Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2013, 4(6):142-149

ISSN: 0976-8610
CODEN (USA): AASRFC

On some fixed point theorems in complete 2-metric spaces

Rajendra Kumar Dubey[2], Rajesh Shrivastava[1] and Pankaj Tiwari[1]

[1]*Department of Mathematics, Govt. Science & Commerce College Benazir Bhopal (M.P.), India*
[2]*Department of Mathematics, Govt. Science P. G. College Rewa (M.P.), India*

ABSTRACT
In the present paper we prove some common fixed theorems for four self mappings in complete 2- metric spaces. This theorem is a version of many fixed point theorems in complete metric spaces, given by many authors announced in the literature.

Keywords: *Fixed point, weakly compatible mappings, 2-metric space.*
Mathematics Subject classification (2010): *Primary 47H10; Secondary 54H25*

INTRODUCTION

The concept of 2-metric space is a natural generalization of the metric space. Initially, it has been investigated by Gähler [3] and has been developed broadly by Gähler [4, 5] and more. After this number of fixed point theorems have been proved for 2-metric spaces by introducing compatible mappings, which are more general than commuting and weakly commuting mappings. Jungck and Rhoades defined the concepts of d-compatible and weakly compatible mappings as extensions of the concept of compatible mapping for single-valued mappings on metric spaces. Several authors used these concepts to prove some common fixed point theorems. Iseki [8, 9] is well-known in this literature which also include Cho et. al., [1,2], Imdad et.al.[10], Murthy et.al.[12], Naidu and Prasad [13], Pathak et.al. [14]. Vishal Gupta et. al. [6] also prove some common fixed point theorems for a class of A-contraction on 2- metric space. Various authors [15, 16, 17] used the concepts of weakly commuting mappings, compatible mappings of type (A) and (P) and weakly compatible mappings of type (A) to prove fixed point theorems in 2-metric space. Commutability of two mappings was weakened by Sessa [15] with weakly commuting mappings. Jungck extended the class of non-commuting mappings by compatible mappings.

In this paper, we establish common fixed point theorems for four self mapping in complete 2- metric spaces.

Definition 2.1: A sequence $\{x_n\}$ said to be a Cauchy sequence in 2-metric space X, if for each $a \in X$,
$$\lim_{(m,n\to\infty)} d(x_n, x, a) = 0$$
Definition 2.2: A sequence $\{x_n\}$ in 2-metric space X is convergent to an element $x \in X$ if for each $a \in X$,
$$\lim_{(n\to\infty)} d(x_n, x, a) = 0$$
Definition 2.3: A complete 2-metric space is one in which every Cauchy sequence in X converges to an element of X.
Definition 2.4: Let A and S be mappings from a metric space (X, d) in to itself, A and S are said to be weakly compatible if they commute at their coincidence point.
i.e., $Ax = Sx$ for some $x \in X$, then $ASx = SAx$.
Definition 2.5: Two self maps f and g of a metric space (X, d) are called compatible if

$$\lim_{(n\to\infty)} d(fgx_n, gfx_n) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{(n\to\infty)} fx_n = \lim_{(n\to\infty)} gx_n = t$$

for some t in X.

Definition 2.6: Two self maps f and g of a metric space (X, d) are called non compatible if there exists at least one sequence $\{x_n\}$ such that

$$\lim_{(n\to\infty)} fx_n = \lim_{(n\to\infty)} gx_n = t$$

for some t in X but

$$\lim_{(n\to\infty)} d(fgx_n, gfx_n)$$

is either non zero or nonexistent.

Definition2.7: Maps f and g are said to be commuting if $fgx = gfx$ for all $x \in X$

Definition2.8: Let f and g be two self maps on a set X, if $fx = gx$ for some x in X then x is called coincidence point of f and g.

Throughout this paper X is stand for complete 2-metric space.

