

## Numerical Analysis of Induced Siphonage in P Trap

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### Abstract

Seal water in trap plays a crucial role in preventing foul-smelling toxic gas in drainage pipes from entering indoors. Induced siphonage is the most important of the phenomena associated with seal break and seal loss. This phenomenon occurs when seal water level changes rapidly in response to air pressure fluctuations in drain and gets lost. Though there have been several studies on numeric analyses and motion equations of seal water fluctuation, none of them addressed the issue of seal water fluctuation analysis in response to air pressure fluctuation in drain. In this study, the authors derived a motion equation for induced siphonage in P trap, and examined the validity of the equation by analyzing seal water fluctuation using Excel BVA based on the force of vibration in drain.

**Keywords:** Induced siphonage; Trap; Drainage system; Simulation

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### Introduction

Water is used in a number of ways in buildings. Its main use involves appliances for water usage such as sanitary fixtures. Water, together with wastes, is discharged through drainage pipe into the sewer or septic tanks. The drainage pipe is usually filled with foul-smelling toxic drainage gas and if such gas enters indoors through drain outlets of sanitary fixture, it may contaminate air and cause health damage. In order to prevent this from happening, fixture drainage pipes are equipped with traps, which contain seal water. Seal water plays an important role to stop drainage gas from entering the room. However, seal water may be lost for many reasons leading to a condition called seal break. Induced siphonage is one of the most important seal break phenomena. In induced siphonage, air pressure inside drainage pipe fluctuates when discharge is made, and seal water also starts to fluctuate in response to pressure fluctuation precipitating seal loss and seal break. To prevent seal break due to this phenomenon, various precautions such as an addition of vent pipes and the use of appropriate diameter pipes are stipulated in the design method. The design method is based on a proportional relation that regards the causal relationship of discharge flow rate and air pressure fluctuations to seal loss as a static phenomenon. However, their relationship must be understood as a dynamic phenomenon as pressure and seal water fluctuates constantly in reality. Though there have been several studies published on numeric analyses and motion equation of dynamic seal water behaviors, not a single one of them analyzed

seal water fluctuations as a response phenomenon of pressure fluctuation in pipe [1-9].

In this study the authors derived a motion equation for seal water fluctuations in response to pressure fluctuations in P trap and analyzed the validity of the equation based on pressure and seal water fluctuation data collected from a discharge experiment conducted in a 15-story experimental tower.

### Motion Equation for Seal Water Fluctuation

Induced siphonage can be considered as a single degree freedom forced vibration phenomenon created by the force of pressure in drain. Following the conventional procedures of vibration analysis, first we derived a motion equation of free vibration and then that of forced vibration.

#### Free vibration

The law of conservation of momentum can be applied to seal water vibration on the premise that the sum of inertia, damping force and power of resistance is constant. As shown in **Figure 1**, the falling mass of water that amounts to the water levels between the trap legs constitutes the power of resistance. The equation (1) represents the motion equation for seal water fluctuations where the water level is  $y$  and damping coefficient

is  $c$ . The damping coefficient  $c$  is determined from the equation (3) with critical damping coefficient  $c_c$  and damping ratio  $\zeta$ . The damping ratio  $\zeta$  is obtained from logarithmic decrement  $\sigma$  in the equation (4) (Figure 1).

The damping ratio  $\zeta$  is obtained from water level fluctuation patterns in the seal water free vibration experiment. For instance, free vibrations of P trap (trap with the same diameter: Seal depth: 50 mm, ratio of cross sectional area of leg: 1.0, initial difference of water level: 20 mm) were incorporated into the equation. The free vibration wave is shown in Figure 2. Based on the wave patterns, the logarithmic decrement is calculated to be  $\sigma=0.139$ . The critical damping coefficient is  $c_c=2.26$ . Therefore, damping coefficient  $\zeta=0.0222$  and damping coefficient  $c=0.0502$  can be obtained from the equations (3) and (4). Natural frequency  $f$  is obtained from equation (5), which, in the case of P trap, is 1.96 Hz. Natural frequency obtained from the water level fluctuation patterns ( $ALd^2=0.0919\text{kg}$ ,  $\rho AL=13.9\text{N/m}$ ) in Figure 2 is 1.94 Hz, which roughly matches the result of the calculation (Figure 2).

