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Nuclear structure of the germanium nuclei in the interacting Boson model (IBM)

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ABSTRACT

The structure of some even-even Ge isotopes have been studied within the framework of the interacting boson model. The positive parity states, B(E2), B(M1) and δ(E2/M1) values of the above nuclei have been calculated. The IBM-2 results obtained for Ge have been compared with the previous experimental and theoretical values obtained on the basis of the interacting boson model (IBM-2). The sufficient aspects of model leading to the E(5) symmetry have been proved by presenting E(5) characteristic of the Ge nuclei .

Keywords: nuclear phase transition, critical symmetry, even $^{64-80}$Ge, Interacting Boson Model.

INTRODUCTION

This paper presents a computational study in the field of nuclear structure. The declared goal of the paper is to identify features of the E(5) dynamical symmetry in the critical point of the phase/shape transition for the even-even $^{64-80}$Ge isotopes. The topic of dynamical symmetries (E(5) and X(5)) was theoretically predicted by[1]. And found in real nuclei by [2], just beside experiment [3-5]. A systematic search of nuclei exhibiting this features along the nuclide chart is underway in several structure labs around the world. This subject is of highly scientific interest in the present days investigations. Three dynamical symmetry limits known as Harmonic oscillator, deformed rotor and asymmetric deformed rotor are labeled by U(5), SU(3) and O(6) respectively[6]. And they form a triangle known as the Casten triangle representing the nuclear phase diagram[7] . In the original Bohr Hamiltonian, the U(5) and O(6) symmetry limits are connected by assuming potentials only depend on β. On the other hand, the U(5) and SU(3) limits are connected by separating the $V(\beta,\gamma)$ potentials into variables in the Hamiltonian. In those cases, one can use the Davidson-like potentials instead of β-dependent part of the potential [8].

In the original Bohr Hamiltonian, the U(5) and O(6) symmetry limits are connected by assuming potentials only depend on β. On the other hand, the U(5) and SU(3) limits are connected by separating the $V(\beta,\gamma)$ potentials into variables in the Hamiltonian. In those cases, one can use the Davidson-like potentials instead of β-dependent part of the potential. As it was also described in[9-11] .Most of the shell-model studies of nuclei with Z, N≤50 assume a $^{88}_{38}Sr$ inert core and restrict the valence-proton particles or neutron holes to the $p_{1/2}$ and $g_{9/2}$ orbital [12] . In principle, the inclusion of these orbital will also make the model space adequate in principle to describe nuclei with Z≤38 as well as to possibly account for the high spin states that have recently been established in the Zr isotopes. Another motivation for considering such a large model space is to allow from one to more accurate calculation of double-beta decay transitions in *Kr*, *Se* and *Ge* nuclei [12]. For the nuclei with Z>28, N<50 protons and neutrons are allowed to occupy $g_{9/2}$, $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ orbital. It is obvious that in such a description the use of $^{88}_{38}Sr$ as a core is no longer convenient. A theoretical explanation of the shape coexistence phenomena has been given by the presence of intruder levels in the neutron or the proton valence shell [13] . The evidence for an extensive region of nuclei near

A~80 is consistent with the definition of three dynamical symmetry limits. The even-even *Ge* isotopes are the members of the chain situated away from both the proton closed shell number at 28 and neutron closed shell at 50. In this study, we have carried out the level scheme of the transitional nuclei $^{64-80}Ge$ showing the characteristic *E*(5) pattern in its some low-lying bands. The positive parity states of even-mass *Ge* nuclei also stated within the framework of the Interacting Boson Model-2 (IBM-2). By comparing transitional behavior in the Ge nuclei with the predictions of an E(5) Critical symmetry, an achievable degree of agreement has been investigated[14]. Interacting Boson Models IBM-1and IBM-2, have been used to calculate energy levels and nuclear properties of the even-even *Ge* isotopes from $A = 64$ to $A = 80$. Energy levels of the low lying states of these nuclei were produced, the electric quadruple reduced transition probabilities $B(E2)$ were calculated as well. Mixing ratios $\delta(E2/M1)$ for transitions with $\Delta I = 0, I \neq 0$ were calculated. All the results are compared with available experimental data and other IBM versions and calculations. Satisfactory agreements were produced.

The aim of this work is to calculate the energy levels and electromagnetic transitions probabilities B(E2) and B(M1), multipole mixing ratios transitional *Ge* isotopes, using the IBM-2, and to compare the results with the experimental data.

1.1 The Interacting Boson Model

In the IBM-2 the structure of the collective states in even-even nuclei is calculated by considering a system of interacting neutron (ν) and proton (π) boson s ($l = 0$) and d ($l = 2$). The boson Hamiltonian can be written as [15]:

$$H = \varepsilon_d (n_{d\nu} + n_{d\pi}) + KQ_\nu^{(2)}.Q_\pi^{(2)} + V_{\pi\pi} + V_{\nu\nu} + M_{\nu\pi} \text{..........................(1)}$$

where $Q_\rho = (s_\rho^{+} d_\rho + d_\rho^{+} s_\rho)^{(2)} + \chi_\rho (d_\rho^{+} d_\rho)^{(2)} \qquad \rho = \nu, \pi$................(2)

κ is the quadrupole-quadrupole strength and $V_{\rho\rho}$ is the boson-boson interaction, which is given by the equation:

