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New algorithms for minimization of non linear functions by numerical methods

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ABSTRACT

In this paper, we propose three new algorithms namely, Orthogonal Circle Approach Algorithm, Orthogonal Circle-Tangent Approach Algorithm and Family of quadratic convergence algorithms for minimization of non-linear functions. Some illustrations are discussed by using the first two new algorithms with Newton's algorithm. Then comparative study among the new algorithms is established by using few nonlinear functions.

Key Words: Non-linear functions, Newton's method, Orthogonal Circle Approach Algorithm, Orthogonal Circle-Tangent Approach Algorithm**,** Family of quadratic convergence Algorithms.

INTRODUCTION

Optimization problems with or without constraints arise in various fields such as science, engineering, economics, management sciences, etc., where numerical information is processed. In recent times, many problems in business situations and engineering designs have been modeled as an optimization problem for taking optimal decisions. In fact, numerical optimization techniques have made deep in to almost all branches of engineering and mathematics.

Several methods [10,12,13,15,16,21,22,27-32]are available for solving unconstrained minimization problem. To solve unconstrained nonlinear minimization problems arising in the diversified field of engineering and technology, we have several methods to get solutions. For instance, multi- step nonlinear conjugate gradient methods [8], a scaled nonlinear conjugate gradient algorithm[1], a method called, ABS-MPVT algorithm [17] are used for solving unconstrained optimization problems. Newton's method [19] is used for various classes of optimization problems, such as unconstrained minimization problems, equality constrained minimization problems.

In this article [4] global convergence for first- and second-order stationary points of a class of derivative-free trust-region methods for unconstrained optimization is established. These methods are based on the sequential minimization of quadratic models built from evaluating the objective function at sample sets. The article [5], shows how the bounds on the error between an interpolating polynomial and the true function can be used in the convergence theory of derivative free sampling methods. These bounds involve a constant that reflects the quality of the interpolation set .The article [6], present an introduction to a new class of derivative free methods for unconstrained optimization. It starts by discussing the motivation for such methods first and then reviews the past developments in this field, before introducing the features that characterize the newer algorithms. In the context of a trust region framework, focus on techniques that ensure a suitable "geometric quality" of the considered models and the article [2] discuss the scope and functionality of a versatile environment for testing small-and large-scale nonlinear optimization algorithms.

In the paper[7] explains an algorithm for finding the roots of non-linear equations is developed by introducing a weight in the formula of the Bisection method. In [24] illustrate the application of numerical optimization methods for non quadratic functional defined on non-Hilbert Sobolev spaces. These methods use a gradient defined on a norm-reflexive and hence strictly convex normed linear space and in the paper[18], any cluster point of a sequence defined by a steepest descent algorithm in a general normed vector space is a critical point.

A new algorithm [11] is used for solving unconstrained optimization problem with the form of sum of squares minimization. A derivative based algorithm [14] is used for a particular class of mixed variable optimization problems. Vinay Kanwar et al**.** [25] introduced new algorithms called, external touch technique and orthogonal intersection technique for solving the non linear equations. Further, they did the comparative study of the new algorithms and Newton's algorithm. Also, Xinyuan Wu and Dongsheng Fu [26] introduced a family of new quadratic convergence algorithms which is a new high order convergence iteration method without employing derivatives for solving non linear equations.

In this paper, we propose initially two new algorithms namely, Orthogonal Circle Approach Algorithm and Orthogonal Circle-Tangent Approach Algorithm for minimization of non linear functions and compared the new algorithms with Newton's algorithm. Then, we introduce another new algorithm called the Family of quadratic convergence algorithm for minimization of non linear functions. Finally, the comparative study among the new algorithms is established using numerical examples.

NEW ALGORITHMS

In this section, we introduce two new numerical algorithms namely, Orthogonal Circle Approach Algorithm and Orthogonal Circle Tangent Approach Algorithm for minimizing nonlinear real valued and twice differentiable real functions using the concept of external touch and orthogonal intersection of numerical algorithms [25].

Consider the nonlinear optimization problem :

Minimize $\{ f(x),\ x \in R,\ f: R \to R \}$

where f is a non-linear twice differentiable function.