$$(\rho AL)d^2y/dt^2 + cdy/dt + 2(\rho Ag)y=0 \quad (1)$$

$$c_c=2((\rho AL) \cdot (2\rho Ag))^{1/2} \quad (2)$$

$$c=\zeta \cdot c_c \quad (3)$$

$$\zeta=\sigma/2\pi \quad (4)$$

$$f=((2\rho Ag)/(\rho AL)/(2\pi))^{1/2} \quad (5)$$

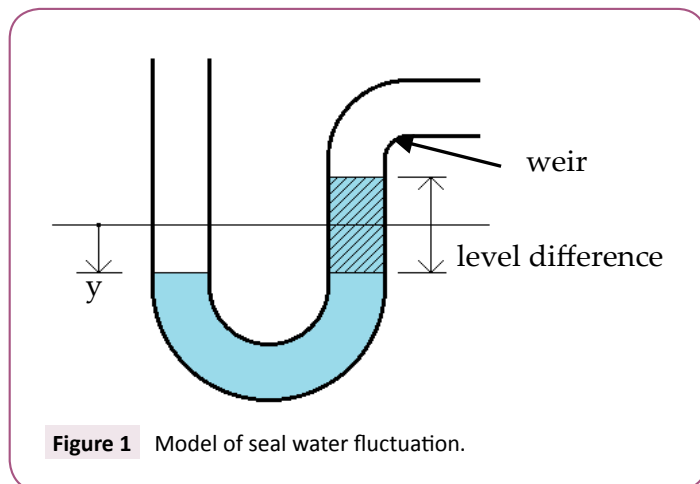


Figure 1 Model of seal water fluctuation.

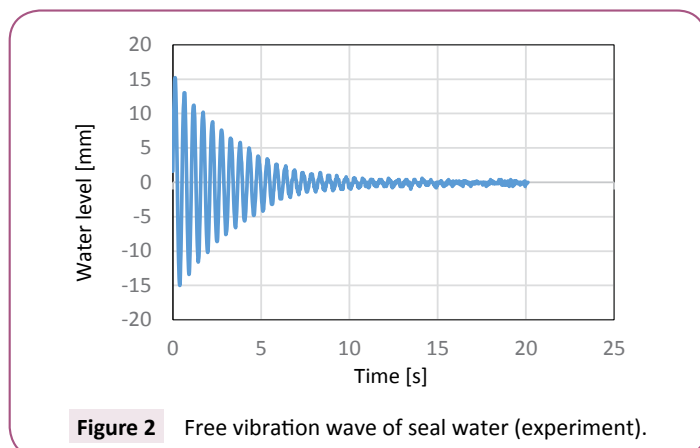


Figure 2 Free vibration wave of seal water (experiment).

## Motion equation for forced vibration

Fluctuations of seal water in trap is a forced vibration phenomenon that changes in response to air pressure fluctuations caused by discharged water and can be expressed in equation (6).

$$(\rho AL)d^2y/dt^2 + cdy/dt + 2(\rho Ag)y=AP \quad (6)$$

## Numerical Analysis of Motion Equation

Seal loss occurs when the top of seal water in the outlet leg overflows the weir of a trap. When this happens, the mass of seal water is reduced and it must be dealt with as an unsteady phenomenon (transient phenomenon). As a general solution cannot be obtained for an unsteady phenomenon, the numerical calculation method must be applied. We applied the Runge-Kutta method as a numerical calculation method, and used EXCEL BVA. The damping coefficient  $c$  of 0.076 obtained from the preliminary experiment was applied and seal water fluctuation was simulated using air pressure fluctuations data from the experiment.

## Time-step

Time-step plays an important role in attaining accuracy in analysis. Therefore, appropriate time-step must be established based on the free vibration wave patterns of seal water. Table 1 shows the results of analysis at time-steps  $t_s=0.01, 0.05, 0.075, 0.1$  s, logarithmic decrement  $\sigma$  and damping ratio  $\zeta$  obtained from the equation (6). As  $\zeta$  when  $t_s=0.25$  was the closest to the experimental results,  $t_s=0.025$  was used in subsequent calculation (Table 1).

