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Natural frequency analysis of a box- type satellite structure using Bubnov-Galerkin's method

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ABSTRACT

This study is concerned with the understanding of the free flexural vibrations of a box-type satellite structure. Specifically, an equation of motion for the determination of the fundamental natural frequency of the box satellite was formulated using Bubnov-Galerkin's Method as a tool. The box satellite structure is considered to be built up from six rectangular plates, each of which is assumed to be perfectly elastic, homogeneous, and isotropic and of uniform thickness. By using the properties of symmetric boundary conditions, only one-eighth of the box was idealized and analyzed using the Galerkin's method. The formulated equation of motion for the fundamental natural frequency of the box was then tested with the data used by Lin and Pan (2009) in their box structure analyses. The outcome of the result shows a good agreement between the present study and the works of Lin and Pan using Displacement Finite Element Method software package.

Key words: Vibration, Box / Satellite Structure, Natural/Fundamental Frequency, Galerkin's Method.

INTRODUCTION

Satellite technology is very important for rapid national development. Both geostationary and non-geostationary satellites are used for earth observation and communication applications. For Nigeria to join the league of space explorers, indigenous expertise in satellite technology is indispensible.

Usually, satellite structure support parts of any spacecraft and accommodates units, instruments and deployable components such as solar array and booms. In designing and building engineering and flight models of satellites, there is need to guarantee structural stability and check the decoupling of mounted gadgets, instruments and attachments during the launching and operation of the satellite.

Launching of satellite into space generates vibration of the satellite structure in question. During such launching of the satellite, the vibration of the satellite structure may build up to dangerous magnitude with severe consequences. In general, the consequences of structural vibration include stressing and collapse of structures, cracking, damage to safety-related equipment, fatigue and adverse human responses (Smith, 1988). These are likely to occur when a satellite structure vibrates at natural frequency, leading to structural resonance. At this point, the satellite structure may become unstable and mounted gadgets and instruments become decoupled. It is therefore necessary to determine the natural frequency at which a satellite structure can vibrate during its launching and operation.

However, the core subject matter of this study on "Frequency Analysis of a Box-Type Satellite Structure" does not appear to have been studied. Previous work on vibration of box type structures focused on establishing analytical methods to obtain the natural frequencies and mode shapes. Dickson and Warburton (1967) developed a series solution and also gave a comprehensive survey of the possible analytical approaches. However, the series approach becomes impracticable for the analysis of complex structures, whereas, the introduction of structural discontinuities presents no inherent difficulty with the finite element method.

Popplewell (1971) was able to overcome the shortcomings encountered by Dickson and Warburton when he applied the use of displacement based finite element method to analyze the same box structure used by Dickson and Warburton. However, Popplewell relied on the already existing finite element software's and could not develop a handy equation for easy evaluation of natural frequency. The present study however employed the Galerkin's Method to develop a handy equation for the determination of the fundamental natural frequency of the box-type satellite structure.

MATERIALS AND METHODS

2.1 System Description

Equipment on board satellites are very often mounted on light weight panels. This study considers one such mass loaded panel in the form of a box structure. The equipment mounted inside the box panels will be modeled as a set of distributed masses and like any other box structure, there is need to determine the fundamental natural frequency at which the entire box structure vibrates at the absence of external loads. The box structure whose geometry is shown in Fig. 1 is assumed to be made of rectangular plates and its vibrational analysis will be carried out using Bubnov-Galerkin's Method.

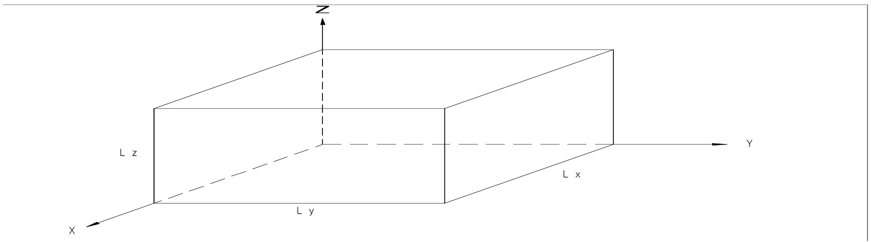

Fig. 1 Geometry of the Box Structure

2.2. Idealization of the Box-Type Satellite Structure

The basic assumption in the frequency analysis of this box, is that the box is assumed to be built up from six rectangular plates folded together to form a box. Each folded plate is considered to be perfectly elastic, homogeneous, isotropic and of uniform thickness. The displacement of each plate is assumed small compared with the wavelength of flexural vibrations. Also, the membrane displacements of a box structure are assumed to be much smaller than the bending displacements. Therefore, they may be neglected. This means that lines of intersection of two faces cannot deform and only rotation about such a line is possible. In addition neither displacements nor rotations are possible at corners.