2.1. Orthogonal Circle Approach Algorithm

Consider the function $G(x) = x - (g(x)/g'(x))$ where $g(x) = f'(x)$. Here $f(x)$ is the function to be minimized. $G'(x)$ is defined around the critical point x^* of $f(x)$ if $g'(x^*) = f''(x^*) \neq 0$ and is given by

$$G'(x) = g(x)g''(x)/g'(x).$$

If we assume that $g''(x^*) \neq 0$, we have $G'(x^*) = 0$ iff $g(x^*) = 0$.
Consider the equation

$$g(x) = 0 \qquad \text{-- (1)}$$

whose one or more roots are to be found. $y = g(x)$ represents the graph of the function $g(x)$ and assume that an initial estimate x_0 is known for the desired root of the equation $g(x) = 0$.

A circle C_1 of radius $g(x_0 + h)$ is drawn with centre at any point $(x_0 + h, g(x_0 + h))$ on the curve of the function $y = g(x)$ where h is small positive or negative quantity. Another circle C_2 with radius $g(x_0 - h)$ and centre at $(x_0 - h, g(x_0 - h))$ is drawn on the curve of the function g(x) such that C_2 intersects C_1 orthogonally.

Let $x_1 = x_0 + h$, $|h| < 1$. Since the circles C_2 and C_1 intersects orthogonally, we have

$$g^2(x_0 + h) + g^2(x_0 - h) = (x_0 + h - x_0 + h)^2 + (g(x_0 + h) - g(x_0 - h))^2$$

$$4h^2 = 2\, g(x_0 + h)g(x_0 - h)$$

Expanding $g(x_0 + h)$ and $g(x_0 - h)$ by Taylor's series (omitting fourth and higher powers of h)

$$4h^2 - 2\{\, \{g(x_0) + h\,g'(x_0) + \frac{h^2\, g''(x_0)}{2!} + ...\} \times \{g(x_0) - h\,g'(x_0) + \frac{h^2 g''(x_0)}{2!} - .....\}\} = 0 \qquad \text{-----(I)}$$

Simplifying, we can conclude that (by retaining the first two terms of the series)

$$h = \pm \frac{g(x_o)}{\sqrt{2 + g'^2(x_o)}} \qquad \text{---- (2)}$$

where h can be taken positive or negative according as x_0 lies in the left or right of true root or slope of the curve at $(x_0, g(x_0))$ is positive or negative. If x_0 lies in the left of true root, then h is taken as positive otherwise negative. Therefore, we get the first approximation to the root as

$$x_1 = x_0 \pm h$$

That is, $$x_1 = x_0 \pm \frac{g(x_o)}{\sqrt{2 + g'^2(x_o)}}$$

Since $g(x) = f'(x)$, it follows that

$$x_1 = x_0 \pm \frac{f'(x_o)}{\sqrt{2 + f''^2(x_o)}}$$

The general iteration formula for successive approximate minimizing point of the non-linear function f is

$$x_{n+1} = x_n \pm \frac{f'(x_n)}{\sqrt{2 + f''^2(x_n)}} \qquad \text{----(3)}$$

Now, we use this Orthogonal Circle Approach Algorithm for finding the minimizing point of the non-linear function of the real valued real functions. Suppose given a non linear function $f(x)$. Find a and b such that $f'(a)$ and $f'(b)$ are of opposite signs such that $a < b$. Initialize for $x_0, \in, f'(x), f''(x), f'''(x)$

Algorithm-I:

1. Let $n = 0$
2. If we choose $x_0 = a$, then go to step 3
   otherwise if we choose $x_0 = b$, then go to step 9
3. Repeat
4. $x_{n+1} = x_n + \dfrac{f'(x_n)}{\sqrt{2 + f''^2(x_n)}}$
5. $n \leftarrow n+1$
6. Until $|x_n - x_{n-1}| < \in$
7. Optimal solution $x^* \leftarrow x_n$
8. End
9. Repeat
10. $x_{n+1} = x_n - \dfrac{f'(x_n)}{\sqrt{2 + f''^2(x_n)}}$
11. $n \leftarrow n+1$
12. Until $|x_n - x_{n-1}| < \in$
13. Optimal solution $x^* \leftarrow x_n$
14. End

2.2. Orthogonal Circles Tangent Approach Algorithm

Consider the function $G(x) = x - (g(x)/g'(x))$ where $g(x) = f'(x)$. Here $f(x)$ is the function to be minimized. $G'(x)$ is defined around the critical point x^* of $f(x)$ if $g'(x^*) = f''(x^*) \neq 0$ and is given by

$$G'(x) = g(x)g''(x)/g'(x).$$

If we assume that $g''(x^*) \neq 0$, we have $G'(x^*) = 0$ iff $g(x^*) = 0$.