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_\rho^L \left(\left[d_\rho^{+} d_\rho^{+} \right]^{(L)} . \left[\tilde{d}_\rho \tilde{d}_\rho \right]^{(L)} \right).$$

and

$$M_{\nu\pi} = \tfrac{1}{2} \xi_2 (s_\nu^{+} d_\pi^{+} - d_\nu^{+} s_\pi^{+})^{(2)} . (\tilde{s}_\nu \tilde{d}_\pi - \tilde{d}_\nu \tilde{s}_\pi)^{(2)} - \sum_{k=1,3} \xi_k (d_\nu^{+} . d_\pi^{+})^{(k)} . (\tilde{d}_\nu . \tilde{d}_\pi)^{(2)} \text{.........(3)}$$

The Majorana term $M_{\pi\nu}$ shits the states with mixed proton-neutron symmetry with respect to the totally symmetric ones. Since little experimental information is known about such states with mixed symmetry , we did not attempt to fit the parameters appearing in eq. (3) , but rather took constant values for all *Ge* isotopes. :

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu \text{.......................(4)}$$

The quadrupole moment Q_ρ is in the form of equation (2), for simplicity, the χ_ρ has the same value as in the Hamiltonian. This is also suggested by the single j-shell microscopy, e_π and e_υ are proton and neutron boson effective charges respectively. In general, the *E*2 transition results are not sensitive to the choice of e_ν and e_π , whether $e_\nu = e_\pi$ or not.

The reduced electric quadrupole transition probability $B(E2)$ is given by:

$$B(E2; I_i \to I_j) = \frac{1}{2I_i + 1} \left(\left| < I_f \| T(E2) \| J_i > \right|^2 \right) \text{..........(5)}$$

The M1 transition operator is given :

$$T(M1) = \sqrt{3/\pi} (g_\pi L_\pi + g_\upsilon L_\upsilon) \qquad \text{................ (6)}$$

where $L_\upsilon(L_\pi)$ is the neutron and (proton) angular momentum operator

$$L_p^{(1)} = \sqrt{10} (d^{+} d)^{(1)}$$

where g_π and g_ν are the effective boson (proton, neutron) geomagnetic –factors. The T(M1) operator can be written alternatively as

$$T(M1) = \left[3/4\pi\right]^{1/2}\left[\frac{1}{2}(g_\pi + g_\nu)\left(L_\pi^{(1)} + L_\nu^{(1)}\right) + \frac{1}{2}(g_\pi - g_\nu)\left(L_\pi^{(1)} + L_\nu^{(1)}\right)\right] ....(7)$$

The direct measurement of B(M1) matrix elements should be normally difficult, so the M1 strength of gamma transition may be expressed in terms of the multiple mixing ratio which can be written as [16] :

$$\delta(E2/M1) = 0.835 E_\gamma (MeV)\frac{\langle I_f \| T(E2) \| I_i \rangle}{\langle I_f \| T(M1) \| I_i \rangle} ....(8)$$

where E_γ is the transition energy.

1.2 E(5) Symmetry Critical Point

Nuclear shapes have always been a point of discussion. In general, an atomic nucleus is believed to have an ellipsoidal shape. The shape of the nucleus is determined by five independent quantities, the two shape parameters (β and γ) and the three Euler angles (θ, φ and ψ). It is believed to have perfect spherical shape when the neutron number or the proton number of the nucleus is one of the magic number as predicted by the Shell Model (for e.g. ^{40}Ca, ^{208}Pb). However as the number of these nucleons changes the shape of the nucleus also changes and it no longer remains spherical. Thus shape transitions are to be seen in nuclei. These shape transitions in atomic nuclei were studied extensively in the early 80‟s in the framework of the IBM.

Dynamical symmetries of nuclear Hamiltonian are an inherent feature of Interacting Boson Model (IBM) , whose *U(6)* group structure leads to subgroup chains denoted by *U*(5), *SO*(6) and *SU*(3) , which describe vibrational, γ-soft rotational and axially symmetric rotational, respectively. These three symmetries are depicted as the three vertices of a (symmetry) triangle. Typical partial level schemes of these symmetries are shown at their respective vertex. Most nuclei do not directly manifest these symmetries exactly; however these symmetries provide a sort of bench mark of structure and allow for a simple mapping procedure to locate any collective nucleus in the triangle.

The basic idea is embodied in the Ising-like Hamiltonian:

$H = H_{sph} + \kappa H_{def}$........................(9)

where H_{sph} denotes the Hamiltonian of a higher symmetry (e.g. a spherical vibrator) with the coupling constant ε, whereas H_{def} has a lower symmetry of the deformed field with coupling constant κ. The resultant structure of the system is determines solely by the ratio ε/κ. If this ratio is large, the spherical solution dominates and if this ratio is small then the nucleus is said to be deformed. The transition in shapes takes place at a critical value $(\varepsilon/\kappa)_{crti}$. The IBM Hamiltonian in case of consistent Quantum formulation (*CQF*) can be written as:

$H = n_d - Q.Q$……………….(10)

the Hamiltonians described above has variation with respect to only one parameter ε/κ, thus only giving two extremes. The third dynamical symmetry is incorporated as the quadruple operator ***Q*** is dependent on an internal parameter χ, which determines the axial symmetry and its stiffness. With these two parameters any point in the symmetry triangle can be labeled. This is done in terms of polar coordinate, where ζ which is related to ε/κ represents the radial coordinate and χ represents the angular coordinate. The Hamiltonian, described in the above equation, along with the dependence on these parameters also depends on the boson number N_B, defined as half the number of valence nucleons.

