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On Magnetorotatory Thermosolutal Convection Problems of Veronis and Stern Type Configurations in the Presence of Dufour-effect

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ABSTRACT

The instability problem of magnetorotatory thermosolutal convection of the Veronis and Stern type is examined taking in to account the Dufour effect. Semi-circle theorems are derived, that prescribe upper limits for complex growth rate of oscillatory motions of neutral or growing amplitude in such a manner that it naturally culminates in sufficient conditions precluding the non- existence of such motions for an initially bottom heavy as well as an initially top heavy configurations. Further, results for Dufour-driven thermosolutal convection problems with or without the individual effects of a rotation or magnetic field follow as a consequence.

Keywords: Dufour-driven thermosolutal convection; Rayliegh numbers; Lewis numbers; Prandtl numbers; Taylor number; Chandrasekhar number.
MSC 2000 No.: *76E06, 76E07, 76E99*

INTRODUCTION

The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects [1, 2]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, arises when the mass flux contains a term that depends upon the temperature gradient. The analogous effect that arises from a concentration gradient dependent term in the heat flux is called the Dufour effect. Although it is clear that the themosolutal and Soret-Dufour problems are quite closely related, their relationship has never been carefully elucidated. They are in fact, formally identical and this is done by means of a linear transformation that takes the equations and boundary conditions for the latter problem into those for the former. [3] have studied the comparison between experimental results and theoretical prediction using Flory's theory for binary liquid mixtures and have calculated u.s.velocity in mixture at temperature 303.15K of molecules of different assigned shapes and a good agreement between experiment and theory has been observed. The effects of flow parameters on the velocity field, temperature field and concentration distribution have been studied by [4] and results are presented graphically and discussed quantitatively on the problem of viscous dissipation effects on unsteady free convection and mass transfer flow past an accelerated vertical porous plate with suction. [5] have investigated the problem on hydromagnetic natural convection flow of an incompressible viscoelastic fluid between two infinite vertical moving and oscillating parallel plates.

Two fundamental configurations have been studied in the context of thermosolutal instability problem, the first one by [6] wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and the second one by [7] wherin the temperature gradient is destabilizing and the concentration gradient is stabilizing. The main results derived by Stern and Veronis for their respective configurations are that both allow the occurrence of a stationary pattern of motions or oscillatory motions of growing amplitude provided the destabilizing concentration gradient or the temperature gradient is sufficiently large. However, stationary pattern of motion is the preferred mode of setting instability in case of Stern' configuration whereas oscillatory motions of growing amplitude are preferred in Veronis configuration. Further, these results are independent of the initially gravitationally stable or unstable character of the two configurations. In view of the above discussion, thermosolutal configurations of Veronis and Stern types can therefore be further classified into the following two classes:

The first class in which thermosolutal instability manifests itself when the total density field is initially bottom heavy and

(i) The second class in which thermosolutal instability manifests itself when the total density field is initially top heavy.

[8] has derived a characterization theorem for Dufour-driven thermosolutal convection problem of Veroins type that disapproves the existence of oscillatory motions growing amplitude in an initially bottom heavy configuration if

(i) $\frac{\tau}{\sigma} \leq 1$ and

(ii) $R_S \leq \frac{27\pi^4\left(1+\frac{\tau}{\sigma}\right)(1-\tau)}{R_3\gamma B}$,where $\tau, \sigma, \gamma, B, R_3, and\ R_S$ respectively denote the Lewis number, the thermal Prandtl number, the Dufour number, the ratio of solute gradient to temperature gradient, and, the concentration Rayliegh number. The restriction (i) in the above results may be physically justifiable in certain situations, however, it is not mathematically palpable. Further, the sufficient character of condition (ii) coupled with the nature of their mathematical analysis strongly suggest the possibility of the derivation of an upper bound for the modulus of the complex growth rate of an arbitrary oscillatory perturbation which may be neutral or unstable that will automatically take care of condition (ii) and yield the above results without the restriction (i) and will also be uniformly applicable for an initially bottom heavy as well as an initially top heavy configurations.