Consider the equation

$$g(x)=0 \qquad \text{-- (5)}$$

whose one or more roots are to be found. $y=g(x)$ represents the graph of the function $g(x)$ and assume that an initial estimate x_0 is known for the desired root of the equation $g(x)=0$.

A circle C_1 of radius $g(x_0+h)$ is drawn with centre at any point $(x_0+h, g(x_0+h))$ on the curve of the function $y=g(x)$ where h is small positive or negative quantity. Another circle C_2 with radius $g(x_0-h)$ and centre at $(x_0-h, g(x_0-h))$ is drawn on the curve of the function g(x) such that it touches the circle C_1 externally.

Let $x_1=x_0+h$, $|h|<1$. Since the circles C_2 and C_1 touches externally, we have

$$h^2=g(x_0+h)g(x_0-h).$$

Expanding $g(x_0+h)$ and $g(x_0-h)$ by Taylor's series (omitting fourth and higher powers of h) and simplifying, we can conclude that

$$h^2-\{\{g(x_0)+h\,g'(x_0)+\frac{h^2\,g''(x_0)}{2!}+.....\}\times\{g(x_0)-h\,g'(x_0)+\frac{h^2 g''(x_0)}{2!}-.....\}\}=0 \qquad \text{------(II)}$$

Simplifying, we can conclude that (by retaining first two ters of the series)

$$h=\pm\frac{g(x_o)}{\sqrt{1+g'^2(x_o)}} \qquad \text{---- (6)}$$

where h can be taken positive or negative according as x_0 lies in the left or right of true root or slope of the curve at $(x_0, g(x_0))$ is positive or negative. If x_0 lies in the left of true root, then h is taken as positive otherwise negative. Therefore, we get the first approximation to the root as

$$x_1=x_0\pm h$$

That is,

$$x_1=x_0\pm\frac{g(x_o)}{\sqrt{1+g'^2(x_o)}}$$

Since $g(x)=f'(x)$, it follows that

$$x_1=x_0\pm\frac{f'(x_o)}{\sqrt{1+f''^2(x_0)}}$$

The general iteration formula for successive approximate minimizing point of the non-linear function f is

$$x_{n+1}=x_n\pm\frac{f'(x_n)}{\sqrt{1+f''^2(x_n)}} \qquad \text{----(7)}$$

Now, we will use this Circles Touch Approach Algorithm for finding the minimizing point of the non-linear function of the real valued real functions. Suppose given a non linear function $f(x)$. Find a and b such that $f'(a)$ and $f'(b)$ are of opposite signs such that $a < b$. Initialize for $x_0, \in, f'(x), f''(x), f'''(x)$

Algorithm-II:

1. Let $n = 0$
2. If we choose $x_0 = a$, then go to step 3
   otherwise if we choose $x_0 = b$, then go to step 9
3. Repeat
4. $$x_{n+1} = x_n + \frac{f'(x_n)}{\sqrt{1 + f''^2(x_n)}}$$
5. $n \leftarrow n+1$
6. Until $|x_n - x_{n-1}| < \in$
7. Optimal solution $x^* \leftarrow x_n$
8. End
9. Repeat
10. $$x_{n+1} = x_n - \frac{f'(x_n)}{\sqrt{1 + f''^2(x_n)}}$$
11. $n \leftarrow n+1$
12. Until $|x_n - x_{n-1}| < \in$
13. Optimal solution $x^* \leftarrow x_n$
14. End