Observables such as $R_{4/2}$, defined as the ratio of level energy for the 2^+ and 4^+ levels, vary systematically across the triangle. The sudden change in the value for $R_{4/2}$ has been described in terms of phase transitional behavior, leading to a new class of critical point symmetries that describe nucleus at the phase transitional point. These are denoted by E(5) for a second order vibrator to γ-soft rotor transition] and X(5) (for a first order vibrator to axial rotor phase transition].

RESULTS AND DISCUSSION

2.1 Interaction Parameters

The Tables 1 contain the IBM-2 Hamiltonians' parameters (in MeV) used in the present study to calculate the energies of the positive parity low-lying levels of $^{64-80}$Ge. N_π =2 and N_ν changes from 4 to 7 for $^{64-72}$Ge and finally varies from 6 to 3 for $^{74-80}$Ge. The Hamiltonian parameter values of IBM-2 were estimated by fitting to the experimental energy levels and it was made by allowing one parameter to vary while keeping the others constant. This procedure was carried out iteratively until an overall fit was achieved.

The computer program NPBOS [17] , was used to make the Hamiltonian diagonal. In principle, all parameters can be varied independently in fitting the energy spectrum of one nucleus. As a results calculations, we find that the structure of the spectra determined almost by four quantities $\varepsilon, \kappa, \chi_\pi$ and χ_ν. These quantities may in general depend both on the proton boson number N_π and neutron boson number N_ν. Guided by the microscopic calculations of [18] . We have assumed that only ε and κ depend on N_π and N_ν i.e., $\varepsilon(N_\pi, N_\nu)$, $\kappa(N_\pi, N_\nu)$ while χ_π depend only on N_π constant for all isotopes and χ_ν on N_ν. Thus a set of isotopes have the same value of χ_π. The parameterization allows one to correlate a large number of experimental data. Similarly , when a proton-proton interaction $V_{\pi\pi}$ and neutron-neutron interaction $V_{\nu\nu}$ is added, the coefficients C_L are taken as $C_L^\pi(N_\pi)$ and $C_L^\nu(N_\nu)$ i.e., the proton-proton interaction will only depend on N_π and neutron-neutron on N_ν.

Table 1: IBM-2 Hamiltonian parameters, all parameters in MeV units except and χ_π are dimensionless. χ_ν

Isotopes	ε	κ	χ_ν	χ_π	$C_{0\nu}$	$C_{2\nu}$	$C_{4\nu}$	$C_{0\pi}$	$C_{2\pi}$	$C_{4\pi}$	$\xi_1 = \xi_3$	ξ_2
Ge-62	1.200	-0.200	1.200	-0.7	0.0	0.0	-0.31	0.0	0.0	0.0	0.061	-0.060
Ge-64	1.235	-0.220	1.250	-0.7	0.0	0.0	-0.33	0.0	0.0	0.0	0.061	-0.060
Ge-66	1.370	-0.235	1.200	-0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.061	-0.055
Ge-68	1.401	-0.200	1.225	-0.7	-1.50	0.0	0.21	0.0	0.0	0.0	0.051	-0.040
Ge-70	1.425	-0.195	1.325	-0.7	-0.19	-0.38	0.17	0.0	0.0	0.0	0.022	-0.039
Ge-72	1.300	-0.245	1.150	-0.7	-2.41	0.0	0.16	0.0	0.0	0.0	0.021	-0.030
Ge-74	1.090	-0.210	1.100	-0.7	-1.21	-1.21	-0.22	0.0	0.0	0.0	-0.021	-0.029
Ge-76	0.945	-0.215	1.100	-0.7	0.0	0.0	-0.90	0.0	0.0	0.0	-0.021	-0.021
Ge-78	0.930	-0.215	1.100	-0.7	0.0	0.0	-0.22	0.0	0.0	0.0	-0.011	-0.018
Ge-80	1.200	0.225	1.00	-0.7	0.0	-0.9	0.0	0.0	0.0	0.0	-0.100	-0.013