Motivated by these considerations, the present paper investigates the combined effect of rotation and magnetic field on Dufour-driven thermosolutal convection problems of the Veronis and Stern type and derives semi -circle theorems that prescribe upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude in such a manner that. it naturally culminates in sufficient conditions precluding the non- existence of such motions for initially bottom heavy as well as top heavy configurations.. Further, results for Dufour-driven thermosolutal convection problems with or without the individual effects of a rotation or a magnetic field follow as a consequence.

2. Mathematical Formulation and Analysis

The relevant governing non-dimensional linearized perturbation equations of Dufour-driven thermosolutal convection of the Veronis' type in the presence of a uniform vertical rotation and magnetic field with slight change in notations are [8,9]

$$(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)w = R_T a^2\theta - R_S a^2\phi + TD\zeta - QD(D^2-a^2)h_z \ , \qquad (2.1)$$

$$(D^2-a^2-p)\theta + R_3\gamma(D^2-a^2)\phi = -w \ , \qquad (2.2)$$

$$[\tau(D^2-a^2)-p]\phi = -w, \qquad (2.3)$$

$$\left(D^2-a^2-\frac{p\sigma_1}{\sigma}\right)h_z = -Dw \ , \qquad (2.4)$$

$$\left(D^2-a^2-\frac{p}{\sigma}\right)\zeta = -Dw - QD\xi \ , \qquad (2.5)$$

and

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)\xi = -D\zeta\,, \tag{2.6}$$

where $R_T = \frac{g\alpha\beta d^4}{\kappa\upsilon}$, $\beta > 0$ and $R_s = \frac{g\alpha'\beta' d^4}{\kappa\upsilon}$, $\beta' > 0, R = \frac{\beta'}{\beta}$,

and $\gamma = \frac{D_{01}}{C_v \kappa}$, $\left(\frac{D_{01}}{C_v} \text{ is called the Dufour coefficient}\right)$,

with

$$w = 0 = \theta = \phi = h_z = Dw = \zeta = D\xi \text{ at } z = 0 \text{ and } z = 1. \tag{2.7}$$

(both boundaries rigid and perfectly conducting)

In the above equations z is the real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is the differentiation with respect to z, $a^2 > 0$ is a constant, $\sigma > 0$ is a constant, $\sigma_1 > 0$ is a constant, $\tau > 0$ is a constant, $\gamma > 0$ is a constant, R_T and R_S are positive constants, $T > 0$ is a constant, $Q > 0$ is a constant, $p = p_r + i\,p_i$ is a complex constant and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi(z) = \phi_r(z) + i\phi_i(z)$ $h_z(z) = h_{z_r} + ih_{z_i}$, $\zeta(z) = \zeta_r(z) + i\zeta_i(z)$ and $\xi(z) = \xi_r(z) + i\xi_i(z)$ are complex valued functions of real variable z. The meaning of symbols from the physical point of view are as follows: z is the vertical coordinate, $\frac{d}{dz}$ is the differentiation along the vertical direction, a^2 is the square of the wave number, σ is the Prandtl number, σ_1 is the magnetic Prandtl number, τ is the Lewis number, γ is here referred to as Dufour number, R_T is the thermal Rayleigh number, R_S is the concentration Rayleigh number, T is the Taylor number, Q is the Chandrasekhar number, p is the complex growth rate, w is the vertical velocity, θ is the temperature, ϕ is the concentration, h_z is the vertical magnetic field, ζ is the vertical vorticity, and ξ is the vertical current density.

The system of equations (2.1-2.6) together with the boundary conditions (2.7) constitute an eigen value problem for complex growth rate $p = p_r + ip_i$ for given values of the other parameters, namely, $a^2, \sigma, R_T, R_S, Q, T, \sigma_1 and\, \tau$ and a given state of the system is stable, neutral or unstable according as p_r is negative, zero or positive. Further,

(a) $p_i \ne 0$ *and* $p_r \ge 0$ describe oscillatory motions of neutral or growing amplitude;

(b) $R > 0$ *and* $R_S > 0$ *and either* $Q = 0 = T$ *or* $Q = 0$ *or* $T = 0$ respectively describe Dufour- driven Veronis thermosolutal configuration (DDVTC) or rotatory DDVTC or hydromagnetic DDVTC;

(c) $R_T < 0$ *and* $R_S < 0$ and either Q = 0= T or Q = 0 or T = 0 respectively describe Dufour- driven Stern thermosolutal configuration (DDSTC) or rotatory DDSTC or hydromagnetic DDSTC;

(d) $\Gamma = \frac{R_T}{R_S} \le 1 \left\{\hat{\Gamma} = \frac{|R_S|}{|R_T|} \le 1\right\}$ describes an initially bottom heavy DDVTC (DDSTC) and

(e) $\Gamma \ge 1 \left(\hat{\Gamma} \ge 1\right)$ describes an top heavy DDVTC (DDSTC).