2.3. Convergence analysis for Orthogonal Circle Approach and Orthogonal Circle-Tangent Approach Algorithm

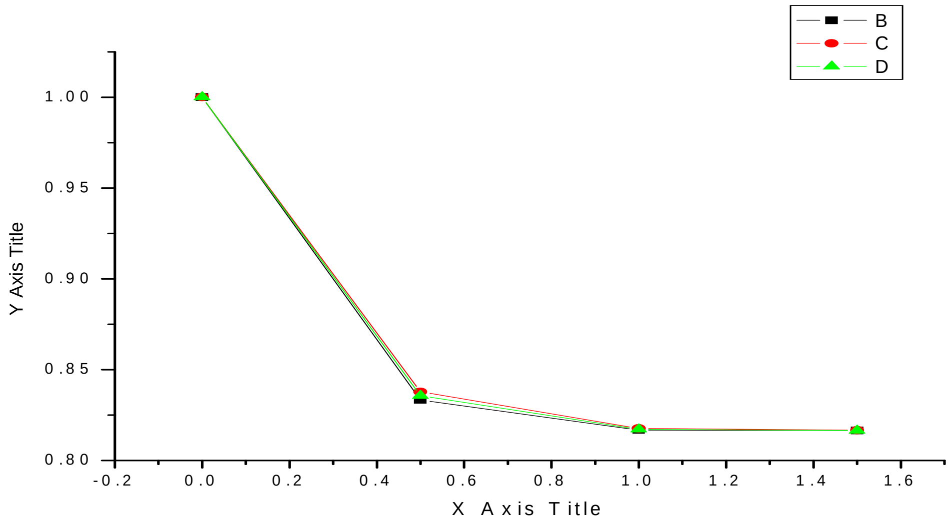
Both these algorithms is similar to the algorithms of special circle formulae[18]. Since the order of convergence of the special circles formulae has linear convergence if $|f'(x_n)| < 1$, otherwise quadratic convergence. Hence we have the order of convergence of new algorithms.

By Newton's algorithm, Orthogonal Circle Approach and Orthogonal Circle Tangent Approach Algorithm, a minimizing point for the given function are obtained and then, the comparative study of the above said algorithms have been established by means of examples.

2.4. Numerical Illustrations:

Example 2.4.1: Consider the function $f(x) = x^3 - 2x - 5,\ x \in R$. Then, minimizing point of the function is equal to 0.8165 which is obtained in 4 iterations by Newton's Algorithm and the same value is obtained by the Orthogonal Circle Approach Algorithm and Orthogonal Circle Tangent Approach Algorithm for the initial value $x_0 = 1$ and also seen the iterations for the initial values of 2 and 3.

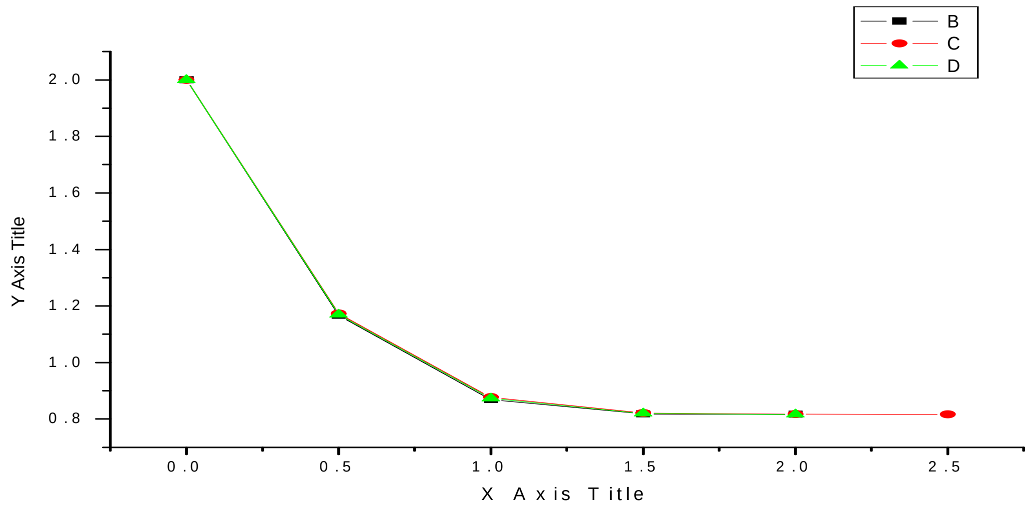
Iterations	Newton's Algorithm	Orthogonal Circle approach algorithm	Orthogonal Circle-Tangent approach algorithm
1	1	1	1
2	0.8333	0.83778	0.835601
3	0.8167	0.81755	0.817080
4	0.8165	0.81654	0.816509