The box structure is idealized in such a way that the symmetrical properties of a freely vibrating uniform box are adopted. Thus if the box in Fig. 2 has three planes of symmetry, then only one- eighth of the box need to be idealized (**Petyt, 1990**). The idealized box structure is shown in Fig.2.

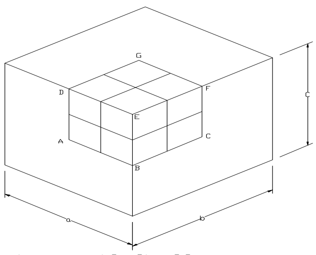

Fig. 2: An idealized box structure.

As a consequence of the above assumptions the box may be treated as a flat plate in 2D as shown in Fig. 3, where X- and Y- axes are lines of symmetry, as are the lines GF and CF.

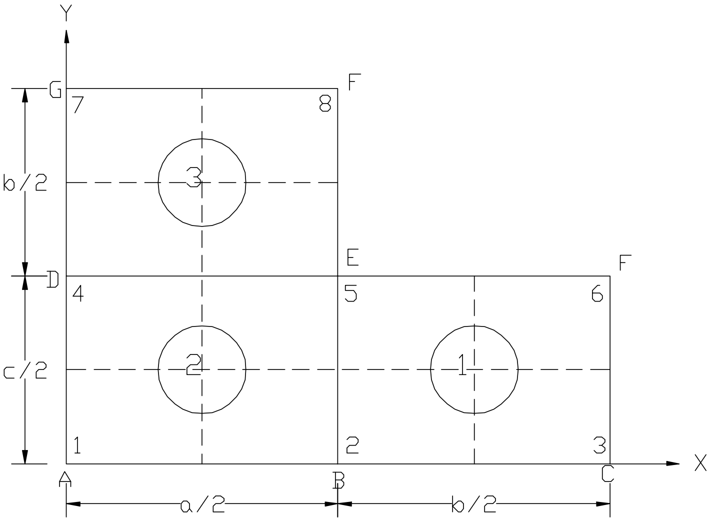

Fig.3: Flat plate idealization of one-eighth of a box (shown in Fig. 2.)

2.3. Governing Dynamic Differential Equations

It will be recalled that the static equilibrium equation of plate is given by:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (1)$$

Where q = External Load.
w = deflection; and x and y are the co-ordinate axes of the plate
D = Bending rigidity /flexural rigidity

However Bezukhov and others (1990) noted that the equations of vibration can be derived by adding the inertia forces of a given system to the external forces appearing in applicable equilibrium equation.

For plate this inertia force is given by Rao (1992) as

$$Z = -\rho h \frac{\partial^2 w}{\partial t^2} \quad (2)$$

Thus adding Eqn. (2) to Eqn. (1) gives the basic equation for the vibrations performed by a plate as:

$$D\nabla^4 w_{(x,y,t)} + \rho h \frac{\partial^2 w_{(x,y,t)}}{\partial t^2} - q\sin \omega t = 0 \quad (3)$$

Now for a free vibration case, , qsin ωt = 0 and Eqn. (3) reduces to

$$D\nabla^4 w_{(x,y,t)} + \rho h \frac{\partial^2 w_{(x,y,t)}}{\partial t^2} = 0 \quad (4)$$

Eqn. (4) is the governing differential equation of the undamped free linear vibrations of plate. A general solution of Eqn.(4) can be obtained by assuming the following solution for the deflection, w (x,y,t).

Let,

$$w(x,y,t) = (A\cos\omega t + B\sin\omega t)\, W(x, y) \quad (5)$$

where ω = natural frequency

W(x, y) = the shape function which describes the modes of vibration and some harmonic function of a time, and A and B are constants

Thus, substituting Eqn. (5) into Eqn. (4) and simplifying yields:

$$D\nabla^4 W(x,y) - \rho h \omega^2 W(x,y) = 0 \quad (6)$$

Dividing through by D, gives

$$\nabla^4 W(x,y) - \frac{\rho h \omega^2}{D} W(x,y) = 0 \quad (7)$$

Let $\lambda = \frac{\rho h \omega^2}{D}$ (8)

Thus Eqn. (7) can be rewritten as:

$$\nabla^4 W(x,y) - \lambda W(x,y) \quad = 0 \qquad (9)$$

Eqn. (9) is the governing dynamic differential equation for each plate in this present work.

where ∇^4 = Bi harmonic differential operator

$$= \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\,\partial y^2} + \frac{\partial^4}{\partial y^4} \qquad (10)$$

W(x,y) = shape function /displacement of plate.

$$D = \frac{Eh^3}{12(1-\mu^2)} \qquad (11)$$

where E, μ, ω, ρ, and h are the young's modulus, poison's ratio, natural frequency, mass density and thickness of the plate respectively.