RESULTS

Theorem 2.1: Let S, T be any two self mappings of a 2- metric space X satisfying the condition

$$d(Su, Tv, a) \leq \alpha_1 \left[\frac{d^2(u, Sw, a) + d^2(u, v, a)}{1 + d(u, Sw, a) + d(u, v, a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(v,Tt,a)+d^2(Sw,Tt,a)}{1+d(v,Tt,a)+d(Sw,Tt,a)}\right]$$

$$+\alpha_3\sqrt{d(v, Sw, a).d(u, Tt, a)}$$

$$+\alpha_4[d(u, v, a)] \quad (1)$$

for all $u, v, w, t \in X$ *where* $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ *are non negative reals such that* $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 < 1$*, then* S, T *have a unique common fixed point.*

Proof: *Let* x_0 *be an arbitrary element of* X *and we construct a sequence* $\{x_n\}$ *defined as follows*

$$Sx_{n-1} = x_n, Tx_n = x_{n+1}, Sx_{n+1} = x_{n+2}, Tx_{n+2} = x_{n+3}$$

and $TSx_{n-1} = x_{n+1}, STx_n = x_{n+2}, TSx_{n+1} = x_{n+3}, STx_{n+2} = x_{n+4}$

where $n = 1, 2, 3, \ldots$

Now putting $u = Ty, v = Sx, w = x$ *and* $t = y$ *in* (1) *then we have*

$$d(STy, TSx, a) \leq \alpha_1 \left[\frac{d^2(Ty,Sx,a)+d^2(Ty,Sx,a)}{1+d(Ty,Sx,a)+d(Ty,Sx,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(Sx,Ty,a)+d^2(Sx,Ty,a)}{1+d(Sx,Ty,a)+d(Sx,Ty,a)}\right]$$

$$+\alpha_3\sqrt{d(Sx, Sx, a).d(Ty, Ty, a)}$$

$$+\alpha_4[d(Ty, Sx, a)]$$

$$d(STy, TSx, a) \leq 2\alpha_1 d(Sx, Ty, a)$$

$$+2\alpha_2 d(Sx, Ty, a)$$

$$+\alpha_4 d(Sx, Ty, a). \tag{2}$$

Now putting $x = x_{n-1}$ *and* $y = x_n$ *in* (2) *then we have*

$$d(STx_n, TSx_{n-1}, a) \leq 2\alpha_1 d(Sx_{n-1}, Tx_n, a)$$
$$+2\alpha_2 d(Sx_{n-1}, Tx_n, a)$$
$$+\alpha_4 d(Sx_{n-1}, Tx_n, a)$$
$$d(x_{n+2}, x_{n+1}, a) \leq 2\alpha_1 d(x_n, x_{n+1}, a)$$
$$+2\alpha_2 d(x_n, x_{n+1}, a)$$
$$+\alpha_4 d(x_n, x_{n+1}, a) \tag{3}$$

from (3) *we conclude that* $d(x_{n-1}, x_n, a)$ *decreases with* n*.*

i.e., $d(x_{n-1}, x_n, a) \to d(x_0, x_1, a)$ *when* $n \to \infty$

If possible let $d(x_0, x_1, a) > 0$ *and taking limit* $n \to \infty$ *on* (3) *then we have*

$$d(x_0, x_1, a) \leq 2\alpha_1 d(x_0, x_1, a) + 2\alpha_2 d(x_0, x_1, a) + \alpha_4 d(x_0, x_1, a)$$
$$= (2\alpha_1 + 2\alpha_2 + \alpha_4) d(x_0, x_1, a)$$
$$< d(x_0, x_1, a)$$

Since $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 < 1$*. Which is not possible hence*

$$d(x_0, x_1, a) = 0\ .$$

Next we shall show that $\{x_n\}$ *is Cauchy sequence.*

Now

$$d(x_m, x_n, a) \leq d(x_m, x_{m+1}, a) + d(x_{m+1}, x_{n+1}, a) + d(x_{n+1}, x_n, a)$$
$$d(x_m, x_n, a) \leq d(x_m, x_{m+1}, a)$$
$$+d(x_n, x_{n+1}, a) + d(Sx_n, Tx_m, a) \tag{4}$$