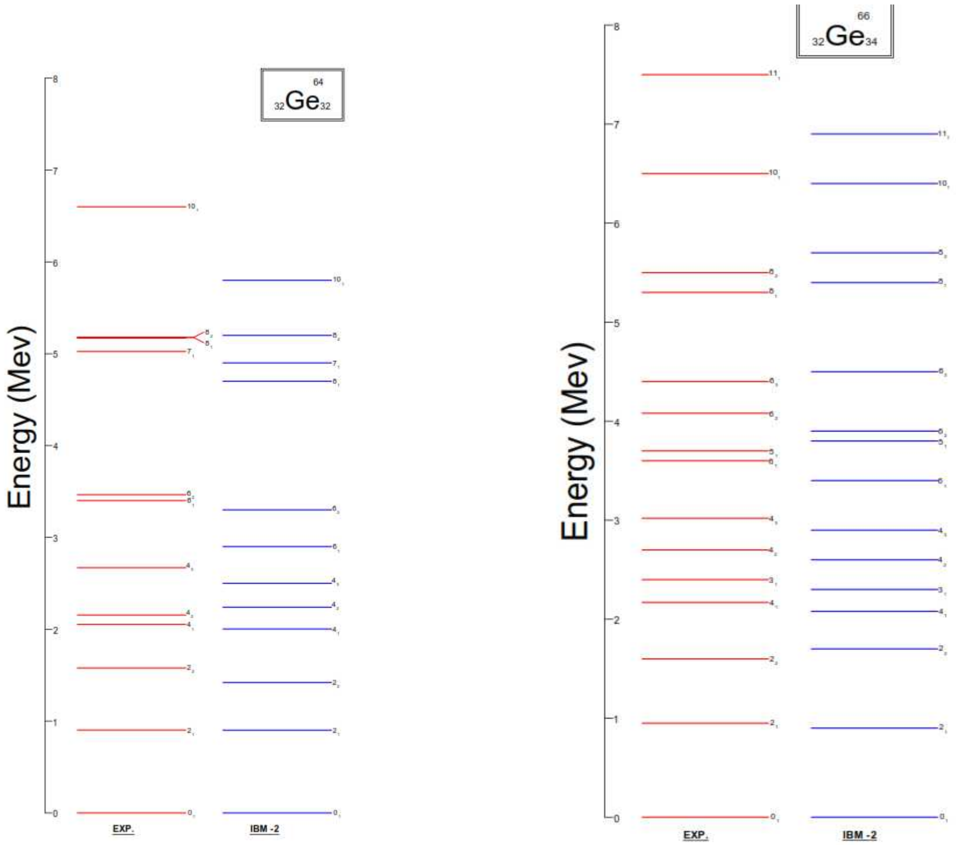
2.2 Energy Levels

We have applied the model describe in the previous section to the calculation of the energy levels of the isotopic chain $^{64-80}_{32}Ge$ in major shell 28 and 50. The results are shown in (figs. 2-9). A detailed comparison with experimental data is shown in the figures.

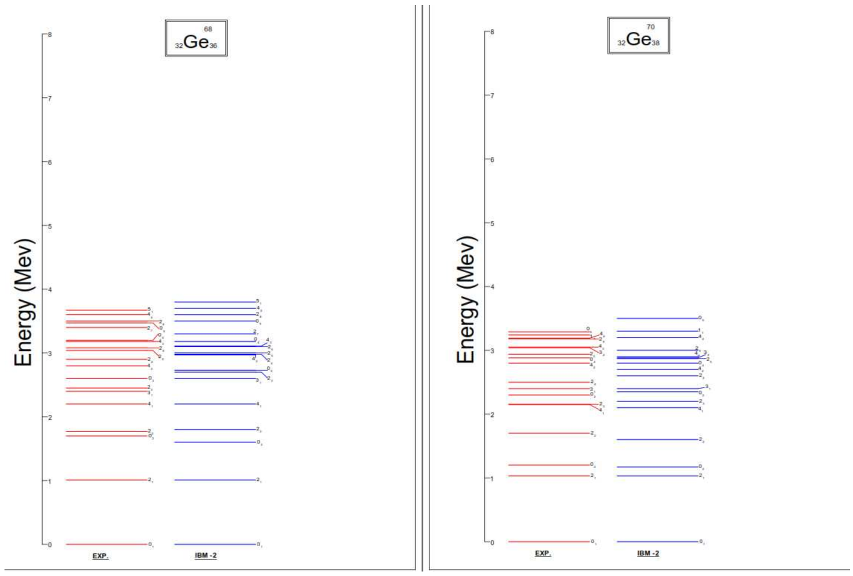
Table 2: Values $E(L_1^+ / 2_1^+)$ for *Ge* Isotopes

Isotopes	$E(4_1^+/2_1^+)$			$E(6_1^+/2_1^+)$			$E(8_1^+/2_1^+)$		
	Exp.[17]	E(5)	IBM-2	Exp.[17]	E(5)	IBM-2	Exp.[17]	E(5)	IBM-2
Ge-64	2.275	2.3	2.225	3.8	5.3	3.318	5.7	5.3	5.253
Ge-66	2.273	2.3	2.184	3.8	5.2	3.623	-	5.2	5.730
Ge-68	2.233	2.2	2.214	3.6	5	3.567	4.8	5.0	4.760
Ge-70	2.072	2.1	2.075	3.5	4.3	3.467	-	4.3	4.542
Ge-72	2.071	2.1	2.465	3.3	4.3	3.913	4.8	4.3	4.732
Ge-74	2.458	2.5	2.508	-	6.6	4.166	-	6.6	7.114
Ge-76	2.508	2.5	2.536	-	7	4.227	-	7	6.478
Ge-78	2.536	2.5	2.560	-	7	4.413	-	7	5.986
Ge-80	2.643	2.5	2.619	-	7	3.607	-	7	4.885