2.4. Method of Analysis

In this present work, the Bubnov-Galerkin's Method is adopted in the vibration analysis of the idealized flat plate (Fig. 3). Before the applicability of the method, the support and symmetric boundary conditions are discussed below.

2.4.1. Support and Symmetric Boundary Conditions

The use of symmetry has enabled the box structure to be idealized as a flat plate shown in Fig. 3. Thus, the following symmetric boundary conditions would ensure that the plate has a behavior of the box structure. Fig. 3 can be assumed to consist of three main plates viz; plate 1 (with Node 2,3,6,5); plate 2 (with Nodes 1,2,5,6,) and plate 3 (with Nodes 4, 5,8,7).

Therefore the symmetric boundary conditions are:

$$\left.\begin{aligned} (\theta x)_1 = (\theta x)_2 = (\theta x)_3 = (\theta x)_7 = (\theta x)_8 = (\theta x)_5 = 0 \\ (\theta y)_1 = (\theta y)_4 = (\theta y)_7 = (\theta y)_3 = (\theta y)_6 = (\theta y)_5 = 0 \end{aligned}\right\} \qquad (12a)$$

where θx and θy are rotation in x- and y- directions respectively

Because the edges of the box cannot deform, then, we have that

$$w_4 = w_5 = w_6 = w_2 = w_8 = 0 \qquad (12b)$$

Now for this flat plate to represent the behavior of a box, the displacement at node 6 will be constrained to have the same displacement as node 8

i.e $(\theta y)_8 = (\theta x)_6$ (13)

Plate Support Conditions

The idealization of one-eighth of the box in Fig. 3 resulted into three finite plates namely, plates 1, 2 and 3. And from the Fig.3, the following support conditions will be employed in the determination of the natural frequency of the system.

i. Plate 1 .

The assumed support condition is that of two opposite sides being clamped and the other two, simply supported. See Fig.4

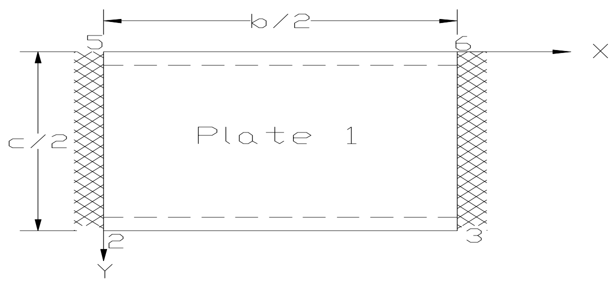

Fig. 4 Assumed support condition for plate 1

i.e Simply Supported at y= 0 and y=c/2
clamped at x = 0 and x= b/2 (14)

ii Plate 2
A clamped support is assumed for this plate element so that the symmetric boundary conditions stated in Eqn. (12) will be met. See Fig. 5

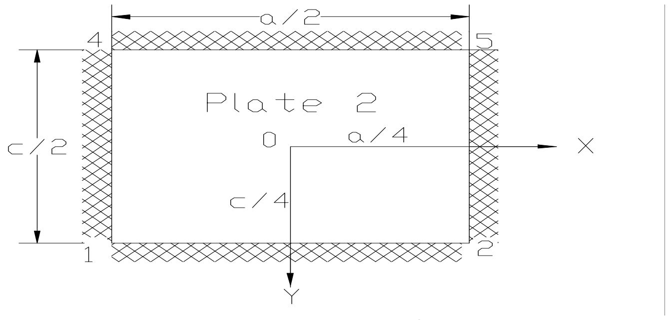

Fig. 5: Assumed Support Condition for Plate 2

iii. **Plate 3**
The assumed support condition for plate 3 is the same as that of plate 1, and this is shown in Fig.6

