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Nuclear structure and electromagnetic transition probability of Hf isotopes by means IBM-1

Saad N. Abood*, Laith A. Al-Ani and Ammar A. Alrawi

Phys. Dept. College of Science AL-Nahrain University Baghdad - Iraq

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ABSTRACT

In this work, the energy levels and transition probabilities B(E2) and B(M1) of some even-even Hf have been investigated by using the interacting boson model-1 (IBM-1). The results were compared with experimental values. It was seen that an acceptable degree of agreement between the predictions of the model (IBM-1) and the experiment is achieved.

Keywords: Interacting boson model, Electromagnetic Transition probability, Energy level.

INTRODUCTION

The nucleus consists of many nucleons, each nucleon interacts with all other and moving within a complex structure. The structure could be described by the analytical solutions of their wave function. As consequence, the excitation energies of collective quadrupole excitation states in nuclei near the closed shell are strongly depended on the number of nucleons outside the closed shells [1]. The interacting boson model of Arima and Iachello [2-7] has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model it was assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number *N* of bosons. Each boson could occupy two levels one with angular momentum $L = 0$ (*s*-boson) and another, usually with higher energy, with $L = 2$ (*d*-boson). In the original form of the model known as IBM-1, proton and neutron-boson degrees of freedom are not distinguished.

We investigate the dynamical symmetry of $^{172\text{-}180}$Hf isotopes and energy spectra and the electromagnetic transition probability B(E2) of these isotopes (Z = 72) within the framework of IBM-1.

The IBM-1 Hamiltonian

The model has an inherent group structure, associated with it. In terms of *s*- and *d*-boson operators the most general IBM Hamiltonian can be expressed as [3]:

$$H^{\wedge} = \varepsilon\left(n_s^{\wedge} + n_d^{\wedge}\right) + a_0 P^{\wedge}.P^{\wedge} + a_1 L^{\wedge}.L^{\wedge} + a_2 Q^{\wedge}.Q^{\wedge} + a_3 T_3^{\wedge}.T_3^{\wedge} + a_4 T_4^{\wedge}.T_4^{\wedge} \ldots\ldots\ldots(1)$$

where ε is the boson energy, and the operators are:

$$n_s^{\wedge} = s^{\wedge +}.s^{\wedge \sim} \;,\; n_d^{\wedge} = d^{\wedge +}.d^{\wedge \sim} \;,\; P^{\wedge} = \frac{1}{2}\left(d^{\wedge \sim}.d^{\wedge \sim}\right) - \frac{1}{2}\left(s^{\wedge}.s^{\wedge}\right)$$

$$L^{\wedge} = \sqrt{10}\left[d^{\wedge +} \times d^{\wedge \sim}\right]^{(1)} \;,\; Q^{\wedge} = \sqrt{5}\left[\left(d^{\wedge +} \times s^{\wedge \sim}\right) + \left(s^{\wedge +} \times d^{\wedge \sim}\right)\right]^{(2)} + \chi\left[d^{\wedge +} \times d^{\wedge \sim}\right]$$

$$T_3^{\wedge} = \left[d^{\wedge +} \times d^{\wedge \sim}\right]^{(3)} \;,\; T_4^{\wedge} = \left[d^{\wedge +} \times d^{\wedge \sim}\right]^{(4)}$$

The phenomenological parameters $a_0, a_1, (a_2, \chi), a_3, a_4$, represents the strengths of the paiuring angular momentum, quadrupole, octupole and hexadecopoule interaction between bosons, respectively.

The interferences between these three dynamical symmetries give three transitional regions. These regions are as follows **SU (3) $\rightarrow$ SU (5):** This transitional region can be treated by breaking SU (3) symmetry in the direction of SU (5) by adding $H^{\wedge} = \varepsilon\left(n_s^{\wedge} + n_d^{\wedge}\right) + a_3 T_3^{\wedge}.T_3^{\wedge} + a_4 T_4^{\wedge}.T_4^{\wedge}$ terms. The Hamiltonian of this region can be written as:

$$H^{\wedge} = \varepsilon\left(n_s^{\wedge} + n_d^{\wedge}\right) + a_1 L^{\wedge}.L^{\wedge} + a_2 Q^{\wedge}.Q^{\wedge} + a_3 T_3^{\wedge}.T_3^{\wedge} + a_4 T_4^{\wedge}.T_4^{\wedge} \text{.....(2)}$$

SU (3) $\rightarrow$ O (6): The nuclei in this transitional region can be treated by breaking SU (3) symmetry in the direction of O (6) by adding $P^{\wedge}.P^{\wedge}, a_3 T_3^{\wedge}.T_3^{\wedge}$ terms. The Hamiltonian of this region can be written as:

$$H^{\wedge} = a_0 P^{\wedge}.P^{\wedge} + a_1 L^{\wedge}.L^{\wedge} + a_2 Q^{\wedge}.Q^{\wedge} + a_3 T_3^{\wedge}.T_3^{\wedge} \text{.....(3)}$$

Associated with the collective states calculated with the IBM are transition operators. In the most general form, the E0, Ml, E2, M3, E4 transition operators are, to leading order, given [54,55,58]:

$$T_m^{(\ell)} = \alpha_\ell \delta_{\ell 2}\left(d^+ s + s^+ d\right)_m^{(2)} + \beta_\ell \left(d^+ d\right)_m^{(\ell)} + \gamma_{l0}\delta_{l0}\delta_{m0}\left(s^+ s\right)_0^{(0)} \text{.....(4)}$$

where l denotes the multipolarity with projection m, and α, β, γ are the coefficients of the different terms of the operator. In particular, for E2 transitions [2-3]:

$$T_m(E2) = \alpha_2\left(d^+ s + s^+ d\right)_m^{(2)} + \beta_2\left(d^+ d\right)_m^{(2)} \text{ ..... (5)}$$

This operator has two parts $\left(d^+ s + s^+ d\right)^{(2)}$: which satisfies the selection rule $\Delta n_d = \mp 1$, and $\left(d^+ d\right)^{(2)}$ which satisfies the selection rule $\Delta n_d = 0$. The coefficients and depend on the limit involved or the appropriate intermediate structure.