Finally, if $p_r = 0 \Rightarrow p_i = 0, \forall a^{2,}$ then the principle of exchange of stabilities is valid, otherwise we have overstability at least when instability sets in as certain modes.

We now prove the following theorems:

Theorem 1 (A semi-circle theorem for Dufour-driven magnetorotatory Veronis thermosolutal convection)
If $(p, w, \theta, \phi, h_z, \zeta, \xi)$, $p = p_r + ip_i$, $p_r \geq 0$, $p_i \neq 0$, is a nontrivial solution of the equations (2.1-2.7),and $R_T > 0$, $R_S > 0$, $\gamma > 0$, $Q \geq 0$ *and* $T \geq 0$, then

$$|p| < \frac{\Gamma R_S B^2 R\gamma}{4\Pi^2(1+\delta)(1-\tau)}\sqrt{\lambda^2 - 1},$$

where $\lambda = \dfrac{4\Gamma R_S R\gamma}{(1-\tau)27\Pi^4(1+\delta)B^2}$, $\delta = \min\left\{\dfrac{\tau}{\sigma}, \dfrac{1}{\sigma_1}, 1\right\}$ and $\Gamma = \dfrac{R_T}{R_S}$.

Proof: Using the transformations

$$\left.\begin{aligned} \hat{\theta} &= \frac{1-\tau}{R\gamma}\theta + \phi \\ \hat{\phi} &= \phi \\ \tilde{w} &= w \\ \tilde{h}_z &= h_z \\ \hat{\zeta} &= \zeta \\ \text{and} \quad \tilde{\xi} &= \xi \end{aligned}\right\}, \tag{2.10}$$

equations (2.1)-(2.8) assume the following forms

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T' a^2\theta - R_s' a^2\phi + TD\zeta - QD(D^2 - a^2)h_z, \tag{2.11}$$

$$(D^2 - a^2 - p)\theta = -Bw \ , \tag{2.12}$$

$$\left(D^2 - a^2 - p/\tau\right)\phi = -\frac{w}{\tau} \ , \tag{2.13}$$

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)h_z = -Dw \ , \tag{2.14}$$

$$\left(D^2 - a^2 - \frac{p}{\sigma}\right)\zeta = -QD\xi - Dw \ , \tag{2.15}$$

and

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)\xi = -D\zeta \ , \tag{2.16}$$

with

$w = 0 = \theta = \phi = h_z = Dw = \zeta = D\xi$ at z = 0 and z = 1,

(both boundaries rigid and perfectly conducting) (2.17)

where $R_T' = \dfrac{R_T R\gamma}{(1-\tau)}(\tau < 1)$, $R_S' = R_S + R_T'$, $B = \dfrac{[1+(1-\tau)]}{R\gamma}$, and the sign '~' has been omitted for simplicity.

Multiplying equation (2.11) by w^* (* indicates complex conjugation) throughout, integrating the resulting equation over the vertical range of z and utilizing (2.12-2.16),we get

$$\int_0^1 w^*\left(D^2-a^2\right)\left(D^2-a^2-\frac{p}{\sigma}\right)wdz+\frac{R_T{}' a^2}{B}\int_0^1 \theta\left(D^2-a^2-p^*\right)\theta^* dz-\tau R_S{}' a^2\int_0^1 \phi\left(D^2-a^2-\frac{p^*}{\tau}\right)\phi^* dz$$
$$+Q\int_0^1 \left(D^2-a^2\right)h_z\left\{D^2-a^2-\frac{p^*\sigma_1}{\sigma}\right\}h_z^* dz$$
$$-T\int_0^1 \zeta\left(D^2-a^2-\frac{p^*}{\sigma}\right)\zeta^* dz-QT\int_0^1 \xi^*\left(D^2-a^2-\frac{p\sigma_1}{\sigma}\right)\xi\, dz \qquad =0$$
(2.18)