## Seal loss rate

Seal loss occurs when water level in the outlet leg flows over the dip. The loss rate depends on how large the water level fluctuations are. As it is difficult to simulate the actual water level conditions, we estimated as follows:  $\gamma=0.001$  when  $y_{\max}<8$  mm,  $\gamma=0.1$  when  $8 \text{ mm} \leq y_{\max}<15$  and  $\gamma=0.8$  when  $y_{\max} \leq 1$ .

## Discharge Experiment on Induced Siphonage

We constructed a stack vent drainage system with special drainage fittings in a 16-story experimental tower and conducted a discharge experiment to obtain data on fluctuations of pressures in drain and seal water in actual drainage situation.

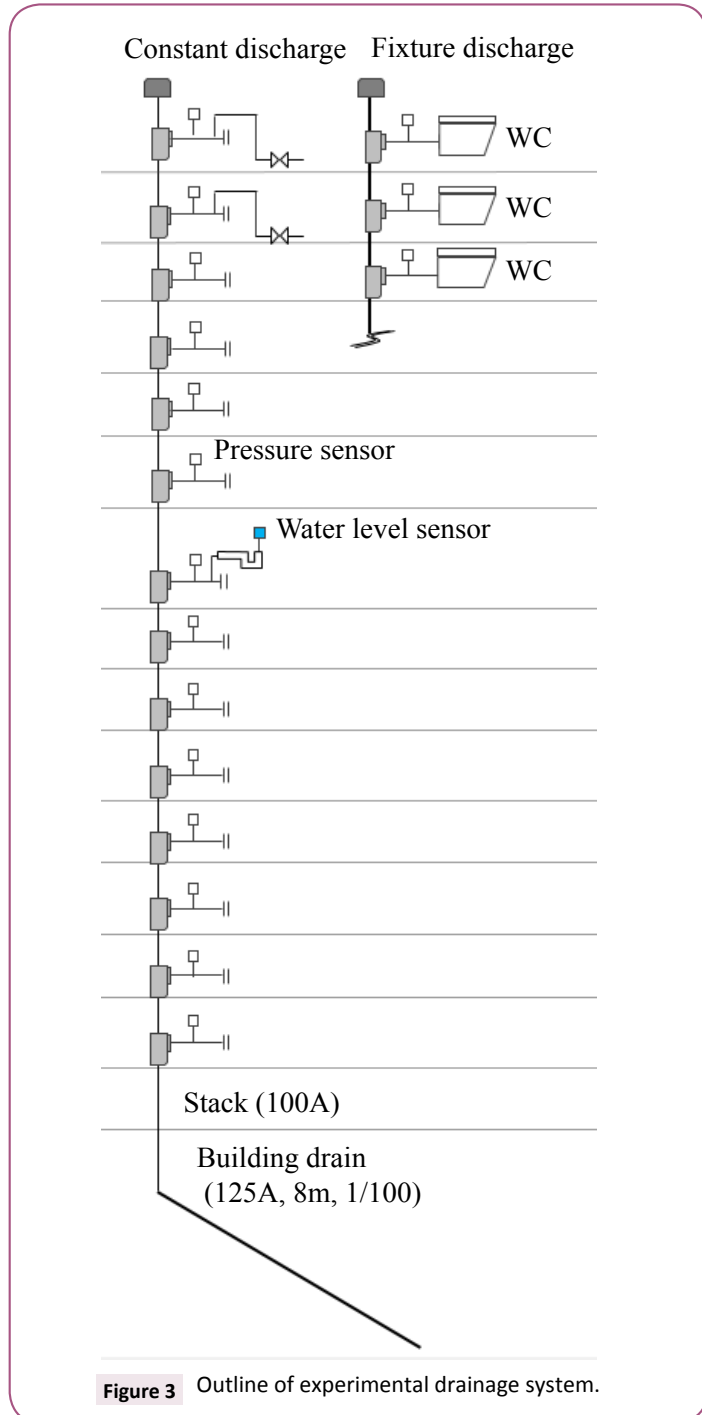
The experimental drainage system is shown in Figure 3. Stacks with 100 Å diameters and horizontal branches with 50 Å diameters were used in the experiment. PVC traps were placed on the 9th floor. Constant discharges (1.5, 4.5 L/s) and fixture discharge (1WC and 3WC) were made constant (Table 2 and Figure 3).

