ SIM(P)}. S_P(u) is the maximal set of objects which are possibly indistinguishable by P with u.

Let U/SIM(P) denote the family sets {S_p(u)| u $\in$ U}, the classification or the knowledge induced by P. A member S_p(u) from U/SIM(P) will be called a tolerance class or an information granule. It should be noticed that the tolerance classes in U/SIM(P) do not constitute a partition of U in general. They constitute a cover of U, i.e., S_p(u) $\neq \phi$ for every u $\in$ U, and $\bigcup_{u\in U} S_p$ (u) = U.

Definition 2.4: Let S = (U,A) be an incomplete information system, P,Q $\subseteq$ A two attribute subsets, X $\subseteq$U, we define a lower approximation of x and a upper approximation of x in U by the following

(2.14) $\underline{P+Q}$ X = $\bigcup$ {x | SIM_P(x) $\subseteq$ X or SIM_Q(x) $\subseteq$ X
and
(2.15) $\overline{P+Q}$(X) = ($\underline{P+Q}$ (X^c))^c

Definition 2.5: A Multi-granulation Rough Set can be classified into following four types
(2.16) If $\underline{P+Q}$(X) $\neq \phi$ and $\overline{P+Q} \neq \cup$, then we say that X is roughly P+Q-definable.
(2.17) If $\underline{P+Q}$(X) = ϕ and $\overline{P+Q} \neq \cup$, then we say that X is internally P+Q-undefinable.
(2.18). If $\underline{P+Q}$(X) $\neq \phi$ and $\overline{P+Q} = \cup$, then we say that X is externally P+Q -undefinable.
(2.19) If $\underline{P+Q}$(X) = ϕ and $\overline{P+Q} = \cup$, then we say that X is totally P+Q –undefinable.

RESULT

In this section we shall find out the types of multigranular rough sets. There are four sets of results accumulated in four tables. The first provides the type of a P+Q rough set from the types of its P and Q rough set types. The second table provides the types of the complement of a multi granular rough set. In the third table we obtain the types for the union of two multi granular rough sets of all possible types. Similarly we establish the types of the intersection of two multi granular rough sets of all possible types. These results will be useful for further studies in approximation of classifications and rule generation.

3.1 Table for type of X with respect to P+Q

	Type of X with respect to Q				
Type Of X With Respect To P		**T-1**	**T-2**	**T-3**	**T-4**
	T-1	**T-1**	**T-1**	**T-1**	**T-1**
	T-2	**T-1**	**T-2**	**T-1**	**T-2**
	T-3	**T-1**	**T-1**	**T-3**	**T-3**
	T-4	**T-1**	**T-2**	**T-3**	**T-4**

3.2 Table for type of X^C with respect to P+Q

X	X^C
T-1	**T-1**
T-2	**T-3**
T-3	**T-2**
T-4	**T-4**

3.3 Table for type of $X \cup Y$ with respect to P+Q

	Type of Y with respect to P+Q				
Type of X with respect to P+Q		**T-1**	**T-2**	**T-3**	**T-4**
	T-1	**T-1/ T-3**	**T-1/ T-3**	**T-3**	**T-3**
	T-2	**T-1/ T-3**	**T-1/ T-2/ T-3/ T-4**	**T-3**	**T-3/ T-4**
	T-3	**T-3**	**T-3**	**T-3**	**T-3**
	T-4	**T-3**	**T-3/ T-4**	**T-3**	**T-3/ T-4**

We shall provide an example to show that for two multigranular rough sets of type 1, the union can be of type 1 or type 3. The other cases can be justified in a similar manner.

Example 3.3

Let U = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$. We assume that

U/ P = $\{\{e_1, e_7\},\{e_2, e_3, e_4, e_5, e_6\},\{e_8\}\}$

U/ Q = $\{\{e_1, e_2\},\{e_3, e_4, e_5\},\{e_6, e_7, e_8\}\}$

Suppose, X = $\{e_1, e_2, e_6, e_8\}$ and Y = $\{e_1, e_2, e_7, e_8\}$. Then X and Y are both of type 1 as $\underline{P+Q}$(X) = $\{e_1, e_2, e_8\} \neq \phi$, $\underline{P+Q}$(Y) = $\{e_8\} \neq \phi$, $\overline{P+Q}$(X) = $\{e_1, e_2, e_6, e_7, e_8\} \neq$ U and $\overline{P+Q}$(Y) = $\{e_1, e_2, e_6, e_7, e_8\} \neq$ U. Now we have $\underline{P+Q}$(XUY) = $\{e_1, e_2, e_6, e_7, e_8\} \neq \phi$ and $\overline{P+Q}$(XUY) = $\{e_1, e_2, e_6, e_7, e_8\} \neq$ U. So, XUY is of type 1.