Exact forms of the E0, M3, and E4 operators exist. It should be noted that *no M1 transitions can occur* in first order [64,65,69]. The reasons lie in the form of the *M*l operator [2,3,4]:

$$T_m(M1) = \beta_1\left(d^+ d\right)_m^{(1)} \text{.....(6)}$$

As discussed in references [54,55,58], the operator $\left(d^+ d\right)^{(1)}$ proportional to the boson angular momentum operator; therefore, Eq. (2-7*a*) may be rewritten as

$$T_m(M1) = g_B I_m^{(1)} \text{.....(7)}$$

where g_B is the effective boson *g*-factor. This form of the operator has no off-diagonal matrix elements, implying that in this approximation Ml transitions are forbidden [2,3,4]. Some of the transition probabilities obtained from perturbation theory are further discussed in Refs. [2-3].

The reduced *E*2 and *M*1 matrix elements were combined in the calculation of the mixing ratio $\delta(E2/M1)$ using the relation [13]:

$$\delta(E2/M1; J_i^+ \rightarrow J_f^+) = 0.835 E_\gamma(MeV) \frac{< J_f^+ \| T(E2) \| J_i^+ >}{< J_f^+ \| T(M1) \| J_i^+ >} \text{.....(8)}$$

RESULTS AND DISCUSSION

1. Hamiltonian interaction parameters

According to the Hamiltonian of IBM-1, the energy of $^{172\text{-}174}$Hf isotopes (total numbers of bosons 14 and 15 respectively) lies in the transitional region SU (3) $\rightarrow$ O (6) (Eq.(2-40a)) and the $^{176\text{-}180}$Hf isotopes (total number of bosons 16,15 and 14 respectively), lies in the dynamical symmetry SU (3), Eq.(2-22a) have been calculated using the angular momentum, quadrupole and octoupole parameters (a_1, $a_2\chi$ and a_3). The best fit values of these parameters are given in Table (1), which show the values of the relevant parameters. These values are obtained by fitting to get results of the energy levels than that the experimental data [8], whereas the first two terms and the last term in Eq.(2-3a)) have now included because they are irrelevant to the case of the fully weakly deformed nuclei (rotational nuclei).

Table (1): IBM-1 Hamiltonian parameters for $^{172\text{-}180}$Hf isotopes

Isotopes	a_1	a_2	a_3	χ
^{172}Hf	0.04	-0.011	-0.07	-0.08
^{174}Hf	0.045	-0.0105	-0.064	-0.06
^{176}Hf	0.0095	-0.113	0	-0.6
^{178}Hf	0.096	-0.0146	0	-0.11
^{180}Hf	0/0101	-0.014	0	-0.26

2 Energy spectra

IBM-1 model has been used in calculating the energy of the positive parity low-lying levels of Hafnium series of isotopes. A comparison between the experimental spectra [8] and our calculations, using the values of the model parameters given in Table (1) for the ground beta and gamma bands, is illustrated in Figures (1) to (5). The agreement between the theoretical and their correspondence experimental values for all the isotopes are in a good agreement but for high spin states are slightly higher but reasonable.

Table (2) given the experimental and theoretical energy ratios, it has been found that the $^{172\text{-}174}$Hf isotopes are in the transitional region SU (3) $\rightarrow$ O (6), and the $^{176\text{-}178}$Hf are deformed isotopes (rotational nuclei) and they have the SU (3) dynamical symmetry respecting to IBM-1.

The obtained results are given in Figures (1) to (5), this figures shows the ground, β and $\gamma-$bands of experimental and IBM-1 calculation for $^{172\text{-}180}$Hf isotopes; It shown that there in good agreement between experimental energy levels and IBM-1 calculations.

Table (2) Energy ratios for $^{172\text{-}180}$Hf isotopes and in IBM-1 Dynamical Symmetries

Isotopes	$E(4_1^+/2_1^+)$			$E(6_1^+/2_1^+)$			$E(8_1^+/2_1^+)$		
	Exp.	**IBM-1**	**IBM-2**	**Exp.**	**IBM-1**	**IBM-2**	**Exp.**	**IBM-1**	**IBM-2**
^{172}Hf	3.245	3.229	3.249	6.596	6.414	6.614	10.892	10.304	10.536
^{174}Hf	3.3	3.3	3.267	6.67	6.66	6.88	11.21	11.3	11.023
^{176}Hf	3.29	3.29	3.481	6.77	6.72	7.094	11.33	11.41	11.932
^{178}Hf	3.3	3.3	3.29	7	7.2	6.784	11.67	11.8	10.774
^{180}Hf	3.3	3.3	3.3011	6.88	6.82	6.881	11.645	11.7	10.739
SU (5)	2			3			4		
O (6)	2.5			4.5			7		
SU (3)	3.33			7			12		

Experimental data are taken from ref. [8].