Series B depicts Newton's algorithm, series C depicts Orthogonal Circle approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

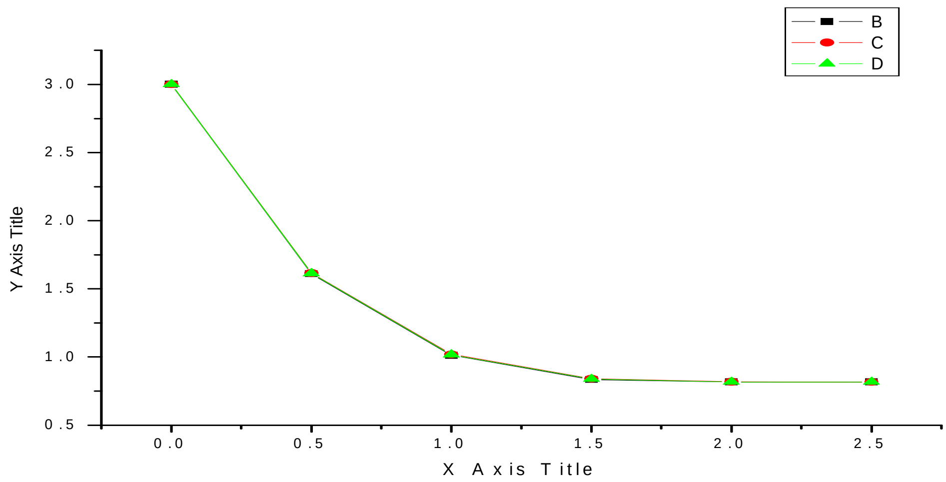
Iterations	Newton's Algorithm	Orthogonal Circle approach algorithm	Orthogonal Circle Tangent approach algorithm
1	2	2	2
2	1.1667	1.172394	1.169545
3	0.8690	0.876437	0.872782
4	0.8181	0.820533	0.819278
5	0.8165	0.816663	0.816557

Example 2.4.2: Consider the function $f(x) = x^4 - x - 10 \quad x \in R$. Then, minimizing point of function is equal to 0.62996 which is obtained in by 5 iterations by Newton' Algorithm and the same value is obtained by Orthogonal Circle approach algorithm and Orthogonal Circle Tangent Approach Algorithm in 6 iterations for the initial value $x_0 = 1$ and also seen the iteration for the initial values of 2 and 3.



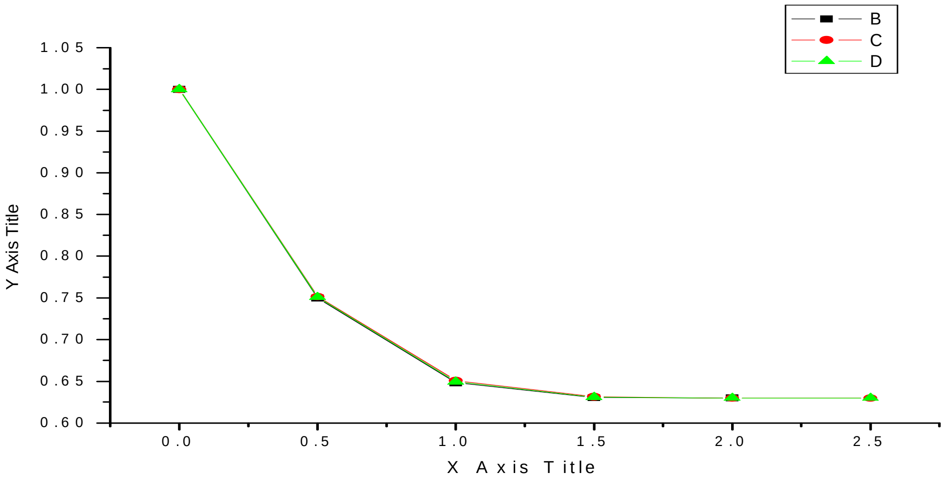
Series B depicts Newton's algorithm, series C depicts Orthogonal Circle Approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