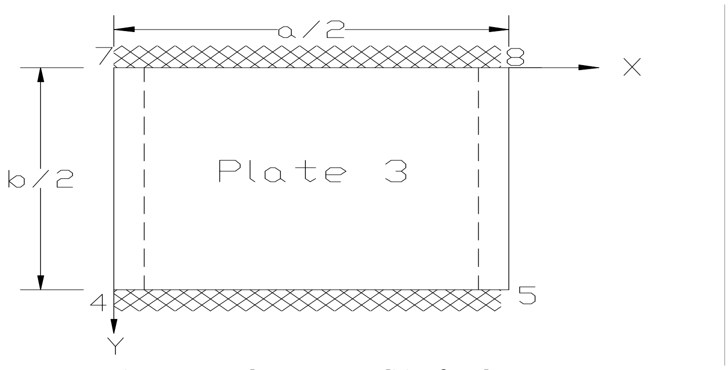

Fig. 6: Assumed Support Condition for Plate 3

i.e Clamped at y = o and y =b/2
Simply supported at x = o and x = a/2 (15)

2.4.2. Bubnov-Galerkin's Method
The governing differential equation (Eqn. 9) can be solved using Bubnov-Galerkin's Method. The preambles to this method have been outlined by Nwaogazie (2004). In this present work, the Galerkin's method as applicable to plates is presented by Szilard (2004) as:

$$\iint_A \left[\sum_{m=1}^{\infty} \nabla^4 c_m W(x,y) - \lambda \sum_{m=1}^{\infty} c_m W(x,y)\right] W(x,y)\, dxdy = 0 \quad (16)$$

Eqn. (16) when compared with Eqn. (9) shows that the shape function is given as:

$$W_{(x,y)} = \sum_{m=0}^{\infty} \boldsymbol{c_m W(x,y)} \quad (17)$$

And for a rectangular plate, this shape function can be presented as:

$$W(x,y) = \sum_{m}^{\infty}\sum_{n}^{\infty} c_{mn} W_{mn}(x,y) \quad (18)$$

Where c_{mn} are unknown coefficients representing the amplitudes of the free vibration modes and $W_{mn}(x, y)$ is the product of the pertinent Eigen functions of lateral beam vibrations, i.e

$$W_{mn}(x,y) = F_m(x)F_n(y) \quad (19)$$

A shape function W(x,y) that will satisfy the prescribed boundary and support conditions will be selected for each plate. Then the results (i.e natural frequencies) of the plates will be super imposed to obtain the overall fundamental natural frequency of the box structure.

Eqn. (16) can be presented in a more compact form for onward derivation of the needed equation of motion in the subsequent section.

$$\iint_A [\nabla^4 W(x,y) - \lambda W(x,y)]W(x,y)dxdy = o \quad (20)$$

3. Frequency Analysis of the Box-Type Satellite Structure

In this section, the required vibration equation for the box satellite structure is developed. This is followed by numerical application, which compares the results obtained with similar works done by other researchers.

It has been established that the vibration problem of the box satellite can be solved by superimposing the results obtained for natural frequency of the individual plate elements of Figs. 4, 5 and 6. Let ω_1, ω_2 and ω_3 represent the natural frequencies of plates 1, 2 and 3 respectively, and then the fundamental natural frequency of the box can be expressed as:

$$\omega_{mn} = \omega_1 + \omega_2 + \omega_3 \quad (21)$$

Each plate will now be analyzed as follow:

3.1. Frequency Analysis of Plate 1

Let us take the shape function for Fig.4 as

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} X_{(x)} Y_{(y)} \quad (22)$$

where $X_{(x)} = \sin \lambda_n x - \sinh {}^{\pi x}/_{a} - b\,(\cos\lambda_n x - \cosh \lambda_n x)$ (23)

$Y_{(y)} = \sin \mu_m y$ (24)

and $\lambda_n = \frac{n\pi}{b}$; $\mu_m = \frac{m\pi}{c}$ (25)

Substituting Eqns. (23), (24) and (25) into Eqn. (22),

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\left[Sin\frac{n\pi x}{b} - \sinh\frac{n\pi x}{b} - bcos\frac{n\pi x}{b} + bcosh\frac{n\pi x}{b}\right] sin\frac{m\pi y}{c} \quad (26)$$

Since we are interested in the fundamental natural frequency, i.e. at m = n = 1, Eqn. (26) becomes:

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{11}\left[Sin\frac{\pi x}{b} - \sinh\frac{\pi x}{b} - bcos\frac{\pi x}{b} + bcosh\frac{\pi x}{b}\right] sin\frac{\pi y}{c} \quad (27)$$

This shape function given by Eqn. (27) satisfies the boundary and support conditions specified in Eqns. (12), (13) and (14).