The root means square deviation (*rmsd)* [9]:

$$rmsd = \left[\frac{1}{N}\sum\left(E_{cal.} - E_{\exp.}\right)^2\right]^{1/2} \quad \text{.....(3-1)}$$

(where *N* is the number of energy levels)

is used to compare the experimental and theoretical energy levels. Tale (3) given the *rmsd* between experimental and theoretical energy levels. In this table we see the ground state levels the best agreement was found in ^{172}Hf isotope

where the smallest value of *rmsd* is equal 0.0039 and equal 0.010 for gamma band in ^{178}Hf isotope. However, *rmsd* = 0.0099 for beta band in ^{180}Hf isotope.

Table (3): The root means square deviations (*rmsd)* between experimental and calculated energy levels for $^{172\text{-}180}$Hf isotopes

Isotopes	root mean square deviations (*rmsd)*					
	ground state band		$\beta-$ band		$\gamma-$ band	
	IBM-1	IBM-2	IBM-1	IBM-2	IBM-1	IBM-2
^{172}Hf	0.0039	0.0031	0.059	0.042	0.014	0.018
^{174}Hf	0.0046	0.0029	0.061	0.040	0.013	0.0131
^{176}Hf	0.0360	0.0030	0.054	0.041	0.012	0.001
^{178}Hf	0.0340	0.0025	0.044	0.0038	0.010	0.09
^{180}Hf	0.0140	0.016	0.0099	0.022	0.019	0.012

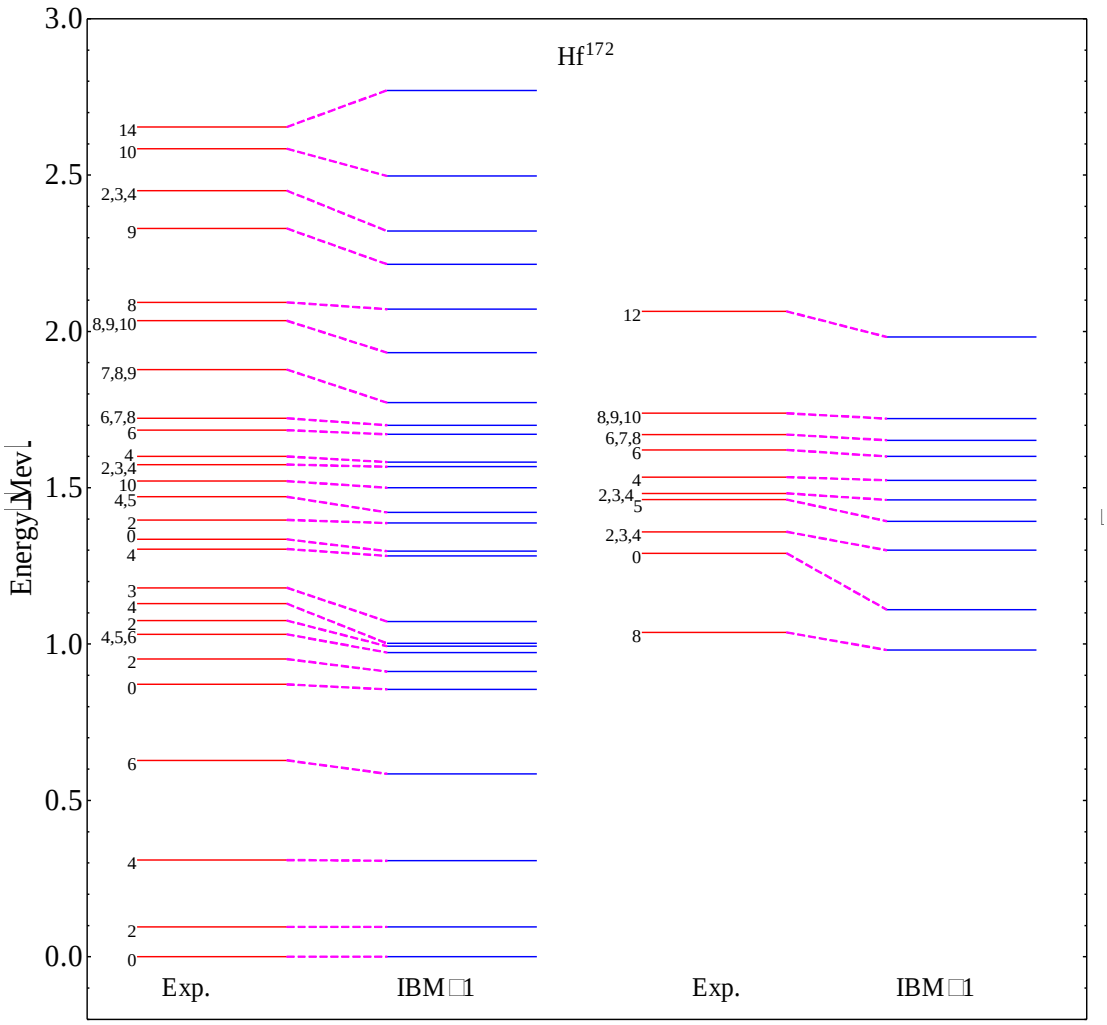

Fig. (1): Comparison between experimental data [8] and IBM-1 calculated energy levels for ^{172}Hf