Iterations	Newton's Algorithm	Orthogonal Circle Approach algorithm	Orthogonal Circle-Tangent Approach algorithm
1	3	3	3
2	1.6111	1.615368	1.613250
3	1.0125	1.020340	1.016424
4	0.8355	0.841566	0.838535
5	0.8167	0.817785	0.817203
6	0.8165	0.816548	0.816511



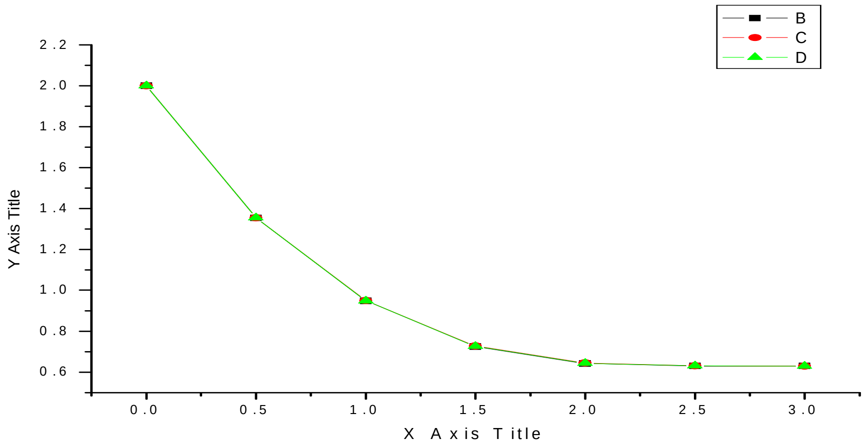
Series B depicts Newton's algorithm, series C depicts Orthogonal Circle Approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

Iterations	Newton's algorithm	Orthogonal Circle Approach algorithm	Orthogonal Circle Tangent Approach Algorithm
1	1	1	1.00000
2	0.750000	0.751718	0.750864
3	0.648148	0.650789	0.649485
4	0.630466	0.631358	0.630901
5	0.629961	0.630028	0.629982



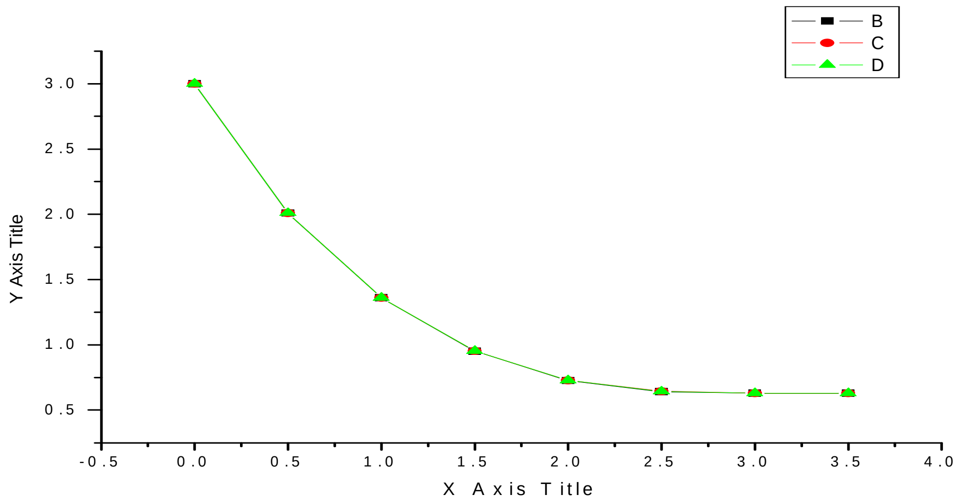
Series B depicts Newton's algorithm, series C depicts Orthogonal Circle Approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

Iterations	Newton's algorithm	Orthogonal Circle Approach Algorithm	Orthogonal Circle Tangent Approach Algorithm
1	2	2	2.00000
2	1.354167	1.354447	1.354307
3	0.948222	0.949225	0.948724
4	0.724830	0.727195	0.726020
5	0.641836	0.644413	0.643139
6	0.630179	0.630819	0.630483
7	0.629961	0.629997	0.629972