Next, we take X = $\{e_6, e_8\}$ and Y = $\{e_1, e_8\}$. So, X and Y are both of Type 1 as

$\underline{P+Q}$ **(X)** = {e_8} $\neq \phi$, $\underline{P+Q}$(Y) = {e_8} $\neq \phi$, $\overline{P+Q}$(X) = {e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8} = U and
$\overline{P+Q}$(Y) = {e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8} = U.
Now, $\underline{P+Q}$(XUY) = {e_8} $\neq \phi$ and $\overline{P+Q}$(XUY) = {e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8} = U. So XUY is of type 3.

Hence both the cases in the table position (1, 1) are possibilities.

Proof of entry (1, 3)
Let both X and Y be of Type 1 and Type 3. Then from the properties of type 1 and type 3

$\underline{P+Q}$ **(X)** $\neq \phi$, $\underline{P+Q}$(Y) $\neq \phi$, $\overline{P+Q}$(X) $\neq$ U and $\overline{P+Q}$(Y) = U. So, using (2.10) 1nd (2.12) we get
$\underline{P+Q}$(XUY) $\neq$ ϕ and $\overline{P+Q}$(XUY) = U. Hence XUY is of type 3 only.

3.4 Table for type of $X \cap Y$ with respect to P+Q

	Type of Y with respect to P+Q				
		T-1	T-2	T-3	T-4
	T-1	T-1/T-2	T-2	T-1/ T-2	T-2
Type of X with respect to P+Q	T-2	T-2	T-2	T-2	T-2
	T-3	T-1/T-2	T-2	T-1/ T-2/ T-3/ T-4	T-2/ T-4
	T-4	T-2	T-2	T-2/ T-4	T-2/ T-4

We shall provide an example to show that for two multigranular rough sets such that one is of type 1 and the other one is of type 3.The intersection can be of type 1 or type 2. The other cases can be justified in a similar manner.

Example 3.4
Let U = {e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8}. So that U/ P = {{ e_1, e_7 },{ e_2,e_3,e_4,e_5,e_6},{ e_8}} and U/ Q = {{ e_1,e_2 },{ e_3,e_4,e_5},{ e_6,e_7,e_8}}. Let us take X = {e_1,e_2,e_6,e_8} and Y = {e_1,e_2,e_7,e_8}. Then
X and Y are of Type 3 and Type 1 respectively. Now, $\underline{P+Q}$(X) = {e_1,e_2,e_8} $\neq \phi$ and $\underline{P+Q}$(Y) = {e_8} $\neq \phi$. Also, $\overline{P+Q}$(X) = {e_1, e_2, e_6, e_7,e_8} $\neq$ U and $\overline{P+Q}$(Y) = {e_1, e_2, e_6, e_7,e_8} $\neq$ U. So that we get $\underline{P+Q}$(X $\cap$Y) = {e_1,e_2,e_8} $\neq \phi$ and $\overline{P+Q}$(X $\cap$Y) = {e_1,e_2,e_6,e_7,e_8} $\neq$ U.
Hence $X \cap Y$ is of type-1.

Again taking X = { e_3,e_4 } and Y = { e_4,e_5}, we find that X and Y are of Type 3 and Type 1 respectively as $\underline{P+Q}$(X) = ϕ, $\underline{P+Q}$(Y) = ϕ, $\overline{P+Q}$(X) = {e_3,e_4,e_5} $\neq$ U and $\overline{P+Q}$(Y) = {e_3,e_4,e_5} $\neq$ U. Now, $\underline{P+Q}$(X $\cap$ Y) = ϕ and $\overline{P+Q}$(X $\cap$ Y) = {e_3,e_4,e_5} $\neq$ U. So, $X \cap Y$ is of type 2. Hence both the cases for intersection operation in position (3,1) can occur.

Proof of entry (2, 1)

Let X and Y be of Type 2 and Type 1 respectively. Then from the properties of type 2 and type 1 multi granular rough sets we get $\underline{P+Q}$(X) = ϕ, $\underline{P+Q}$(Y) = ϕ, $\overline{P+Q}$(X) $\neq$ U and $\overline{P+Q}$(Y) $\neq$ U.

So using properties (2.9) and (2.11) we get $\underline{P+Q}$(X $\cap$ Y) = ϕ and $\overline{P+Q}$(X $\cap$ Y) $\neq$ U. So, $X \cap Y$ is of type 2. This completes the proof. The other cases can be established similarly.

CONCLUSION

In this paper we studied the topological properties of multi granular rough sets with respect to the three set theoretic operations of union, intersection and complementation. The tables show that there are multiple answers to some of the cases as like as the case of basic rough sets. Also, we provided examples in some cases to illustrate the fact that the multiple answers can actually occur. These results can be used in approximation of classifications and rule induction

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