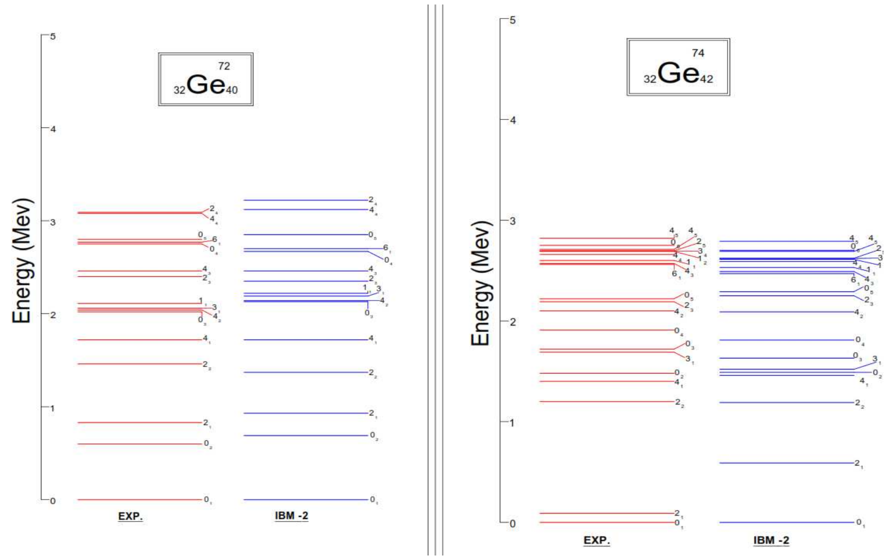
As it can be seen from the figures 2-9, the agreement between the experimental (EnSDF, 2010) [19].And theoretical results are quite good and the general features are reproduced well, especially for the members of the ground-state band. The value of $R_{4/2}$ ratio has the limiting value 2 for a quadrupole vibrator, 2.5 for a non-axial gamma-soft rotor and 3.33 for an ideally symmetric rotor. As it is seen in table 2 it increases gradually from about 2.28 to 2.60. The agreement between the experimental values and IBM-2 for $E(4_1^+/2_1^+)$ ratios of all Ge isotopes and the results show that $R_{4/2}>2$ for all Ge isotopes. It means that their structure seems to be varying from Harmonic Vibrator (HV) to along gamma soft rotor (SU(5)→O(6)). So, the energy levels of the $^{64-80}$Ge nuclei can be situated between the pure vibrational and rotational limit [20], are also trying to get a solution of potentials for the *E*(5) and *X*(5) models of the Bohr Hamiltonian by comparing the findings with the experimental data as well as the previous results.



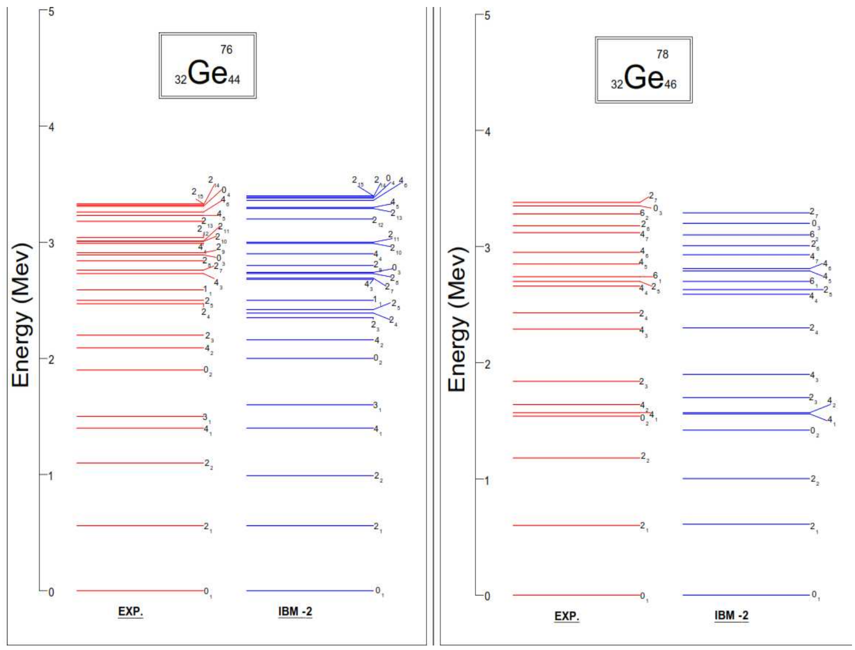
Figures 1&2 : A comparison between the experimental energy levels from IBM-2 calculations for ^{64}Ge, ^{66}Ge [19].



Figures 3&4: A comparison between the experimental energy levels from IBM-2 calculations for ^{68}Ge, ^{70}Ge [19].



Figures 5&6: A comparison between the experimental energy levels from IBM-2 calculations for [72]Ge, [74]Ge [19].



Figures 7&8: A comparison between the experimental energy levels from IBM-2 calculations for [76]Ge, [78]Ge [19].

2.3 Electromagnetic Transition Rates

NPBOS code has been used to calculate the transition matrix elements. Electric quadrupole transition probability B(E2) have been calculated using the effective charge $e_\pi = 0.0253$ eb and $e_\upsilon = 0.279$ eb which have been estimated using the method described in [17] . The results of the calculation of the *B*(*E*2) matrix elements are shown in table 3.

Calculation of electromagnetic properties gives us a good test of the nuclear model prediction. The electromagnetic matrix elements between eigenstates were calculated using program NPBTRN for IBM-2 model.