Integrating the various terms of equations (2.18) by parts for an appropriate number of times and making use of the boundary conditions (2.17)), it follows that

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz+\tau R_S{}' a^2\int_0^1\left(\left|D\phi\right|^2+a^2\left|\phi\right|^2\right)dz$$
$$+Q\int_0^1\left|\left(D^2-a^2\right)h_z\right|^2+QT\int_0^1\left(\left|D\xi\right|^2+a^2\left|\xi\right|^2\right)dz+T\int_0^1\left(\left|D\zeta\right|^2+a^2\left|\zeta\right|^2\right)dz-\frac{R_T{}' a^2}{B}\int_0^1\left(\left|D\theta\right|^2+a^2\left|\theta\right|^2\right)+R_S{}' a^2 p^*\int_0^1\left|\phi\right|^2 dz$$
$$+\frac{Qp^*\sigma_1}{\sigma}\int_0^1\left(\left|Dh_z\right|^2+a^2\left|h_z\right|^2\right)dz-\frac{R_T a^2 p^*}{B}\int_0^1\left|\theta\right|^2 dz+\frac{T\ p^*}{\sigma}\int_0^1\left|\zeta\right|^2 dz+\frac{QT\sigma_1 p}{\sigma}\int_0^1\left|\xi\right|^2 dz=0$$
(2.19)

Equating the real and imaginary parts of (2.19) to zero and canceling $(p_i \neq 0)$ throughout from the imaginary part, we get

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p_r}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz$$
$$+\tau\ R_S{}' a^2\int_0^1\left(\left|D\phi\right|^2+a^2\left|\phi\right|^2\right)dz+Q\int_0^1\left|(D^2-a^2)h_z\right|\,dz+QT\int_0^1\left(\left|D\xi\right|^2+a^2\left|\xi\right|^2\right)\,dz$$
$$+T\int_0^1\left(\left|D\zeta\right|^2+a^2\left|\zeta^2\right|\right)dz-\frac{R_T{}' a^2}{B}\int_0^1\left(\left|D\theta\right|^2+a^2\left|\theta\right|^2\right)dz+$$
$$+p_r\left[R_S{}' a^2\int_0^1\left|\phi\right|^2 dz+\frac{Q\sigma_1}{\sigma}\int_0^1\left(\left|Dh_z\right|^2+a^2\left|h_z\right|^2\right)dz-\frac{R_T{}' a^2}{B}\int_0^1\left|\theta\right|^2 dz+\frac{T}{\sigma}\int_0^1\left|\zeta\right|^2 dz+\frac{Q\sigma_1 T}{\sigma}\int_0^1\left|\xi\right|^2 dz\right]=0$$
(2.20)

and

$$\frac{1}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz-R_s{}'a^2\int_0^1|\phi|^2dz-\frac{Q\sigma_1}{\sigma}\int\left(|Dh_z|^2+a^2|h_z|^2\right)dz+\frac{R_T{}'a^2}{B}\int_0^1|\theta|^2dz$$
$$-\frac{T}{\sigma}\int_0^1\left(|\zeta|^2\right)dz+\frac{QT\sigma_1}{\sigma}\int|\xi|^2dz=0 \tag{2.21}$$

Multiplying equation (2.21) by p_r and adding the resulting equation to (2.20), we obtain

$$\int_0^1\left|(D^2-a^2)w\right|^2+\frac{2p_r}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz+QT\sigma_1\int_0^1|\xi|^2dz+\tau R_S{}'a^2\int_0^1\left(|D\phi|^2+a^2|\phi|^2\right)dz$$
$$+Q\int_0^1\left|(D^2-a^2)h_z\right|^2dz+QT\int_0^1\left(|D\xi|^2+a^2|\xi|^2\right)dz+T\int_0^1\left(|D\zeta|^2+a^2|\zeta|^2\right)dz$$
$$-\frac{R_T{}'a^2}{B}\int_0^1\left(|D\theta|^2+a^2|\theta|^2\right)dz=0 \tag{2.22}$$