Table 1 Logarithmic decrement  $\sigma$  and damping ratio  $\zeta$  according time-step.

| Parameter                          | Simulation |        |        |        |        | Experiment |
|------------------------------------|------------|--------|--------|--------|--------|------------|
|                                    | 0.01       | 0.025  | 0.05   | 0.075  | 0.1    |            |
| Time step $t_s$ (s)                | 0.01       | 0.025  | 0.05   | 0.075  | 0.1    |            |
| logarithmic decrement $\sigma$ (-) | 0.0476     | 0.139  | 0.182  | 0.230  | 0.375  | 0.139      |
| damping ratio $\zeta$ (-)          | 0.00758    | 0.0218 | 0.0290 | 0.0366 | 0.0568 | 0.222      |

**Table 2** Shape parameters of P trap.

| Diameter | Cross-sectional area            | Seal depth | Volume |
|----------|---------------------------------|------------|--------|
| 0.03 m   | $7.1 \times 10^{-4} \text{m}^2$ | 0.05 m     | 150 mL |

**Figure 3** Outline of experimental drainage system.

## Validation of Analysis Results

### Pressure fluctuation in drain and seal water fluctuation

The experimental drainage system is shown in branches with 50 Å diameter were used in the experiment. PVC traps were placed on the 9th floor. Constant discharges (1.5, 4.5 L/s) and fixture discharge (1 WC and 3WC) were made. Constant discharges were

made from the floors 14 and 15, fixture discharges from the floors 13~15. Average flow rate of fixture discharge from WC qd was 2.2 L/s.

Experimental results of air pressure fluctuations (Pa) in drain and experimental and simulated results of seal water fluctuation (mm) with constant discharge of 1.5 L/s and 4.5 L/s and fixture discharge with 1WC and 3WC are shown in **Figure 4**. Maximum and minimum values of air pressure and seal water level and seal loss of those discharges are shown in **Figure 4 and Table 3**.

Air Pressure fluctuations in drain with fixture discharge moved from the negative to positive range while those with constant discharge stayed in the negative range. The reason for the increase in the case of fixture discharge is that air in drain was compressed by initial large loads of large discharge. Experimental and simulated results of seal water fluctuation indicated the similar trend in response to fluctuation of air pressure in drain [10]. While air pressure in drain returns to zero after discharge is completed, seal water level continues to show minute vibration due to oscillation of water surface. In simulation, there was no seal water level fluctuation as the movement of water surface was not computed.

As a whole, maximum and minimum air pressure and water level in simulation were approximately 10% smaller than those in experiment expect constant discharge of 1.5 L/s and fixture discharge with 3WC. However, maximum seal water level with constant discharge of 4.5 L/s only showed larger values than maximum negative pressure in drain (water head value). This seems to indicate that some type of resonance phenomenon had occurred.

### Power spectrum of pressure in drain and seal water fluctuations

Experimental results of pressure fluctuations in drain and experimental and simulated power spectrum distribution of seal water fluctuation with constant discharge of 1.5 L/s and 4.5 L/s and fixture discharge with 1WC are shown in **Figure 5**. Dominant frequencies of pressure fluctuation in drain and seal water fluctuation are shown in **Tables 4 and 5**.

Though natural frequencies of trap varied in response of the extent of seal loss, they stayed within the range of 1.53~1.94 Hz. The first and second dominant frequencies of pressure fluctuation in drain fell in the range of 3.0~3.6Hz except for fixture discharge with 1WC. The fourth dominant frequency with a discharge rate of 4.5 L/s roughly matched trap's natural frequency 1.76 Hz, which indicates partial resonance phenomenon had occurred.