Thus, substituting Eqn. (27) into the Bubnov–Galerkin's equation i.e. Eqn. (20) yields;

$$\iint_A \left[\nabla^4\left\{\mathrm{Sin}\frac{\pi x}{b}\sin\frac{\pi y}{c} - \sinh\frac{\pi x}{b}\sin\frac{\pi y}{c} - \mathrm{b}\cos\frac{\pi x}{b}\sin\frac{\pi y}{c} + \mathrm{b}\cosh\frac{\pi x}{b}\sin\frac{\pi y}{c}\right\} - \lambda\left\{\sin\frac{\pi x}{b}\sin\frac{\pi y}{c} - \sinh\frac{\pi x}{b}\sin\frac{\pi y}{c} - \mathrm{b}\cos\frac{\pi x}{b}\sin\frac{\pi y}{c} + \mathrm{b}\cosh\frac{\pi x}{b}\sin\frac{\pi y}{c}\right\}\right]\left[\sin\frac{\pi x}{b}\sin\frac{\pi y}{c} - \sinh\frac{\pi x}{b}\sin\frac{\pi y}{c} - \mathrm{b}\cos\frac{\pi x}{b}\sin\frac{\pi y}{c} + \mathrm{b}\cosh\frac{\pi x}{b}\sin\frac{\pi y}{c}\right]\mathrm{dxdy} = 0 \quad (28)$$

Simplifying Eqn.(28) gives the natural frequency for plate 1 as :

$$\omega_1 = \left[\frac{16D\pi^4}{\rho h} * \left(\frac{1}{b^4} + \frac{2}{c^4} + \frac{8\pi}{\gamma_{m_1} b^2 c^2}\right)\right]^{1/2} \quad (29a)$$

$$where\ \gamma_{m_1} = -2.98E\ 78b^2 + 2.98E78b - 5.215E78 \quad (29b)$$

3.2. Frequency Analysis of Plate 2

For Plate 2, the shape function that will satisfy the boundary conditions can be assumed to be

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}(x^2 - a^2)^2(y^2 - c^2)^2 + \cdots \quad (30)$$

Since we are interested in the fundamental natural frequency, Eqn. (30) can be taken as :

$$W(x,y) = C_{11}\,(x^2 - a^2)^2\,(y^2 - c^2)^2 \quad (31)$$

Now, substituting Eqn. (31) into Eqn. (20) yields:

$$\iint_A \left[\nabla^4[(x^2 - a^2)^2(y^2 - c^2)^2] - \lambda(x^2 - a^2)^2(y^2 - c^2)^2\right](x^2 - a^2)^2(y^2 - c^2)^2\,dxdy = 0 \quad (32)$$

Simplifying Eqn. (32) gives the final expression of natural frequency of plate 2 as:

$$\omega_2 = \left[\frac{D}{256\rho h a^4 c^4}(31.5a^4 + 18a^2c^2 + 31.5c^4)\right]^{1/2} \quad (33)$$

3.3. Frequency Analysis of plate 3

This plate element has the same boundary conditions as plate element 1, though with different nomenclature of dimensions. Thus, the expression for natural frequency of plate 3 ω_3 can be obtained by substituting a for b and b for c in the expression for natural frequency of plate 1(i.e. Eqn. (29a)).

Then,

$$\omega_3 = \left[\frac{16D\pi^4}{\rho h} * \left(\frac{1}{a^4} + \frac{2}{b^4} + \frac{8\pi}{\gamma_{m_2} a^2 b^2}\right)\right]^{1/2} \quad (34)$$

where,

$$\gamma_{m_2} = -2.98E78a^2 + 2.98E78a - 5.215E78 \quad (35)$$

3.4. Expression for Fundamental Natural Frequency of the Box –Type Satellite Structure

By considering Eqn. (21), the expression for fundamental natural frequency of the box satellite structure is obtained by superposition of Eqns. (29), (33) and (34).

i.e. $\omega_{11} = \omega_1 + \omega_2 + \omega_3$

$$\omega_{11} = \left[\frac{16D\pi^4}{\rho h}\left(\frac{1}{b^4} + \frac{2}{c^4} + \frac{8\pi}{\gamma_{m_1} b^2 c^2}\right)\right]^{1/2} + \left[\frac{D}{256\rho h a^4 c^4}(31.5a^4 + 18a^2c^2 + 31.5c^4)\right]^{1/2} + \left[\frac{16D\pi^4}{\rho h}\left(\frac{1}{a^4} + \frac{2}{b^4} + \frac{8\pi}{\gamma_{m_2} a^2 b^2}\right)\right]^{1/2} \quad (36)$$

$$\text{Let}\ \left(\frac{1}{b^4} + \frac{2}{c^4} + \frac{8\pi}{\gamma_{m_1} b^2 c^2}\right)^{1/2} = \gamma_{m_3} \quad (37)$$