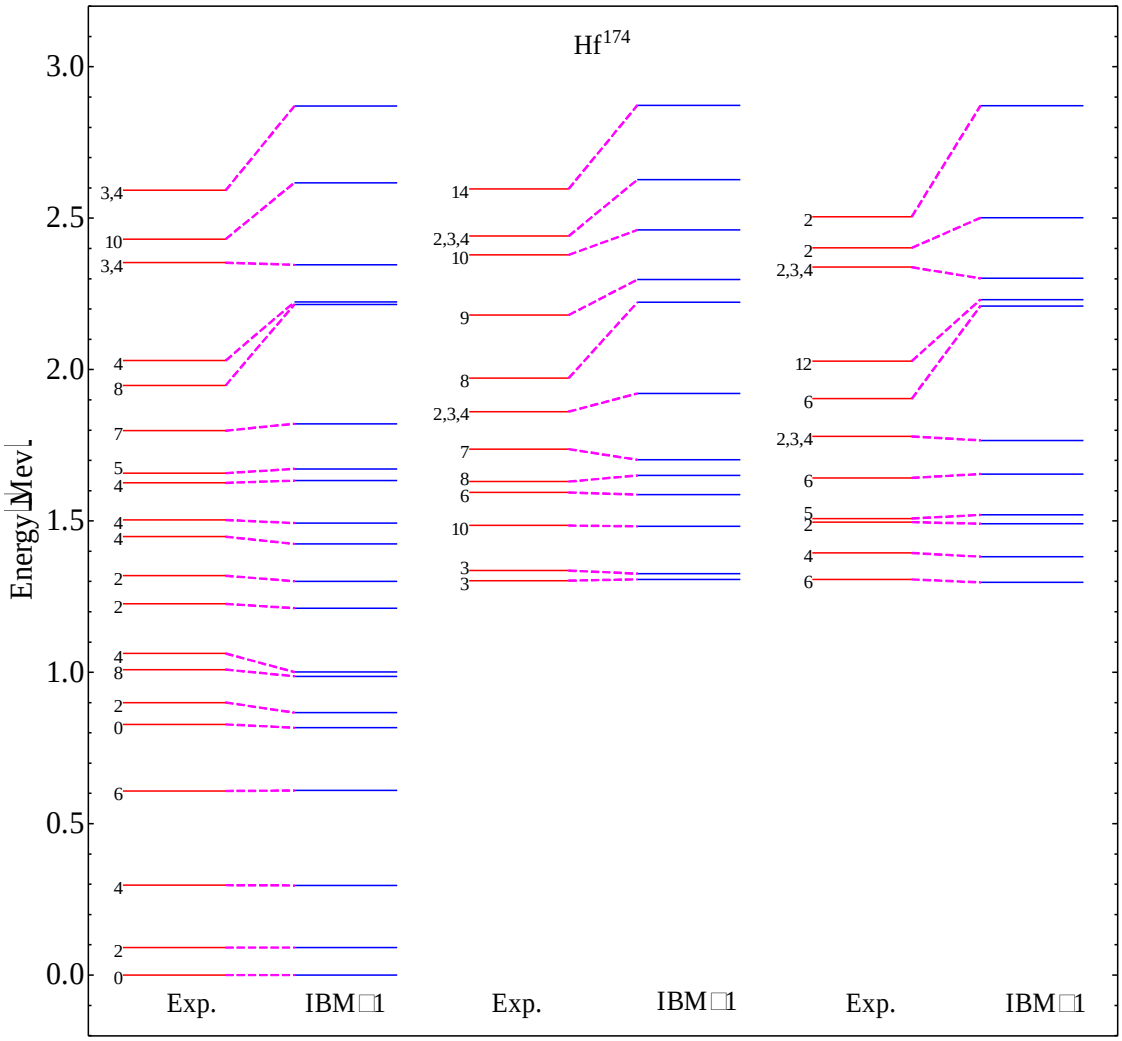

Fig. (2): Comparison between experimental data [8] and IBM-1 calculated energy levels for ^{174}Hf