Series B depicts Newton's algorithm, series C depicts Orthogonal Circle Approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

Iterations	**Newton's algorithm**	**Orthogonal Circle Approach algorithm**	**Orthogonal Circle Tangent Approach Algorithm**
1	3	3.000000	3.000000
2	2.00926	2.009344	2.009302
3	1.36015	1.360479	1.360314
4	0.95181	0.952835	0.952323
5	0.72653	0.728889	0.727715
6	0.64223	0.644824	0.643540
7	0.63019	0.630851	0.630506
8	0.62996	0.629998	0.629973



Series B depicts Newton's algorithm, series C depicts Orthogonal Circle Approach algorithm and series D depicts Orthogonal Circle Tangent Approach Algorithm

2.5. Advantages of the new Algorithms: Comparison of Orthogonal Circle Approach Algorithm and Orthogonal Circle Tangent Approach Algorithm with Newton's Algorithm

Consider the function $f(x) = e^x - 1 - \cos(\pi x)$ in the interval (0,1) and also consider the initial guess at x_0 = 0 with difference 0.1. The comparison made with the convergence of new algorithms namely Orthogonal Circle Approach algorithm, Orthogonal Circle Tangent Approach algorithm with Newton's algorithm. From the following table, it is clear that Newton's algorithm, converges to -0.093538 for the initial values x_0 = 0, 0.1, 0.2, 0.3, 0.7, 0.9 and does not converges to -0.093538 for the initial values x_0 = 0.4, 0.5, 0.6, 0.8, 1.0. But the new algorithms converges to the same value -0.093538 for the initial values x_0 = 0, 0.1, 0.2, 0.3, 0.4, 0.7, 0.8, 0.9 but does not converges to the same value for the initial values x_0 = 0.5, 0.6, 1.0. At the same time, Newton's algorithm takes 23 iterations for the convergence to -0.093538 for the initial value x_0 = 0.7 where as the new algorithms, Orthogonal Circle Approach algorithm takes 8 iterations and Orthogonal Circle Tangent algorithm takes 10 iterations to converge to the value same value. This is one of the example for the numerical instability of Newton's algorithm whereas the new algorithms have less numerical instability. The drawback of these new algorithms is it needs first, second and third derivatives of the given function whereas in Newton's algorithm involves only first and second derivatives.

Sl.No.	Initial Value	Newton's Algorithm Convergence Value No. of Iterations		Orthogonal Circle Approach Algorithm Convergence Value No. of Iterations		Orthogonal Circle Tangent Approach Algorithm Convergence Value No. of Iterations	
1	X_0 = 0	-0.093538	4	-0.093538	**4**	-0.093538	**4**
2	x_0 = 0.1	-0.093538	4	-0.093538	**4**	-0.093538	**4**
3	x_0 = 0.2	-0.093538	4	-0.093538	**5**	-0.093538	**5**
4	x_0 = 0.3	-0.093538	5	-0.093538	**6**	-0.093538	**6**
5	x_0 = 0.4	**-2.012700**	5	-0.093538	**9**	-0.093538	**15**
6	x_0 = 0.5	**-0.960754**	7	**-2.012700**	**6**	**-2.012700**	**4**
7	x_0 = 0.6	**-4.000190**	14	**-2.012700**	**5**	**-2.012700**	**8**
8	x_0 = 0.7	-0.093538	**23**	-0.093538	**8**	-0.093538	**10**
9	x_0 = 0.8	**-2.993680**	16	-0.093538	**5**	-0.093538	**5**
10	x_0 = 0.9	-0.093538	6	-0.093538	**7**	-0.093538	**7**
11	x_0 = 1.0	**-2.012700**	6	**-2.012700**	**7**	**-2.012700**	**9**

3. FAMILY OF NEW QUADRATIC CONVERGENCE ALGORITHM FOR MINIMIZATION

3.1 : Family of new quadratic convergence Algorithm

Consider a family of iteration methods for computing the minimized value of the non linear equation g(x) = 0 ---(1)where $g(x) = f'(x)$ and we restrict to functions of real variable which are real and single valued. The family of iteration formulae with a parameter p under consideration is in the following form

$$x_{n+1} = x_n - \frac{g^2(x_n)}{p.g^2(x_n) + g(x_n) - g(x_n - g(x_n))}, \quad n = 0,1,2,3 \ldots.. \quad \ldots(8)$$

where $p \in R, \ |p| < \infty$

By the reference of the article[26], we have the following theorems which will give the order of convergence of the iterative formula(8)

Theorem 3.1.1. It is assumed that $g(x) \in C^1[a,b]$ and $p.g(x) + g'(x) \neq 0$, $x \in [a, b]$. Then the equation (1) has at most a root in [a, b].