We first note that, since w,θ,ϕ,h_z and ζ satisfy $w(0)=0=w(1)$, $\theta(0)=0=\theta(1)$, $\phi(0)=0=\phi(1)$, $h_z(0)=0=h_z(1)$ and $\zeta(0)=0=\zeta(1)$, we have by [10]

$$\int_0^1|Dw|^2dz\geq\pi^2\int_0^1|w|^2dz\quad, \tag{2.23}$$

$$\int_0^1|D\theta|^2dz\geq\pi^2\int_0^1|\theta|^2dz\quad, \tag{2.24}$$

$$\int_0^1|D\phi|^2dz\geq\pi^2\int_0^1|\phi|^2dz\quad, \tag{2.25}$$

$$\int_0^1|Dh_z|^2dz\geq\pi^2\int_0^1|h_z|^2dz\quad, \tag{2.26}$$

and

$$\int_0^1|D\zeta|^2dz\geq\pi^2\int_0^1|\zeta|^2dz\quad. \tag{2.27}$$

Further,

$$\int_0^1|Dw|^2dz=-\int_0^1w^*D^2wdz\leq\left|-\int_0^1w^*D^2wdz\right|$$
$$\leq\int_0^1\left|w^*D^2w\right|dz\leq\int_0^1|w^*|\left|D^2w\right|dz$$
$$\leq\int_0^1|w|\left|D^2w\right|dz\leq\left(\int_0^1|w|^2dz\right)^{1/2}\left(\int_0^1\left|D^2w\right|^2dz\right)^{1/2}$$

(utilizing Schwartz inequality)

$$\leq \frac{1}{\pi}\left\{\int_0^1 |Dw|^2 dz\right\}^{1/2}\left\{\int_0^1 |D^2 w|^2 dz\right\}^{1/2},$$

So that we have

$$\int_0^1 |D^2 w|^2 dz \ \geq \pi^2 \int_0^1 |Dw|^2 dz \ \geq \pi^4 \int_0^1 |w|^2 dz \quad . \tag{2.28}$$

(using 2.23)

Therefore by utilizing inequalities (2.23) and (2.28), we obtain

$$\int_0^1 \left(|D^2 w|^2 + a^4 |w|^2 + 2a^2 |Dw|^2\right) dz \ \geq (\pi^2 + a^2)^2 \int_0^1 |w|^2 dz \quad . \tag{2.29}$$

Further, (2.12) implies that

$$\int_0^1 |w|^2 dz = \int_0^1 ww^* dz = \frac{1}{B^2}\int_0^1 \left[(D^2 - a^2)\theta - p\theta\right]\left[(D^2 - a^2)\theta^* - p^*\theta^*\right]$$

$$= \frac{1}{B^2}\left[\int_0^1 |(D^2 - a^2)\theta|^2 + 2p_r \int \left(|D\theta|^2 + a^2|\theta|^2\right) + |p|^2 \int_0^1 |\theta|^2\right] \tag{2.30}$$

Second, since $p_r \geq 0$, therefore it follows from (2.30) that

$$\int_0^1 |w|^2 dz \geq \frac{1}{B^2}\left[\int_0^1 |(D^2 - a^2)\theta|^2 dz + |p|^2 \int_0^1 |\theta|^2 dz\right], \tag{2.31}$$

And

$$\int_0^1 |w|^2 dz > \frac{1}{B^2}\int_0^1 |(D^2 - a^2)\theta|^2 dz \tag{2.32}$$

Also, emulating the derivation of inequality (2.28) and (2.29), we have

$$\int_0^1 |(D^2 - a^2)\theta|^2 dz = \int_0^1 \left(|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2\right) dz \geq (\pi^2 + a^2)^2 \int_0^1 |\theta|^2 dz \tag{2.33}$$

Combining inequalities (2.31) and (2.33), we obtain

$$\int_0^1 |w|^2 dz \geq \frac{1}{B^2}\left\{(\pi^2 + a^2)^2 + |p|^2\right\}\int_0^1 |\theta|^2 dz \tag{2.34}$$

Again

$$\int_0^1 |w|^2 dz = \left(\int_0^1 |w|^2 dz\right)^{\frac{1}{2}}\left(\int_0^1 |w|^2\right)^{\frac{1}{2}}$$

$$> \left(\pi^2 + a^2\right)\frac{1}{B^2}\left\{1 + \frac{|p|^2}{\left(\pi^2 + a^2\right)^2}\right\}^{\frac{1}{2}}\left\{\int_0^1 \left|\left(D^2 - a^2\right)\theta\right|^2\right\}^{\frac{1}{2}}\left\{\int_0^1 |\theta|^2\right\}^{\frac{1}{2}}.$$