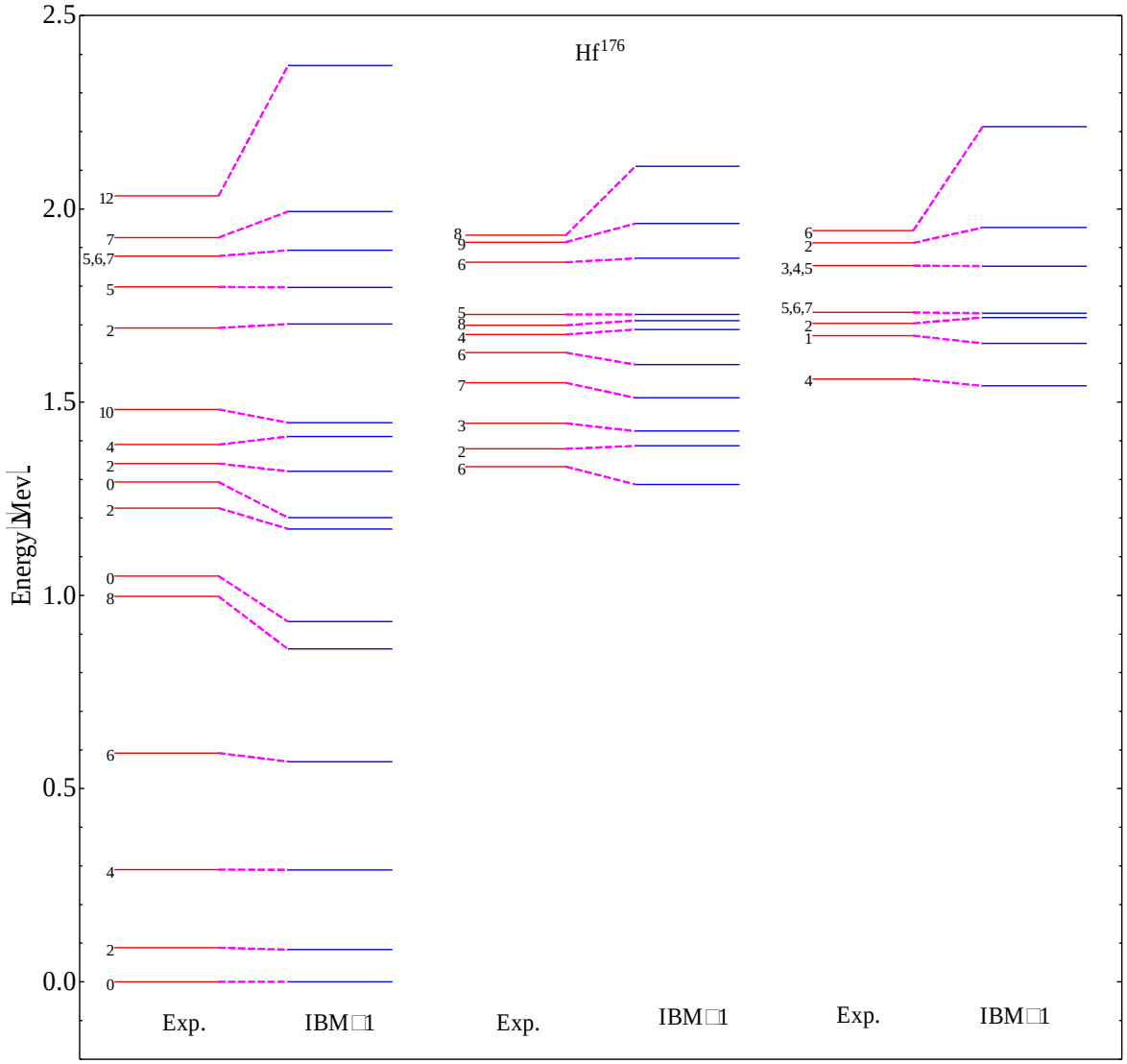

Fig. (3): Comparison between experimental data [8] and IBM-1calculated energy levels for ^{176}Hf

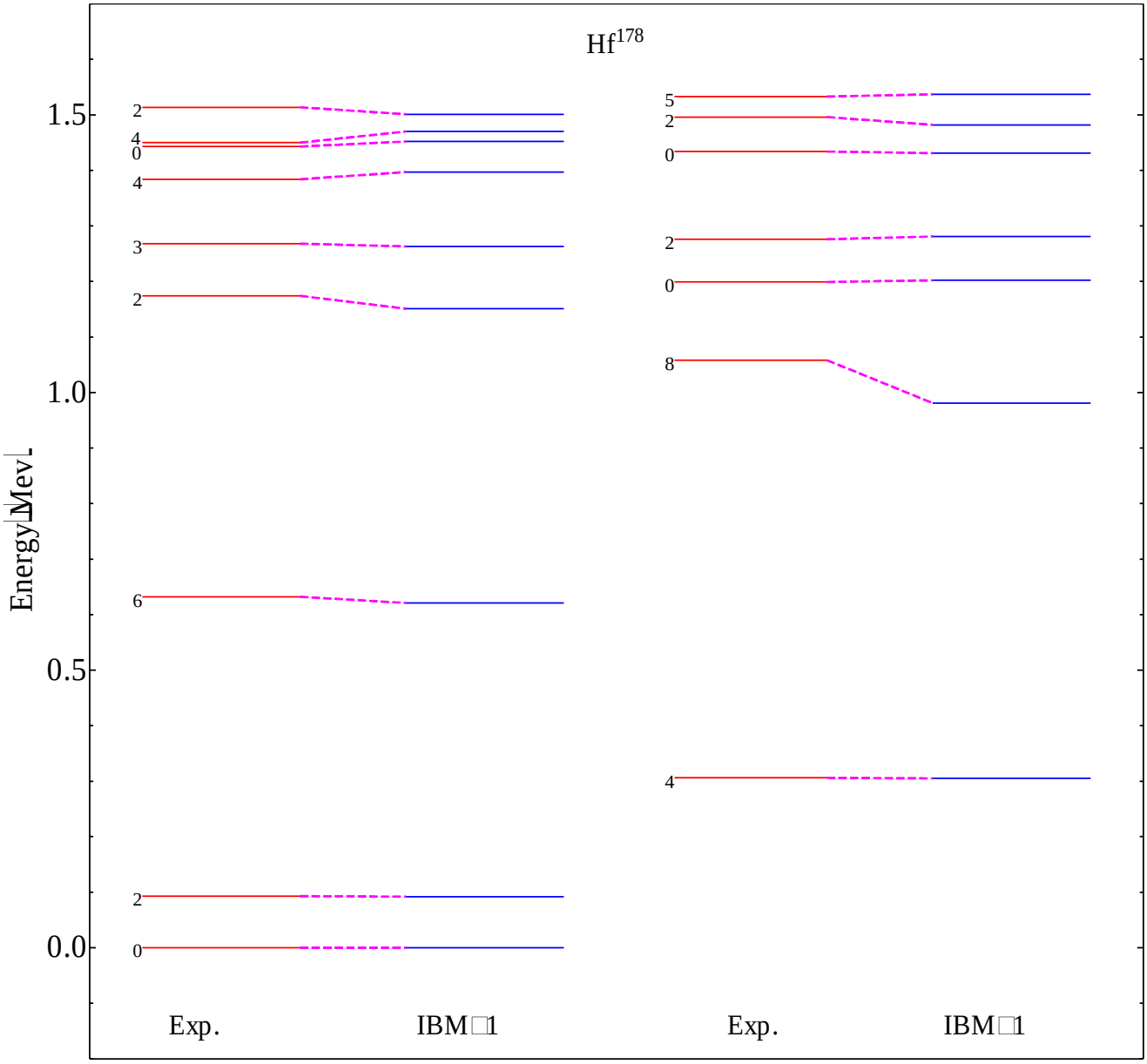

Fig. (4): Comparison between experimental data [8] and IBM-1 calculated energy levels for ^{178}Hf