On putting $u = x_n, v = x_m, w = x_{m-1}, t = x_{n-1}$ *in* (1) *then we have*

$$d(Sx_n, Tx_m, a) \leq \alpha_1 \left[\frac{d^2(x_n, Sx_{m-1}, a) + d^2(x_n, x_m, a)}{1 + d(x_n, Sx_{m-1}, a) + d(x_n, x_m, a)}\right]$$
$$+\alpha_2 \left[\frac{d^2(x_m, Tx_{n-1}, a) + d^2(Sx_{m-1}, Tx_{n-1}, a)}{1 + d(x_m, Tx_{n-1}, a) + d(Sx_{m-1}, Tx_{n-1}, a)}\right]$$
$$+\alpha_3 \sqrt{d(x_m, Sx_{m-1}, a).\, d(x_n, Tx_{n-1}, a)}$$
$$+\ \alpha_4 [d(x_n, x_m, a)]$$
$$= \alpha_1 \left[\frac{d^2(x_n, x_m, a) + d^2(x_n, x_m, a)}{1 + d(x_n, x_m, a) + d(x_n, x_m, a)}\right]$$
$$+\alpha_2 \left[\frac{d(x_m, x_n, a) + d(x_m, x_n, a)}{1 + d^2(x_m, x_n, a) + d^2(x_m, x_n, a)}\right]$$

$$+\alpha_3\sqrt{d(x_m,x_m,a).d(x_n,x_n,a)}+\alpha_4[d(x_n,x_m,a)]$$

$$=2\alpha_1 d(x_n,x_m,a)+2\alpha_2 d(x_n,x_m,a)+\alpha_4 d(x_n,x_m,a)$$

$$d(Sx_n,Tx_m,a)\le(2\alpha_1+2\alpha_2+\alpha_4)d(x_n,x_m,a) \qquad (5)$$

from (4) *and* (5) *we have*

$$d(x_m,x_n,a)\le d(x_m,x_{m+1},a)+d(x_n,x_{n+1},a)$$

$$+(2\alpha_1+2\alpha_2+\alpha_4)d(x_n,x_m,a)$$

Letting $m,n\to\infty$ *then* $d(x_n,x_m,a)\to 0$, *as* $2\alpha_1+2\alpha_2+\alpha_4<1$

Hence $\{x_n\}$ *is a Cauchy sequence.*

Now we prove z *is a common fixed point of* S,T.

On putting $u=z, v=x_{n-1}, w=z$ *and* $t=x_{n-2}$ *in* (1) *we have*

$$d(Sz,Tx_{n-1},a)\le\alpha_1\left[\frac{d^2(z,Sz,a)+d^2(z,x_{n-1},a)}{1+d(z,Sz,a)+d(z,x_{n-1},a)}\right]$$

$$+\alpha_2\left[\frac{d^2(x_{n-1},Tx_{n-2},a)+d^2(Sz,Tx_{n-2},a)}{1+d(x_{n-1},Tx_{n-2},a)+d(Sz,Tx_{n-2},a)}\right]$$

$$+\alpha_3\sqrt{d(x_{n-1},Sz,a).d(z,Tx_{n-2},a)}$$

$$+\alpha_4[d(z,x_{n-1},a)]$$

$$d(Sz,x_n,a)\le\alpha_1\left[\frac{d^2(z,Sz,a)+d^2(z,x_{n-1},a)}{1+d(z,Sz,a)+d(z,x_{n-1},a)}\right]$$

$$+\alpha_2\left[\frac{d^2(x_{n-1},x_{n-1},a)+d^2(Sz,x_{n-1},a)}{1+d(x_{n-1},x_{n-1},a)+d(Sz,x_{n-1},a)}\right]$$