[Using (2.32) and (2.34)]

$$\geq \frac{\left(\pi^2 + a^2\right)}{B^2}\left\{1 + \frac{|p|^2}{\left(\pi^2 + a^2\right)}\right\}^{\frac{1}{2}}\left| - \int_0^1 \theta * \left(D^2 - a^2\right)\theta \right| dz$$

[Using Schwartz inequality]

$$= \frac{(\pi^2 + a^2)}{B^2}\left\{1 + \frac{|p|^2}{\left(\pi^2 + a^2\right)^2}\right\}^{\frac{1}{2}}\int_0^1 \left(|D\theta|^2 + a^2|\theta|^2\right)dz \tag{2.35}$$

Using inequality (2.29) in the first integral, inequalities (2.23)-(2.27) and (2.35) in (2.22) and utilizing the fact that $p_r \geq 0$, we get

$$\left(\pi^2 + a^2\right)^2\int_0^1 |w|^2 dz + \tau R_S' a^2\left(\pi^2 + a^2\right)\int_0^1 |\phi|^2 dz + Q\left(\pi^2 + a^2\right)\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz$$

$$+ T\left(\pi^2 + a^2\right)\int_0^1 |\zeta|^2 dz$$

$$< \frac{R_T' a^2}{B^2\left(\pi^2 + a^2\right)}\left\{1 + \frac{|p^2|}{\left(\pi^2 + a^2\right)}\right\}^{\frac{1}{2}}\int_0^1 |w|^2 dz \tag{2.36}$$

Equation (2.21) upon using (2.23) yields the following inequalities

$$R_S' a^2\int_0^1 |\phi|^2 dz > \frac{\left(\pi^2 + a^2\right)}{\sigma}\int_0^1 |w|^2 dz - \frac{Q\sigma_1}{\sigma}\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz - \frac{T}{\sigma}\int_0^1 |\zeta|^2 dz \tag{2.37}$$

$$Q\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz > \frac{\left(\pi^2 + a^2\right)}{\sigma}\int_0^1 |w|^2 dz - \frac{R_S' a^2\sigma}{\sigma_1}\int_0^1 |\phi|^2 dz - \frac{T}{\sigma}\int_0^1 |\zeta|^2 dz \tag{2.38}$$

and

$$T|\zeta|^2 > \left(\pi^2 + a^2\right)\int_0^1 |w|^2 dz - Q\sigma_1\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz - R_S' a^2\sigma\int_0^1 |\phi|^2 dz \tag{2.39}$$

Inequality (2.36) coupled with each of the inequalities (2.37)-(2.39) yields the following inequalities respectively:

$$\left(\pi^2 + a^2\right)^2\left(1 + \frac{\tau}{\sigma}\right)\int_0^1 |w|^2 dz + Q\left(\pi^2 + a^2\right)\left(1 - \frac{\tau\sigma_1}{\sigma}\right)\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz + T\left(\pi^2 + a^2\right)\left(1 - \frac{\tau}{\sigma}\right)\int_0^1 |\zeta|^2 dz$$

$$< \frac{R_T' a^2}{\left(\pi^2 + a^2\right)B^2}\left\{1 + \frac{|p|^2}{\left(\pi^2 + a^2\right)^2}\right\}^{\frac{1}{2}}\int_0^1 |w|^2 dz \tag{2.40}$$

$$\left(\pi^2+a^2\right)^2\left(1+\frac{1}{\sigma_1}\right)\int_0^1|w|^2dz+R_S{}'a^2\left(\pi^2+a^2\right)\left(\tau-\frac{\sigma}{\sigma_1}\right)\int_0^1|\phi|^2dz+T\left(\pi^2+a^2\right)\left(1-\frac{1}{\sigma_1}\right)\int_0^1|\zeta|^2dz$$

$$<\frac{R_T{}'a^2}{\left(\pi^2+a^2\right)B^2}\left\{1+\frac{|p|^2}{\left(\pi^2+a^2\right)^2}\right\}^{\frac{1}{2}}\int_0^1|w|^2dz \qquad (2.41)$$

and

$$2\left(\pi^2+a^2\right)^2\int_0^1|w|^2dz+Q\left(\pi^2+a^2\right)\left(1-\sigma_1\right)\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz+R_S{}'a^2\left(\pi^2+a^2\right)\left(\tau-\sigma\right)\int_0^1|\phi|^2dz$$

$$<\frac{R_T{}'a^2}{\left(\pi^2+a^2\right)B^2}\left\{1+\frac{|p|^2}{\left(\pi^2+a^2\right)^2}\right\}^{\frac{1}{2}}\int_0^1|w|^2dz \qquad (2.42)$$