Theorem 3.1.2. If $g(x) \in C^1[a,b]$, $g(a)g(b) < 0$ and $p.g(x) + g'(x) \neq 0$, then equation (1) has a unique root $\bar{x}$ in [a, b].

Theorem 3.1.3. Under the assumption of Theorem 2, suppose that $g(\bar{x}) = 0$ and $U(\bar{x})$ is a sufficiently small neighborhood of $\bar{x}$. Let $g''(x)$ be continuous and $p.g(x) + g'(x) \neq 0$ in $U(\bar{x})$ and $f'(\bar{x}) \neq 0$. Then the sequence $\{x_n\}$ generated by the iteration formula (2) with a parameter p are at least quadratically convergent.

3.2. Selection of the parameter p in the iteration formula

In the iterative formula, the value of the parameter p_n is calculated in each step. The value of p_n is chosen in such a way that the denominator of the corresponding iteration formulae does not vanish. The iteration method with the value of p_n is given by

$$x_{n+1} = x_n - \frac{g^2(x_n)}{p.g^2(x_n) + g(x_n) - g(x_n - g(x_n))} \qquad n = 0, 1, 2,3, \ldots. \qquad \textbf{……(9)}$$

where $x_0 \in [a, b]$ and

$$p_n = \begin{cases} 1, & if \ f(x_n) - f(x_n) - f(x_n - f(x_n)) \geq 0 \\ -1, & if \ f(x_n) - f(x_n) - f(x_n - f(x_n)) \leq 0 \end{cases}$$

3.3. Comparative study of the rate of convergence the new algorithms

Example 3.3.1.: Consider the equation $f(x) = x^3 - 2x - 5 = 0, \ x_0 = 1$

Table 1: Number of iterations and minimized value 0.816497 obtained from the following new algorithms

Sl. No	New Methods	Number of iterations	Obtained solution
1.	Orthogonal Circle-Tangent Approach algorithm	4	0.816497
2.	Orthogonal Circle Approach Algorithm	5	0.816497
3.	Family of new quadratic convergence Algorithm	5	0.816497

Example 3.3.2: Consider the equation $f(x) = x^4 - x - 10 = 0, \ x_0 = 1$

Table 2: Number of iterations and minimized value 0.629961 obtained from the following new algorithms

Sl. No	New Methods	Number of iterations	Obtained solution
1.	Orthogonal Circle-Tangent Approach algorithm	5	0.629961
2.	Orthogonal Circle Approach Algorithm	6	0.629961
3.	Family of new quadratic convergence Algorithm	7	0.629961

Example 3.3.3: Consider the equation $f(x) = xe^{x} - 1 = 0, \ x_0 = 1$

Table 3: Number of iterations and minimized value -1 obtained from the following new algorithms

Sl. No	New Methods	Number of iterations	Obtained solution
1.	Orthogonal Circle-Tangent Approach algorithm	15	-1
2.	Orthogonal Circle Approach Algorithm	20	-1
3.	Family of new quadratic convergence Algorithm	Divergent	Divergent

CONCLUSION

In this paper, we have introduced new numerical algorithms namely, Orthogonal Circle approach algorithm, Orthogonal Circle-Tangent approach algorithm and Family of new quadratic convergence algorithms for minimization of nonlinear unconstrained optimization problems. It is clear from the comparative study that among the new algorithms the rate of convergence of Orthogonal Circle-Tangent Algorithm is better than the other two algorithms. In real life problems, the variables can not be chosen arbitrarily rather they have to satisfy certain specified conditions called constraints. Such problems are known as constrained optimization problems. In near future, we have a plan to extend the proposed new algorithms to constrained optimization problems.

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