$$+\alpha_3\sqrt{d(x_{n-1},Sz,a).d(z,x_{n-1},a)}+\alpha_4[d(z,x_{n-1},a)].$$

Letting $n\to\infty$ *then we have*

$$d(Sz,z,a)\le\alpha_1\left[\frac{d^2(z,Sz,a)+d^2(z,z,a)}{1+d(z,Sz,a)+d(z,z,a)}\right]$$

$$+\alpha_2\left[\frac{d^2(z,z,a)+d^2(Sz,z,a)}{1+d(z,z,a)+d(Sz,z,a)}\right]$$

$$+\alpha_3\sqrt{d(z,Sz,a).d(z,z,a)}+\alpha_4[d(z,z,a)]$$

$$\Rightarrow \qquad d(Sz,z,a)\le(\alpha_1+\alpha_2)d(Sz,z,a).$$

$\Rightarrow d(Sz,z,a)<d(Sz,z,a)$ *Since* $2\alpha_1+2\alpha_2+\alpha_3+\alpha_4<1$.

Which gives $d(Sz,z,a)=0\Rightarrow Sz=z$. *Thus* z *is a fixed point of* S.

Similarly we can show that z *is a fixed point of* T. *Hence* z *is a common fixed point of* S,T.

We are taking one another point q *which is not equal to* z *such that* $Sq=q=Tq$.

On putting $u=z, v=q, w=q, t=z$ *in* (1) *then we have*

$$d(Sz, Tq, a) \le \alpha_1 \left[\frac{d^2(z,Sq,a)+d^2(z,q,a)}{1+d(z,Sq,a)+d(z,q,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(q,Tz,a)+d^2(Sq,Tz,a)}{1+d(q,Tz,a)+d(Sq,Tz,a)}\right]$$

$$+\alpha_3 \sqrt{d(q, Sq, a).d(z, Tz, a)}$$

$$+\alpha_4[d(z, q, a)]$$

$$d(z, q, a) \le \alpha_1 \left[\frac{d^2(z,q,a)+d^2(z,q,a)}{1+d(z,q,a)+d(z,q,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(q,z,a)+d^2(q,z,a)}{1+d(q,z,a)+d(q,z,a)}\right]$$

$$+\alpha_3 \sqrt{d(q, q, a).d(z, z, a)} + \alpha_4[d(z, q, a)]$$

$$d(z, q, a) \le (2\alpha_1 + 2\alpha_2 + \alpha_4)d(z, q, a)$$

$$d(z, q, a) < d(z, q, a)$$

Since $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 < 1$. *Which gives* $d(z, q, a) = 0 \Rightarrow z = q$.

Hence z *is unique. This completes the proof of the theorem.*

Theorem 2.2: *Let* S, T, R *be any three self mappings of a 2- metric space* X *satisfying the condition*

$$d(SRu, TRv, a) \le \alpha_1 \left[\frac{d^2(u,SRw,a)+d^2(u,TRt,a)+d^2(u,SRw,a)}{1+d(u,SRw,a)+d(u,TRt,a)+d(u,SRw,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(v,SRw,a)+d^2(u,TRt,a)+d^2(v,TRt,a)}{1+d(v,SRw,a)+d(u,TRt,a)+d(v,TRt,a)}\right]$$

$$+\alpha_3 \sqrt{d(v, SRw, a)d(u, TRt, a)}$$

$$+\alpha_4[d(SRw, TRt, a)] + \alpha_5[d(u, v, a)]. \quad (6)$$

for $u, v, w, t \in X$ *where* $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ *are non negative reals such that* $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1$ *then* SR, TR *have a unique common fixed point.*

Proof: *Let* x_0 *be an arbitrary element of* X *and we construct a sequence* $\{x_n\}$ *defined as follows*

$$SRx_{n-1} = x_n, TRx_n = x_{n+1}, SRx_{n+1} = x_{n+2}, TRx_{n+2} = x_{n+3}$$

and $TRSRx_{n-1} = x_{n+1}, SRTRx_n = x_{n+2}, TRSRx_{n+1} = x_{n+3}, SRTRx_{n+2} = x_{n+4}$.