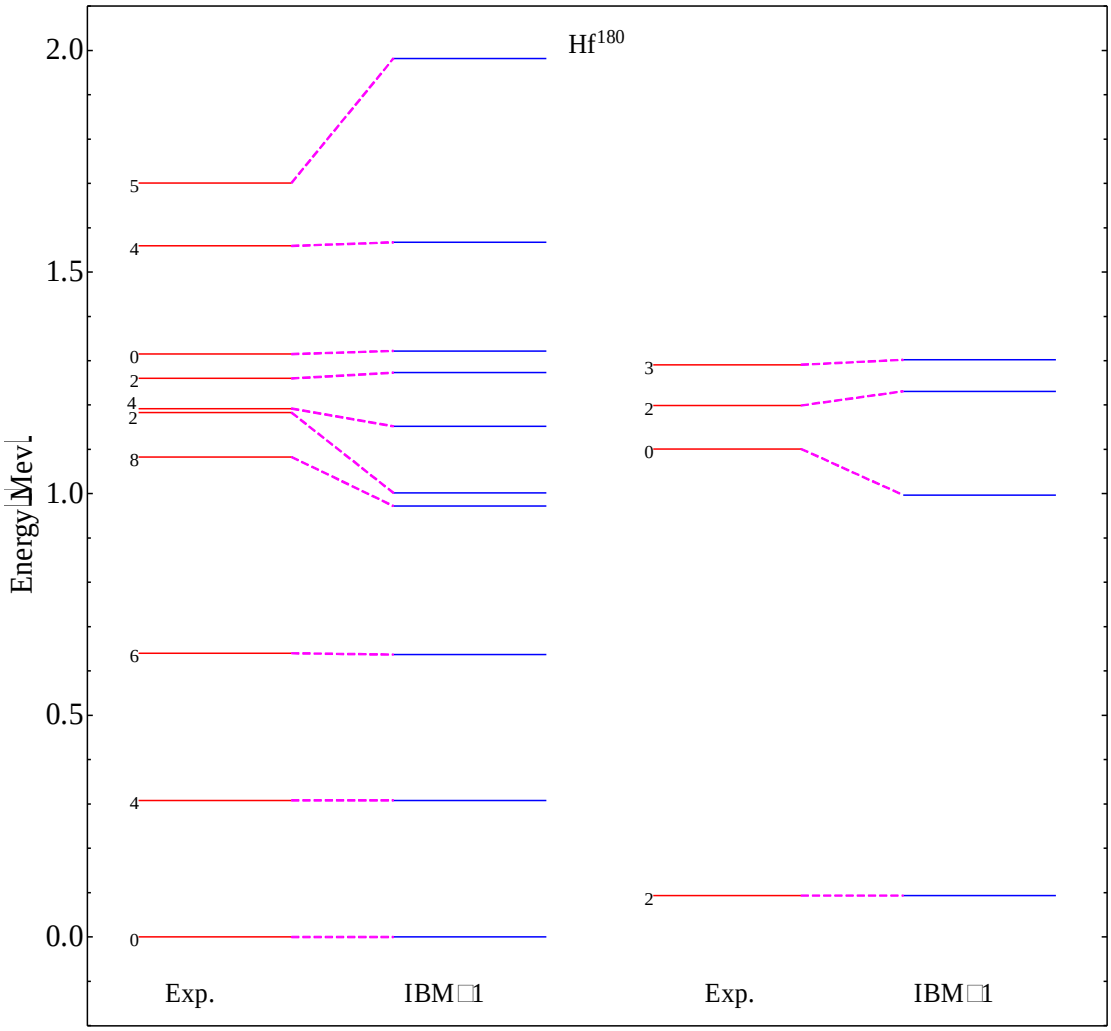

Fig. (5): Comparison between experimental data [8] and IBM-1 calculated energy levels for ^{180}Hf

3. Electric Transition Probability B(E2)

The E2 transitions provide more stringent test of the IBM-1. The general E2 transition operator is given by the Eq. (6). The coefficient α_2 called the boson effective charge is an overall scaling factor for all B(E2) values which is determined from the fit to the $B(E2;2_1^+ \rightarrow 0_1^+)$ value. The coefficient β_2 may be determined from the quadrupole moment Q (2_1^+). The ratio β_2 / α_2 = χ = -1.32 in the SU (3) limit and is reduced to zero in the O (6) limit. In the "**FBEM**" program the corresponding parameters are $\alpha_2 = (E2SD)$ and $\beta_2 = (1/\sqrt{5})(E2DD)$. The used parameters in T(E2) matrix element of $^{172\text{-}180}$Hf isotopes are given in Table (4)**.**

As we noticed in IBM-1 results the B(E2) for $\beta \rightarrow g$ and $\gamma \rightarrow g$ transitions can vanish when these nuclei are treated as SU (3) symmetric nuclei. This problem was solved by breaking this symmetry in the direction of U (5) and employing the ε parameter. The calculated B(E2) values were improved by this attempt.