The power spectrum distribution of simulated seal water fluctuation showed a similar trend to that of pressure fluctuation in drain. Experimental power spectrum distribution of seal water fluctuation showed the dominant frequency of 3.2~3.5Hz, which was identical to that of pressure fluctuation in drain. The first dominant frequency of seal water fluctuation in experiment with a constant discharge flow rate of 4.5 L/s was 1.76 Hz roughly matched the dominant frequency of trap, which also indicates the occurrence of partial resonance phenomenon.

**Table 3** Maximum and minimum values of seal water level and seal loss.

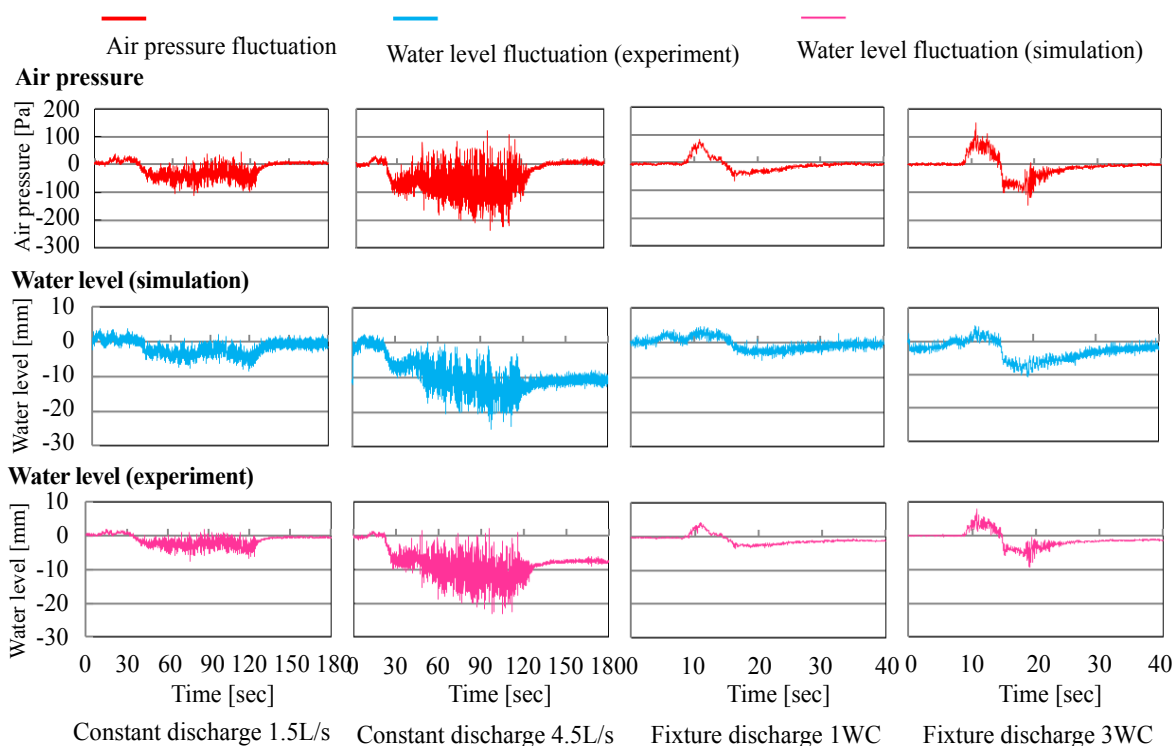
| Discharge load     |         | Air pressure (mm Aq) |       | Water level (mm) |       |            |       | Seal loss (mm) |            |
|--------------------|---------|----------------------|-------|------------------|-------|------------|-------|----------------|------------|
|                    |         |                      |       | Experiment       |       | Simulation |       | Experiment     | Simulation |
|                    |         | Max                  | Min   | Max              | Min   | Max        | Min   |                |            |
| Constant discharge | 1.5 L/s | 4.5                  | -11.0 | 3.8              | -8.5  | 1.9        | -7.5  | 0.0.8          | 0.7        |
|                    | 4.5 L/s | 12.2                 | -23.8 | 2.5              | -24.9 | 2.1        | -23.2 | 11.6           | 6.2        |
| Fixture discharge  | 1WC     | 8.4                  | -6.6  | 4.7              | -5.1  | 4.0        | -3.8  | 0.9            | 0.7        |
|                    | 3WC     | 14.9                 | 14.8  | 4.6              | -10.7 | 6.9        | -9.2  | 1.9            | 1.6        |