where $n = 1, 2, 3, \ldots$

Now putting $u = TRy, v = SRx, w = x$ *and* $t = y$ *in* (6) *then we have*

$$d(SRTRy, TRSRx, a) \le \alpha_1 \left[\frac{d^2(TRy,SRx,a)+d^2(TRy,TRy,a)+d^2(TRy,SRx,a)}{1+d(TRy,SRx,a)+d(TRy,TRy,a)+d(TRy,SRx,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(SRx,SRx,a)+d^2(TRy,TRy,a)+d^2(SRx,TRy,a)}{1+d(SRx,SRx,a)+d(TRy,TRy,a)+d(SRx,TRy,a)}\right]$$

$$+\alpha_3 \sqrt{d(SRx, SRx, a).d(TRy, TRy, a)}$$

$$+\alpha_4[d(SRx, TRy, a)] + \alpha_5[d(TRy, SRx, a)]$$

$$= \alpha_1 d(SRx, TRy, a) + \alpha_2 d(SRx, TRy, a)$$

$$+\alpha_4 d(SRx, TRy, a) + \alpha_5 d(SRx, TRy, a)$$

$$d(SRTRy, TRSRx, a) \leq (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5) d(SRx, TRy, a). \quad (7)$$

Now putting $x = x_{n-1}$ *and* $y = x_n$ *in* (7) *then we have*

$$d(SRTRx_n, TRSRx_{n-1}, a) \leq (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5) d(SRx_{n-1}, TRx_n, a)$$

$$d(x_{n+2}, x_{n+1}, a) \leq (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5) d(x_n, x_{n+1}, a). \quad (8)$$

from (8) *we conclude that* $d(x_{n-1}, x_n)$ *decreases with* n

i.e., $\quad d(x_{n-1}, x_n, a) \to d(x_0, x_1, a)$ *when* $n \to \infty$.

If possible let $d(x_0, x_1) > 0$ *and Taking limit* $n \to \infty$ *on* (8) *then we have*

$$d(x_0, x_1, a) \leq (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5) d(x_0, x_1, a)$$

$$d(x_0, x_1, a) < d(x_0, x_1, a).$$

Since $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1. \Rightarrow d(x_0, x_1, a) = 0.$

Next we shall show that $\{x_n\}$ *is Cauchy sequence.*

Now

$$d(x_m, x_n, a) \leq d(x_m, x_{m+1}, a) + d(x_{m+1}, x_{n+1}, a) + d(x_{n+1}, x_n, a)$$

$$d(x_m, x_n, a) \leq d(x_m, x_{m+1}, a) + d(x_n, x_{n+1}, a) + d(SRx_n, TRx_m, a) \quad (9)$$

On putting $u = x_n, v = x_m, w = x_{m-1}, t = x_{n-1}$ *in* (6) *then we have*

$$d(Sx_n, Tx_m, a) \leq \alpha_1 \left[\frac{d^2(x_n, SRx_{m-1}, a) + d^2(x_n, TRx_{n-1}, a) + d^2(x_n, SRx_{m-1}, a)}{1 + d(x_n, SRx_{m-1}, a) + d(x_n, TRx_{n-1}, a) + d(x_n, SRx_{m-1}, a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_m, SRx_{m-1}, a) + d^2(x_n, TRx_{n-1}, a) + d^2(x_m, TRx_{n-1}, a)}{1 + d(x_m, SRx_{m-1}, a) + d(x_n, TRx_{n-1}, a) + d(x_m, TRx_{n-1}, a)}\right]$$