In Table (5) which shows that the electric transition probability for $\beta \rightarrow g$ and $\gamma \rightarrow g$ are smaller than the electric transition probabilities between $g \rightarrow g$ band, and in this table shows also that, in general, there is a good agreement between the experimental and theoretical B(E2) values in ground state band in $^{174\text{-}180}$Hf isotopes except the transition $6_1^+ \rightarrow 4_1^+$ in $^{174\text{-}180}$Hf, where the experimental and IBM-1 results of this transitions are weak in agreement. The experimental and IBM-1 B(E2) calculations between beta and ground band and between gamma

band in general are weakly in agreement except the transition $2_2^+ \rightarrow 0_1^+$ in ^{176}Hf isotope and $2_3^+ \rightarrow 0_1^+$ in ^{178}Hf isotope which gave a good agreement.

The weak agreement between experimental and theoretical in some B(E2) values in those isotopes can be explained by the fact that many small component of the initial and final wave functions contribute coherently to the value of the reduced E2 transition probability, since these small components are not stable enough against small changes in the model parameters [10]. There are no available experimental data to many transitions in Table (5); therefore, it has been predicted by IBM-1.

Table (4): The reduced matrix element parameters for $^{172-180}$Hf isotopes

Isotopes	$B(E2;2_1^+ \rightarrow 0_1^+)\ (e^2.b^2)$	$\alpha_2(e.b)$	$\beta_2(e.b)$
^{172}Hf	0.92	0.046	-0.22
^{174}Hf	1.0615	0.042	-0.33
^{176}Hf	1.04	0.125	-0.54
^{178}Hf	0.97	0.127	-0.033
^{180}Hf	0.95	0.139	-0.031

Table (5): Electric Transition Probability $B(E2; J_i \rightarrow J_f)$ for $^{172-180}$Hf in e^2. b^2 units

	^{172}Hf		^{174}Hf		^{176}Hf		^{178}Hf		^{180}Hf	
$J_i \rightarrow J_f$	Exp.	IBM-1	Exp.	IBM-1	Exp.	Exp.	IBM-1	Exp.	IBM-1	Exp.
$2_1 \rightarrow 0_1$	0.92	0.982	1.0615	1.081	1.04	1.087	0.97	0.953	0.95	0.93
$4_1 \rightarrow 2_1$	-	0.0371	-	1.426	-	1.427	-	1.42	1.38	1.02
$2_2 \rightarrow 0_1$	-	7×10^{-4}	0.012	0.016	0.00057	0.0007	0.0043	0.0052	0.114	0.097
$2_3 \rightarrow 0_1$	-	3×10^{-4}	0.027	0.029	0.023	0.072	0.023	0.033	0.023	0.031
$2_2 \rightarrow 2_1$	-	0.072	0.0567	0.072	-	0.081	-	0.431	-	0.22
$2_3 \rightarrow 2_1$	-	0.0073	0.042	0.028	-	0.0066	0.026	0.028	0.031	0.052
$2_2 \rightarrow 4_1$	-	-	0.00179	0.001	-	-	0.026	0.031	0.031	0.044
$2_4 \rightarrow 4_1$	-	-	$>3.80\times10^{-4}$	2.2×10^{-4}	-	-	-	-	-	-
$2_4 \rightarrow 0_1$	-	-	-	-	-	-	0.013	0.0157	-	-
$6_1 \rightarrow 4_1$	-	-	-	1.269	-	1.54	1.3	1.472	1.32	1.302
$8_1 \rightarrow 6_1$	-	-	-	-	-	-	1.41	1.627	1.5	1.672
$10_1 \rightarrow 8_1$	-	-	-	-	-	-	1.51	1.596	1.44	1.621
$0_2 \rightarrow 2_1$	-	-	-	-	-	-	-	0.0022	0.0018	0.0022
$1 \rightarrow 2_1$	-	0.00044	-	0.0066	0.0284	0.0007	-	0.00088	-	-
$1 \rightarrow 2_2$	-	0.00023	-	0.053	0.0129	0.0024	-	0.00001	-	-
$1 \rightarrow 2_3$	-	0.0451	-	0.128	0.098	0.096	-	0.086	-	-
$Q(2_1^+)$	-	-0.431	-	-1.872	-2.1	-2.21	-2	-2.11	-2	-1.872

Experimental values are taken from Refs. [8, 11,12]

4- Magnetic Transition Probability and Mixing Ratio δ (E2/M1)

To evaluate the magnetic transition probability B(M1), we depend on Eqs. (6) and (7), where the effective boson *g*-factor is estimated using the fact g = *Z*/*A* is. The form (7) of the operator has no off-diagonal matrix elements, implying that in this approximation Ml transitions are forbidden [2,,3,4]. Some of the transition probabilities obtained from perturbation theory are further discussed in refs. [2,3].

The results shows that the transitions between low-lying collective states are weak. This is because of the increase of antisymmetric component in the wave functions. The magnitude of M1 values increase with increasing spin for $\gamma \rightarrow g$ and $\gamma \rightarrow \gamma$ transitions, see Table (6).