**Table 4** Dominant frequencies of air pressure.

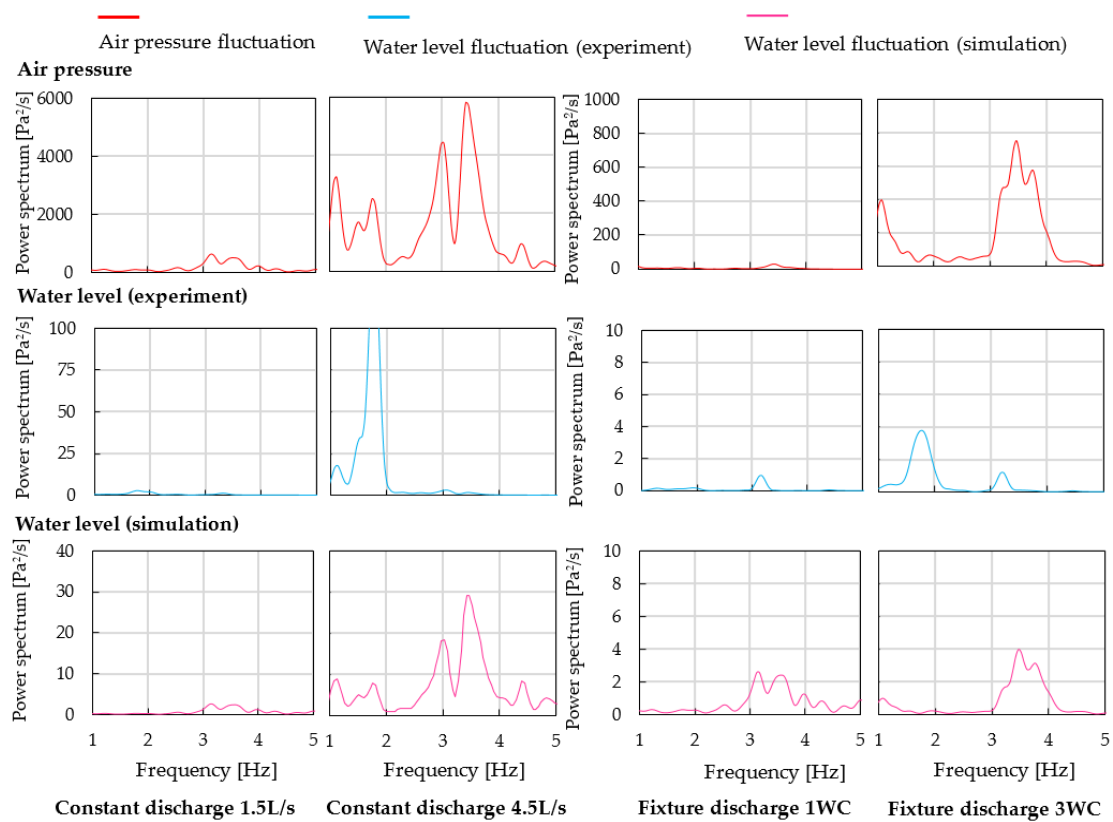
| Dominant frequency (Hz) | Constant discharge |         | Fixture discharge |      |
|-------------------------|--------------------|---------|-------------------|------|
|                         | 1.5 L/s            | 4.5 L/s | 1WC               | 3WC  |
| The first               | 3.17               | 3.42    | 1.22              | 3.47 |
| The second              | 3.56               | 3.03    | 6.59              | 3.22 |
| The third               | 3.96               | 1.12    | 3.47              | 3.76 |
| The fourth              | 2.54               | 1.76    | 2.25              | 1.07 |
| The fifth               | 4.3                | 1.51    | 2.0               | 1.51 |