$$+\alpha_3 \sqrt{d(x_m, SRx_{m-1}, a). d(x_n, TRx_{n-1}, a)}$$

$$+\alpha_4 [d(SRx_{m-1}, TRx_{n-1}, a)]$$

$$= \alpha_1 \left[\frac{d^2(x_n, x_m, a) + d^2(x_n, x_n, a) + d^2(x_n, x_m, a)}{1 + d(x_n, x_m, a) + d(x_n, x_n, a) + d(x_n, x_m, a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_m, x_m, a) + d^2(x_n, x_n, a) + d^2(x_m, x_n, a)}{1 + d(x_m, x_m, a) + d(x_n, x_n, a) + d(x_m, x_n, a)}\right]$$

$$+\alpha_3 \sqrt{d(x_m, x_m, a). d(x_n, x_n, a)}$$

$$+\alpha_4 [d(x_m, x_n, a)] + \alpha_5 [d(x_n, x_m, a)]$$

$$= \alpha_1 d(x_n, x_m, a) + \alpha_2 d(x_n, x_m, a)$$

$$+\alpha_4 d(x_m, x_n, a) + \alpha_5 d(x_n, x_m, a)$$

$$\Rightarrow \quad d(Sx_n, Tx_m, a) \leq (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5) d(x_n, x_m, a) \quad (10)$$

from (9) *and* (10) *we have*

$$d(x_m, x_n, a) \leq d(x_m, x_{m+1}, a) + d(x_n, x_{n+1}, a)$$

$$+(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5)d(x_m, x_n, a)$$

Letting $m, n \to \infty$ *then* $d(x_n, x_m, a) \to 0$ *as* $\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 < 1$.

Hence $\{x_n\}$ *is a Cauchy sequence.*

Now we prove z *is a common fixed point of* SR, TR.

On putting $u = z, v = x_{n-1}, w = z$ *and* $t = x_{n-2}$ *in* (6) *we have*

$$d(SRz, TRx_{n-1}, a) \leq \alpha_1 \left[\frac{d^2(z,SRz,a)+d^2(z,TRx_{n-2},a)+d^2(z,SRz,a)}{1+d(z,SRz,a)+d(z,TRx_{n-2},a)+d(z,SRz,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_{n-1},SRz,a)+d^2(z,TRx_{n-2},a)+d^2(x_{n-1},TRx_{n-2},a)}{1+d(x_{n-1},SRz,a)+d(z,TRx_{n-2},a)+d(x_{n-1},TRx_{n-2},a)}\right]$$

$$+\alpha_3 \sqrt{d(x_{n-1}, SRz, a).d(z, TRx_{n-2}, a)}$$

$$+\alpha_4[d(SRz, TRx_{n-2}, a)] + \alpha_5[d(z, x_{n-1}, a)].$$

Letting $n \to \infty$ *then we have*

$$d(SRz, z, a) \leq \alpha_1 \left[\frac{d^2(z,SRz,a)+d^2(z,z,a)+d^2(z,SRz,a)}{1+d(z,SRz,a)+d(z,z,a)+d(z,SRz,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(z,SRz,a)+d^2(z,z,a)+d^2(z,z,a)}{1+d(z,SRz,a)+d(z,z,a)+d(z,z,a)}\right]$$

$$+\alpha_3 \sqrt{d(z, SRz, a).d(z, z, a)}$$

$$+\alpha_4[d(SRz, z, a)] + \alpha_5[d(z, z, a)]$$

$$d(SRz, z, a) \leq (\alpha_1 + \alpha_3 + \alpha_4)d(SRz, z, a)$$

$$d(SRz, z, a) < d(SRz, z, a)$$

Since $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1$.

Which gives $d(SRz, z, a) = 0 \Rightarrow SRz = z$. *Thus* z *is a fixed point of* SR.

Similarly we can show that z *is a fixed point of* TR.

Hence z *is a common fixed point of* SR, TR.