Now, if $\delta=\min\left(\frac{\tau}{\sigma},\frac{1}{\sigma_1},1\right)$, then depending on the value of δ ,exactly one of the inequalities (2.40)-(2.42) will imply that

$$(1+\delta)\left(\pi^2+a^2\right)^2\int_0^1|w|^2dz<\frac{R_T{}'a^2}{\left(\pi^2+a^2\right)B^2}\left\{1+\frac{|p|^2}{\left(\pi^2+a^2\right)^2}\right\}^{\frac{1}{2}}\int_0^1|w|^2dz\,. \qquad (2.43)$$

Since the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 *is* $\frac{27\pi^4}{4}$, it therefore follows from inequality (2.43) that

$$\frac{27\pi^4}{4}\left((1+\delta)B^2\right)\left\{1+\frac{|p|^2}{\left(\pi^2+a^2\right)^2}\right\}^{\frac{1}{2}}<R_T{}'=\frac{R_TR\gamma}{(1-\tau)}=\frac{\Gamma R_SR\gamma}{(1-\tau)} \qquad (2.44)$$

Inequality (2.44) implies that

$$|p|<\left(\pi^2+a^2\right)\sqrt{\lambda^2-1}\ , \qquad (2.45)$$

where $\lambda=\frac{4R_TR\gamma}{(1-\tau)27\pi^4(1+\delta)B^2}\left(=\frac{4\Gamma R_SR\gamma}{(1-\tau)27\pi^4(1+\delta)B^2}\right)$

Further, it follows from inequality (2.43) that

$$\frac{\left(\pi^2+a^2\right)\left(\pi^2+a^2\right)^2(1+\delta)}{a^2B^2R\gamma}<R_T=\Gamma R_S \qquad (2.46)$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 *is* $4\pi^2$, therefore it follows from inequality (2.46) that

$$\left(\pi^2+a^2\right)<\frac{\Gamma R_SRB^2\gamma}{4\pi^2(1+\delta)(1-\tau)} \qquad (2.47)$$

Combining inequalities (2.45) and (2.47), we finally get

$$|p| < \frac{\Gamma R_S R B^2 \gamma}{4\pi^2 (1+\delta)(1-\tau)} \sqrt{\lambda^2 - 1}.$$

This completes the proof of the theorem.

Theorem 1, from the point of view of hydrodynamic stability theory, may be stated as: the complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of neutral or growing amplitude in Dufour-driven magnetorotatory thermosolutal instability of Veronis' type lies inside a semicircle in the right half of the $p_r p_i$-plane whose centre is at the origin and whose radius is $\frac{\Gamma R_S R B^2 \gamma}{4\pi^2 (1+\delta)(1-\tau)} \sqrt{\lambda^2 - 1} = \frac{R_T R B^2 \gamma}{4\pi^2 (1+\delta)(1-\tau)} \sqrt{\lambda^2 - 1}$. This result is uniformly valid for an initially top heavy as well as initially bottom heavy configuration.

***Corollary* 1**: If, $(p, w, \theta, \phi, h_z, \zeta, \xi)$, $p = p_r + ip_i, p_i \neq 0$, is a non trivial solution of equations (2.11)-(2.17) and

$$R_T > 0, R_S > 0 \; and \; \Gamma < \frac{27\pi^4 (1-\tau)(1+\delta) B^2}{4 R_S R \gamma}, \; then \; p_r < 0.$$

Proof: Follows from theorem 1.

Cor. 1 implies that oscillatory motions of growing amplitude are not allowed in Dufour –driven magnetorotatory thermosolutal instability of Veronis' type if the initial stability parameter Γ does not exceed the value $\frac{27\pi^4 (1-\tau)(1+\delta) B^2}{4 R_S R \gamma}$. Further, this result is uniformly valid for initially top heavy as well as initially bottom heavy configuration.