The *E*2/*M*1 multiple mixing ratios for $^{172\text{-}180}$Hf isotopes, $\delta(E2/M1)$, were calculated for some selected transitions between states of $\Delta J = 0$. The sign of the mixing ratio must be chosen according to the sign of the reduced matrix elements. The equations used are (7) for M1 transitions and (8) for the mixing ratios. The results are listed in Table (7). The agreement with available experimental data [8] is more than good especially in the sign of the mixing ratio. However, there is a large disagreement in the mixing ratios of $3^+ \rightarrow 2^+$, due to the small value of M1 matrix elements.

The present high-precision measurements indicate some disagreements and these would not change significantly if the δ value recommended by Lange *et a1*., [13] were used. The most serious disagreement occurs for the $3^+ \rightarrow 4^+$ transition which has the same initial state as the $3^+ \rightarrow 2^+$ reference transition. A possible conclusion is that one or both of the ground-state band levels contain admixtures. The difference between the measured and deduced δ values for the $2^+ \rightarrow 2^+$ transition may be due to mixing in either or both of the levels.

Band mixing, and in particular a $K = 1$ admixture within the $K = 0$ ground state band, has previously been considered necessary in order to explain the M1 component in transitions linking the γ and ground-state bands. An analysis following the Mikhailov formulation and involving the lower-spin states indicates a substantial $K = 1$ admixture. The approximately equal value δ **(**E2/M1) obtained for all such transitions suggests that this mixing is uniform within the ground-state band.

The IBM-1 formalism predicts essentially the same spin dependence for M1 transitions in $^{172\text{-}180}$Hf isotopes as does a geometrical approach, and is thus capable of giving at least an equally good description of the data. In addition, the IBM-1 model yields the simple prediction that $\Delta(E2/M1)$ values of $\gamma \rightarrow \gamma$ and $\gamma \rightarrow g$ transitions should be equal for the same initial and final spins, and this prediction seems to be borne out empirically. It has been shown that different signs for $\beta \rightarrow g$ and $\gamma \rightarrow g$ $\Delta(E2/M1)$ values can be reproduced by the IBM-1 model.

Table (6): Magnetic Transition Probability $B(M1; J_f^+ \rightarrow J_i^+)$ for $^{172\text{-}180}$Hf isotopes in μ_N^2

$J_i^+ \rightarrow J_f^+$	^{172}Hf	^{174}Hf	^{176}Hf	^{178}Hf	^{180}Hf
$2_2 \rightarrow 2_1$	0.0038	0.028	0.0077	0.0035	0.0279
$2_3 \rightarrow 2_1$	0.0773	0.024	0.0139	0.079	0.0732
$3_1 \rightarrow 2_1$	0.0034	0.0091	0.0089	0.00078	0.00039
$3_1 \rightarrow 2_2$	0.037	0.025	0.0022	0.0062	0.00062
$3_2 \rightarrow 2_1$	0.017	0.0031	0.0065	0.0067	0.00048
$4_2 \rightarrow 4_1$	0.0035	0.0037	0.006	0.0045	0.0029
$1_1 \rightarrow 0_1$	0.814	0.822	0.768	0.834	0.88
$1_1 \rightarrow 2_1$	0.0018	0.0037	0.0173	0.002	0.0026
$1_1 \rightarrow 2_2$	0.0138	0.0271	0.00162	0.0052	0.0031
$1_1 \rightarrow 2_3$	0.0971	0.093	0.023	0.0237	0.012

Table (7): Mixing Ratio $\delta(E2/M1)$ for $^{172\text{-}180}$Hf in eb/μ_N units

$J_j^+ \rightarrow J_f^+$	^{172}Hf		^{174}Hf		^{176}Hf		^{178}Hf		^{180}Hf	
	Exp.	**IBM-1**	**Exp.**	**IBM-1**	**Exp.**	**IBM-1**	**Exp.**	**IBM-1**	**Exp.**	**IBM-1**
$2_2 \rightarrow 2_1$	-	0.0431	-2^{+2}_{-2}	-3.779	$\lvert\delta\rvert \geq 4$	6.431	0.410	0.621	$9.8^{+3.6}_{-3.2}$	10.761
$2_3 \rightarrow 2_1$	-	10.227	-	2.551	-	10.2	$\delta < 32$	20	$6.8^{+3.6}_{-3.2}$	5.257
$3_1 \rightarrow 2_1$	-	4.762	-	4.530	-	2.098	-	-0.471	-	1.334
$3_1 \rightarrow 4_1$	-	-	-	1.320	-	-	-	-	-	
$4_2 \rightarrow 4_1$	-	0.007	$-2.5^{+13}_{-0.7}$	-18.5	$\lvert\delta\rvert \geq 0.7$	1.22	-	-	4.5(1.1)	6.573
$2_4 \rightarrow 4_1$	-	-	0.00039	0.0752	0.022	2.87	$-0.74^{+0.19}_{-0.12}$	-0.931	-	-
$6_2 \rightarrow 6_1$	-	3.272	-0.92(8)	-7.420	0.04	0.0073	-	0.761	-	32.12

CONCLUSION

Theoretical calculations of $^{170-180}$Hf (with Z=72) were performed by using IBM-1. The $^{172-174}$Hf total numbers of bosons 14,15 respectively (weakly deformed) lies in the transitional region SU (3) $\longrightarrow$ O (6) and the $^{176-180}$Hf isotopes (total number of bosons 16,15 and 14 respectively), lies in the dynamical symmetry SU (3) (deformed nuclei).

The low-lying levels and electromagnetic transition probabilities of the $^{172-180}$Hf isotopes by using interacting boson model-1 are compared with the available experimental data. A good agreement was obvious.

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