Now we are taking one another point q *which is not equal to* z *such that*

$$SRq = q = TRq$$

On putting $u = z, v = q, w = q, t = z$ *in* (6) *then we have*

$$d(SRz, TRq, a) \leq \alpha_1 \left[\frac{d^2(z,SRq,a)+d^2(z,TRz,a)+d^2(z,SRq,a)}{1+d(z,SRq,a)+d(z,TRz,a)+d(z,SRq,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(q,Sq,a)+d^2(q,Tz,a)+d^2(Sq,Tz,a)}{1+d(q,Sq,a)+d(q,Tz,a)+d(Sq,Tz,a)}\right]$$

$$+\alpha_3 \sqrt{d(q,SRq,a).d(z,TRz,a)}$$

$$+\alpha_4[d(SRq,TRz,a)] + \alpha_5[d(z,q,a)]$$

$$d(z,q,a) \leq \alpha_1 \left[\frac{d^2(z,q,a)+d^2(z,z,a)+d^2(z,q,a)}{1+d(z,q,a)+d(z,z,a)+d(z,q,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(q,q,a)+d^2(z,z,a)+d^2(q,z,a)}{1+d(q,q,a)+d(z,z,a)+d(q,z,a)}\right]$$

$$+\alpha_3 \sqrt{d(q,q,a).d(z,z,a)}$$

$$+\alpha_4[d(q,z,a)] + \alpha_5[d(z,q,a)]$$

$$d(z,q,a) \leq (\alpha_1+\alpha_2+\alpha_4+\alpha_5)d(z,q,a)$$

$$d(z,q,a) < d(z,q,a)$$

Since $\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5 < 1$. *Which gives* $d(z,q,a) = 0 \Rightarrow z = q$.

Hence z is unique.

This completes the proof of theorem.

CONCLUSION

In this present article we prove some common fixed point theorem satisfying new rational contractive conditions in 2- metric spaces. In fact our main result is more general then other previous known results.

Acknowledgement
The authors thank the referees for their careful reading of the manuscript and for their valuable suggestions.

REFERENCES

[1] Cho Y.J, *Math. Japonica*, **1993**, 38(3), 497-508.
[2] Cho Y.J, Khan M.S, Singh S.L, Common fixed points of weakly commutating Mappings, *univ. Novom Sadu, Sb. Rd. Prirod-mat. Fak.Ser.Mat*., **1988**, 18(1), 129-142.
[3] Gahler S, *Math.Nachr*, **1963**, 26, 115-148.
[4] Gahler S, *Math.Nachr*, **1965**, 28, 235-244.
[5] Gahler S, Zur *Math. Pures*, **1966**, Appl.11, 665-667.
[6] Gupta V, Kour R, *International Journal of pure and applied mathematics*, **2012**, 79(1).
[7] Gupta V, *Advances in Applied Science Research*, **2012**, 3 (5), 2733-2738.
[8] Iseki K, *Math. Sem. Notes, Kobeuni*, **1975**, 3(1) 133-136.
[9] Iseki K, Sharma P.L, Sharma B.K, *Math. Japonica*, **1976**, 21, 67-70.
[10] Imdad M, Kumar M.S, Khan M.D, *Math., Japonica*, **1991**, 36(5), 907-914.
[11] Jungck G, *Far East. J. Math. Sci*., **1996**, 4(2), 199-215.
[12] Muthuy P.P, Chang S.S, Cho Y.J, Sharma B.K, *Kyungpook Math*, J.,**1992**, 32(2), 203-216.
[13] Naidu S.V.R, Prasad J.R, *Indian J. Pure Appli.Math*. **1986**, 17(8), 974-993.
[14] Pathak H.K, Kang S.M, Back J.H, *Kyungbook Math.J.*,**1995**, 35, 345- 359.
[15] Sessa S, *Publ. Inst. Math. (Beograd)*, **1982**, 32(46), 146-153.
[16] Tan D, Liu Z, Kim J. K, *Nonlinear Funct. Anal. Appl*., **2003**, 8(2), 215-232.
[17] Wang W.Z, *Honam Math. J*., **2000**, 22, 91-97.