**Table 5** Dominant frequencies of seal water.

| Dominant frequency (Hz) | Constant discharge |      |         |      | Fixture discharge |      |       |      |
|-------------------------|--------------------|------|---------|------|-------------------|------|-------|------|
|                         | 1.5 L/s            |      | 4.5 L/s |      | 1WC               |      | 3WC   |      |
|                         | Expt.              | Sim. | Expt.   | Sim. | Expt.             | Sim. | Expt. | Sim. |
| The first               | 1.81               | 3.17 | 1.76    | 3.42 | 3.17              | 3.47 | 1.75  | 3.47 |
| The second              | 3.32               | 3.56 | 1.12    | 3.03 | -                 | 1.22 | 3.22  | 3.76 |
| The third               | 1.22               | 4.0  | 3.08    | 1.12 | -                 | 2.25 | 1.22  | 1.07 |
| The forth               | -                  | 4.3  | 2.29    | 4.39 | -                 | 4.05 | -     | 1.95 |
| The fifth               | -                  | 2.54 | 3.42    | 1.76 | -                 | 4.88 | -     | 1.51 |



**Figure 4** Experimental results of air pressure fluctuations and experimental and simulated results of seal water fluctuation.



**Figure 5** Power spectrum distribution of air pressure and seal water fluctuation.

**Table 6** Dominant frequencies of air pressure.

|   |
|---|
| $A$ : cross-sectional area of trap leg ( $m^2$ )              |
| $c$ : <b>damping</b> coefficient ( $N \cdot s/m$ )            |
| $c_c$ : critical <b>damping</b> coefficient ( $N \cdot s/m$ ) |
| $g$ : <b>gravitational acceleration</b> ( $m/s^2$ )           |
| $L$ : length of seal water (m)                                |
| $m$ : mass (kg)   |
| $P$ : air pressure (Pa), (mmAq)                               |
| $q_d$ : average flow rate of fixture discharge (L/s)          |
| $t$ : time (s)  |
| $t_s$ : time step (s)   |
| $y$ : water level (m), (mm)                                   |
| $y_{max}$ : <b>maximum water level (m), (mm)</b>              |
| $\gamma$ : seal loss rate (-)                                 |
| $\zeta$ : damping ratio (-)                                   |
| $\rho$ : <b>density</b> ( $kg/m^3$ )                          |
| $\sigma$ : logarithmic decrement (-)                          |

The power spectrum distribution of simulated seal fluctuation corresponds to that of pressure in drain. Experimental dominant frequencies in the seal water power spectrum distribution 3.2~3.5 Hz is roughly identical to those of pressure in drain. The first dominant frequency of seal water fluctuation at the constant discharge flow rate of 4.5 L/s was 1.76 Hz, which roughly

corresponds to the natural frequency of trap. This also confirms that partial resonance phenomena had occurred (**Table 6**).

## Conclusion

The authors derived a motion equation for simulated induced siphonage in P trap and examined the validity of the equation by analyzing seal water fluctuation using EXCEL BVA based on the force of vibration in drain and by comparing with experimental data. The results of analysis can be summarized as follows:

1. The trend of simulated seal water fluctuation roughly corresponded to experimental data.
2. Simulated maximum and minimum seal water level, and seal depth were 10 to 20% smaller than experimental data.
3. The first and second dominant frequencies of pressure in drain fluctuation fell in the range of 3.0~3.6 Hz except for fixture discharge with 1WC.
4. The simulated power spectrum distribution of seal water fluctuation resembled to that of pressure in drain.
5. Partial resonance phenomena seem to have occurred in constant discharge load of 4.5 L/s as the maximum water level exceeded the maximum negative pressure (water head) in experiment. This has been confirmed by the analysis of the

power spectrum, but the simulation analysis failed to give any supportive evidence to this finding.

Based on these we can safely conclude that our simulation was

validated in its application. As for (2) and (5), small damping coefficient may have contributed to the results. Along with seal loss rate, it prompts future studies.

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