Remarks: The following remarks, now, deserve attention:

(a) If $0 < R_T < R_S \leq \frac{27\pi^4 (1-\tau)(1+\delta) B^2}{4 R \gamma}$ and $p_i \neq 0$, then cor.1 implies that $p_r < 0$.

(b) If $0 < R_T \leq \frac{27\pi^4 (1-\tau)(1+\delta) B^2}{4 R \gamma} < R_S$ and $p_i \neq 0$, even then cor.1 implies that $p_r < 0$

It is easy to see that

(i) $\delta = \frac{\tau}{\sigma} \; for \; DDVTC, (ii) \; \delta = \min\left(\frac{\tau}{\sigma}, \frac{1}{\sigma_1}\right) for \; Dufour-driven \; hydromagnetic VTC, \; and$

(iii) $\delta = \min\left(\frac{\tau}{\sigma}, 1\right)$ for Dufour-driven rotatory VTC. Consequently, the characterization theorem of [8] can easily be averred from (a).

Theorem 2 (A Semi-circle theorem for Dufour-driven magnetorotatory Stern thermosolutal convection):

If $(p, w, \theta, \phi, h_z, \zeta, \xi)$ $p = p_r + ip_i, p_r \geq 0, p_i \neq 0,$ is a non trivial solution of equations (2.11)-(2.17) and $R_T < 0, R_S < 0, \; then$

$$|p| < \frac{(\hat{\Gamma} + \frac{R\gamma}{1-\tau})|R_T|}{4\pi^2(1+\hat{\delta})}\sqrt{\hat{\lambda}^2 - 1} \quad ,$$

$$\hat{\lambda} = \frac{4\left(\hat{\Gamma} + \frac{R\gamma}{(1-\tau)}\right)|R_T|}{27\pi^4(1+\hat{\delta})\tau}, \hat{\delta} = \min\left\{\frac{1}{\sigma},\frac{1}{\sigma_1},1\right\} \; \hat{\Gamma} = \frac{|R_S|}{|R_T|}.$$

Proof: Replacing R_T *and* R_S, by $-|R_T|$ *and* $-|R_S|$ respectively in equation (2.11) and proceeding exactly as in theorem 1, *mutatis mutandis*, we get the desired result.

***Corollary* 2:** If, $(p, w, \theta, \phi, h_z, \zeta, \xi)$, $p = p_r + ip_i,\ p_i \neq 0,$ is a nontrivial solution of equations(2.11)-(2.17) and

$$R_T < 0, R_S < 0 \text{ and } \hat{\Gamma} < \left[\frac{27\pi^4\tau(1+\hat{\delta}) - \frac{4R\gamma}{(1-\tau)}}{4|R_T|}\right], \text{then } p_r < 0.$$

Proof: Follows from Theorem 2.

The essential contents of Theorem 2 and cor.2 from the point of view of hydrodynamic stability are similar to those of Theorem 1. However now they pertain this time to Dufour -driven magnetorotatory thermosolutal instability of Stern type .Further remarks similar to those after cor.1 hold here also. To be specific, cor. 2 implies that:

(a) If

$$0 < |R_S| < |R_T| \leq \frac{27\pi^4\tau(1+\hat{\delta}) - \frac{4R\gamma}{(1-\tau)}}{4}$$ and $p_i \neq 0$, then $p_r < 0$.

(b) If

$$0 < |R_S| \leq |R_T| \leq \frac{27\pi^4\tau(1+\hat{\delta}) - \frac{4R\gamma}{(1-\tau)}}{4} < |R_T|$$ and $p_i \neq 0$, even then $p_r < 0$.

Further, it is easy to see that

(i) $\hat{\delta} = \frac{1}{\sigma}$ *for DD STC*, *(ii)* $\hat{\delta} = \min\left(\frac{1}{\sigma}, \frac{1}{\sigma_1}\right)$ *for Dufour – driven hydromagnetic STC, and*

(iii) $\hat{\delta} = \min\left(\frac{1}{\sigma}, 1\right)$ for Dufour–driven rotatory STC. Consequently, one can easily write down from cor.2 the characterization theorem for DDSTC with or without the individual effects of a rotation and a magnetic field.

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