The $B(E2;2_1^+ \rightarrow 0_1^+)$ decreased for $^{64\text{-}68}Ge$ as neutron number increased and increased as neutron number increases toward the middle of the shell for the $^{70\text{-}74}Ge$. While for the $^{76\text{-}82}$Ge as the value is decreased toward the closed shell. of $B(E2;2_2^+ \rightarrow 2_1^+)$ has small value because contains admixture of $M1$. As a consequence of possible $M1$ admixture, this quantity is rather difficult to measure. The values of $B(E2;2_2^+ \rightarrow 0_1^+)$,

$B(E2;2_3^+ \rightarrow 0_1^+)$ and $B(E2;2_3^+ \rightarrow 2_1^+)$ is small because this transition from quasi-beta band to ground state band (cross over transition).

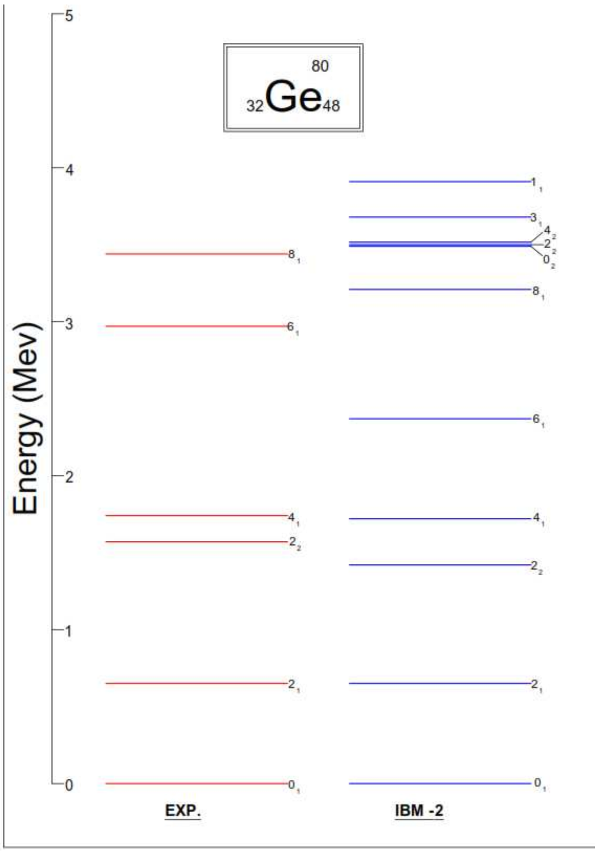


Figure 9: A comparison between the experimental energy levels from IBM-2 calculations for ^{80}Ge [19].

The magnetic transition operator $T(M1)$ were calculated using equations (6), and the gyromagnetic ratios by making use of equation [21] :

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \text{.................(11)}$$

$$g = \frac{Z}{A} \text{.................(12)}$$

where Z is atomic number, A- atomic mass number.

and having fit E2 matrix elements, one can then use them to obtain M1 matrix elements and then the mixing ratio $\delta(E2/M1)$, and compare them with the prediction of the model using the operator (6). If they had not been measured in the case of Ge isotopes, factors g_π and g_ν have to be estimated. In phenomenological studies g_π and g_ν are treated as parameters and kept constant for a whole isotope chain. The total g factor is defined by Many relations could be obtained for a certain mass region and then the average g_π and g_ν values for this region could be calculated, and one of the experimental B(M1) values. It is found that $g_\pi - g_\nu = 0.176\ \mu_N$. The estimated values of the parameter are $g_\pi = 0.562\ \mu_N$ and $g_\nu = 0.397\ \mu_N$. These were used to calculate the magnetic transition probability $B(M1)$ (see table 4) These values were then generalized for all Ge isotopes. They are different from those of the rare–earth nuclei, $(g_\pi - g_\nu = 0.65\mu_N)$, suggested by[15] . However they also used $g_\pi = 1$ and $g_\nu = 0$ to reduce the number of the model parameters in their calculation of M1 properties in deformed nuclei. The results of our calculation are listed in table 4. There is experimental data to compare with the *IBM*-2 calculations. As can be seen from the table yields to a simple prediction that *M*1 matrix elements values for gamma to ground and transitions should be equal for the same initial and final spin. Also the size of gamma to ground matrix elements seems to decrease as the mass number increases.

Table 3: Electric Transition probability *B*(*E*2) for Ge isotopes in e^2b^2 units

Isotopes	$J_i^+ \to J_f^+$	Exp. [19]	Present Work	Subber [14]
Ge-64	$2_1 \to 0_1$	0.0410 (60)	0.0351	0.0125
	$2_2 \to 0_1$	0.00015(5)	0.0012	0.0028
	$2_2 \to 2_1$	0.0620 (210)	0.0523	0.0166
	$2_3 \to 0_1$	-	0.0033	0.0018
	$2_3 \to 2_1$	-	0.0027	0.0012
	$4_1 \to 2_1$	-	0.020	0.0121
	$6_1 \to 4_1$	-	0.119	-
Ge-66	$2_1 \to 0_1$	0.01896(362)	0.0129	0.0212
	$2_2 \to 0_1$	0.00016(6)	0.0014	0.0029
	$2_2 \to 2_1$	0.02686(1264)	0.0310	0.0283
	$2_3 \to 0_1$	-	0.0024	0.0018
	$2_3 \to 2_1$	-	0.0281	0.0225
	$4_1 \to 2_1$	-	0.0335	0.0325
	$6_1 \to 4_1$	-	0.127	-
Ge-68	$2_1 \to 0_1$	$\geq$0.01517	0.0182	0.0273
	$2_2 \to 0_1$	0.02912(329)	0.0371	0.0048
	$2_2 \to 2_1$	0.00023(4)	0.0004	0.0406
	$2_3 \to 0_1$	0.00086(34)	0.00077	0.0038
	$2_3 \to 2_1$	-	0.0082	0.0076
	$4_1 \to 2_1$	-	0.0529	0.0446
	$6_1 \to 4_1$	-	0.129	-
Ge-70	$2_1 \to 0_1$	0.02287(29)	0.0321	0.0340
	$2_2 \to 0_1$	0.03593(68)	0.0301	0.0069
	$2_2 \to 2_1$	0.00171(85)	0.00232	0.0500
	$2_3 \to 0_1$	0.0497(189)	0.0618	0.0030
	$2_3 \to 2_1$	-	0.0015	0.0010
	$4_1 \to 2_1$	0.04112(11)	0.0681	0.0579
	$6_1 \to 4_1$	-	0.134	-
Ge-72	$2_1 \to 0_1$	0.040(3)	0.039	0.0330
	$2_2 \to 0_1$		0.0076	0.0099
	$2_2 \to 2_1$	0.114(12)	0.129	0.0478
	$2_3 \to 0_1$	-	0.0024	0.0017
	$2_3 \to 2_1$	-	0.018	0.0190
	$4_1 \to 2_1$	0.0641(71)	0.048	0.0565
	$6_1 \to 4_1$	-	0.141	-
Ge-74	$2_1 \to 0_1$	0.060(3)	0.065	0.028(5)
	$2_2 \to 0_1$	$\leq$0.078	0.0671	0.0055
	$2_2 \to 2_1$	0.0997(203)	0.0897	0.0470
	$2_3 \to 0_1$	-	0.0014	0.0017
	$2_3 \to 2_1$	-	0.0047	0.0056
	$4_1 \to 2_1$	0.0664(55)	0.0605	0.0464
	$6_1 \to 4_1$	-	0.147	-
Ge-76	$2_1 \to 0_1$	0.046(3)	0.0498	0.026
	$2_2 \to 0_1$	-	0.0032	0.0041
	$2_2 \to 2_1$	0.0746(96)	0.0687	0.0308
	$2_3 \to 0_1$	-	0.0019	0.0011
	$2_3 \to 2_1$	-	0.0013	0.000
	$4_1 \to 2_1$	0.073(13)	0.0587	0.0373
	$6_1 \to 4_1$	-	0.152	-
Ge-78	$2_1 \to 0_1$	0.044(30)	0.0402	0.0230
	$2_2 \to 0_1$		0.0041	0.0033
	$2_2 \to 2_1$	0.0396(238)	0.0298	0.0164
	$2_3 \to 0_1$	-	0.0037	0.0040
	$2_3 \to 2_1$	-	0.00066	0.0007
	$4_1 \to 2_1$	$\geq$0.0218	0.029	0.0160
	$6_1 \to 4_1$	-	0.160	-
Ge-80	$2_1 \to 0_1$	0.028(5)	0.021	0.034
	$2_2 \to 0_1$	-	0.0019	0.0012
	$2_2 \to 2_1$	-	0.0023	0.0019
	$2_3 \to 0_1$	-	0.00167	0.000
	$2_3 \to 2_1$	-	0.00023	0.000
	$4_1 \to 2_1$	-	0.0042	0.0036
	$6_1 \to 4_1$	-	0.163	-
Ge-82	$2_1 \to 0_1$	$9.467\text{x}10^{-3}$	0.0008	-
	$2_2 \to 0_1$	-	0.0023	-
	$2_2 \to 2_1$	-	0.0025	-

	$2_3 \rightarrow 0_1$	-	0.0021	-
	$2_3 \rightarrow 2_1$	-	0.00034	-
	$4_1 \rightarrow 2_1$	-	0.0028	-
	$6_1 \rightarrow 4_1$	-	0.0135	-

Table 4: Reduced transitions probability *B*(*M*1) in μ_N^2 units for *Ge* isotopes

Transitions	**B(M1)**								
	^{64}Ge	**^{66}Ge**	**^{68}Ge**	**^{70}Ge**	**^{72}Ge**	**^{74}Ge**	**^{76}Ge**	**^{78}Ge**	**^{80}Ge**
$2_2 \rightarrow 2_1$	0.0662	0.0288	0.0132	0.00043	0.0043	0.000421	0.00987	0.00011	0.00003
$2_3 \rightarrow 2_1$	0.0378	0.0191	0.0002	0.002	0.0052	0.00051	0.0057	0.00005	0.00262
$2_3 \rightarrow 2_2$	0.0561	0.0442	0.0251	0.0045	0.00022	0.0020	0.0209	0.00012	0.00005
$3_1 \rightarrow 2_1$	0.0662	0.0432	0.00145	0.0389	0.00081	0.000421	0.00987	0.00011	0.00003
$1_1 \rightarrow 0_1$	0.451	0.560	0.755	0.799	0.823	0.896	0.9022	0.910	0.943

The δ(*E2*/*M*1) mixing ratios for some selected transitions in *Ge* isotopes are calculated from the useful equations as above and with the help of *B*(*E*2) and *B*(*M*I) values which are obtained from NPBEM (computer code which is subroutine of *NPBOS* package program) (Otsuka and Yoshida,1985) , the results are given in table 5. In general, the calculated electromagnetic properties of the *Ge* isotopes do not differ significantly from those calculated in experimental and theoretical work, However, there is a large disagreement in the mixing ratios of $\delta(2_2^+ \rightarrow 2_1^+)$ and $\delta(3_1^+ \rightarrow 2_1^+)$, due to the small value of M1 matrix elements.

Table 5: Mixing ratios δ(*E2*/*M*1) for *Ge*$^{64\text{-}80}$ in eb/μ_N units

Isotopes	$J_i^+ \rightarrow J_f^+$	**Exp.[19,21]**	$IBM-2$	**Subber [14]**
Ge-64	$2_2 \rightarrow 2_1$	–	-4.450	-5.6
	$2_3 \rightarrow 2_1$	–	3.764	2.3
	$3_1 \rightarrow 2_1$	–	12	10.74
	$3_1 \rightarrow 2_2$	–	0.0921	-2.027
Ge-66	$2_2 \rightarrow 2_1$	-3.5^{+18}_{-26}	2.276	-1.591
	$2_3 \rightarrow 2_1$	–	-2.980	-1.56
	$3_1 \rightarrow 2_1$	–	17.98	20.9
	$3_1 \rightarrow 2_2$	–	3.220	2.61
Ge-68	$2_2 \rightarrow 2_1$	-0.2(0.1)	-0.811	-1.934
	$2_3 \rightarrow 2_1$	–	-2.0	-1.734
	$3_1 \rightarrow 2_1$	-0.2(0.1)	-1.77	-36.78
	$3_1 \rightarrow 2_2$	-0.2(0.3)	-0.33	-0.31
Ge-70	$2_2 \rightarrow 2_1$	-5.0(3.0)	-10.19	-1.76
	$2_3 \rightarrow 2_1$	–	0.011	-5.78
	$3_1 \rightarrow 2_1$	-2.2(+5-3)	-2.86	-0.35
	$3_1 \rightarrow 2_2$	-0.05(8)	-0.087	-3.45
Ge-72	$2_2 \rightarrow 2_1$	-10.3(13)	-13.4	-3.89
	$2_3 \rightarrow 2_1$	–	10.32	-7.88
	$3_1 \rightarrow 2_1$	–	11.6	3.92
	$3_1 \rightarrow 2_2$	≈+4.0	5.22	-3.67
Ge-74	$2_2 \rightarrow 2_1$	+3.4(4)	3.96	-1.222
	$2_3 \rightarrow 2_1$	-2.8(3)	-3.21	7.44
	$3_1 \rightarrow 2_1$	0.34(5)	0.661	3.02

	$3_1 \rightarrow 2_2$	+1.3(4)	2.4	-5.789
Ge-76	$2_2 \rightarrow 2_1$	+3.5(15)	5.21	3.50
	$2_3 \rightarrow 2_1$	–	3.4	-11.58
	$3_1 \rightarrow 2_1$	–	17.2	2.44
	$3_1 \rightarrow 2_2$	–	-7.34	-6.87
Ge-78	$2_2 \rightarrow 2_1$	–	1.456	0.98
	$2_3 \rightarrow 2_1$	–	21.90	29.5
	$3_1 \rightarrow 2_1$	–	2.11	1.96
	$3_1 \rightarrow 2_2$	–	-2.56	-1.2
Ge-80	$2_2 \rightarrow 2_1$	–	-2.64	-1.6
	$2_3 \rightarrow 2_1$	–	0.002	-1.37
	$3_1 \rightarrow 2_1$	–	-0.414	-0.511
	$3_1 \rightarrow 2_2$	–	0.0115	

CONCLUSION

In this work, it has been searched that the nuclear structure and electromagnetic transitions of the nuclei $^{64\text{-}80}Ge$ in IBM-2 and E(5) symmetry, shows the characteristic E(5) pattern or not in the ground state and some other low-lying bands by using two different approaches. Transitional behavior in *Ge* nuclei is compared with the results of E(5), critical symmetry and then an acceptable degree of agreement is proved **.** We may conclude that the general characteristics of the *Ge* isotopes are well satisfied in this study and are not expected to be deformed. We have investigated an acceptable degree of agreement between the predictions of the model and experiment. The good agreement between the theoretical and experimental energy spectra , electromagnetic transition probabilities values, and mixing ratios support the hypothesis of phase transitions between vibrational to gamma unstable in these nuclei.

Calculated and experimental multipole mixing ratios ($\delta(E2/M1)$) are mostly in agreement with each other. The variations in sign of the *E2/M1* mixing ratios from nucleus to nucleus for the same class transitions and within a given nucleus for transitions from different spin states suggest that a microscopic approach is needed to explain the data theoretically. For that reason, we did not take into consideration the sign of mixing ratios. Sign convention of mixing ratios had explained in detail by Lange *et al*.,(1982) [22] .

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