Let

$$(31.5a^4 + 18a^2c^2 + 31.5c^4)^{1/2} = \gamma_{m_4} \quad (38)$$

$$\text{Let } \left(\frac{1}{a^4} + \frac{2}{b^4} + \frac{8\pi}{\gamma_{m_2} a^2 b^2}\right)^{1/2} = \gamma_{m_5} \quad (39)$$

Substituting Eqns. (37) – (39) into Eqn. (36) gives:

$$\omega_{11} = \sqrt{\frac{D}{\rho h}} * [4\pi^2 (\gamma_{m3} + \gamma_{m5}) + \frac{1}{16a^2c^2}\gamma_{m4} \;] \quad (40)$$

Eqn. (40) is the formulated equation for determining fundamental natural frequency of a box-type satellite structure.
where:
a, b, c are the dimensions of the box structure;
D, ρ and h are as defined before.

4. Numerical Example

As a way of demonstrating the applicability of the derived Equation, a numerical study is conducted using the data outlined by Lin and Pan (2009).The aim is to determine the fundamental frequency of a box structure which is as a rectangular parallelepiped box whose dimension is as shown in Fig.7

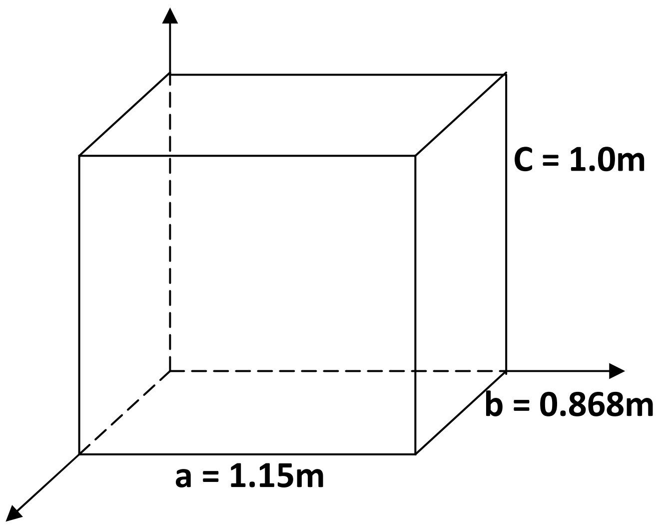

Fig. 7: Data for the Numerical Example

Lin and Pan (2009) assumed that all the plate panels have the uniform thickness of 2.5mm, and are made of aluminum (with Young Modulus E = 7.1 x 10^{10} N/m^2, density ρ = 2700Kg/m^3 Poison's ratio, μ = 0.3.)
Solving the problem by Eqn. (40), yields

ω_{11} = 19.25Hz

This result obtained is compared with that obtained by Lin and Pan (2009) in Table 1.

Table 1. Comparison of Predicted and Existing fundamental natural frequencies of a box structure

Mode No:	Existing	Predicted	% Difference
1	18.0Hz	19.25Hz	5.3

DISCUSSION

From the numerical studies conducted, there is a slight difference (though insignificant) between the fundamental frequency evaluated using the equations for natural frequency formulated in this work and that by Lin and Pan (2009). This difference could result from the assumed support conditions. For instance, Lin and Pan (2009) assumed clamped support conditions all round the edges, whereas in the present study, the satellite box considered has one of the plate element (i.e. plate 2) fully clamped, while the other two plate elements (I.e. plate 1 and 3) are partially fixed and partially simply supported. However, the derived equation would facilitate the evaluation of the needed fundamental natural frequency without resort to determining the natural frequencies at other modes. The result can be accepted within the limit range of 5.5%.

CONCLUSION

The Bubnov-Galerkin's Method is presented and used to obtain the solution of the undamped free flexural vibration analysis of a box-type satellite structure. Using the symmetric boundary conditions in the three planes of symmetry of the box structure, the finite element solution of the box satellite vibrations was simplified by idealizing only the one-eighth of the box structure, which represents three finite plates. Two of the plates were assumed to be simply supported at opposite edges and the other two clamped while one was assumed to be clamped all round to reflect the symmetric boundary conditions.

Thus, the study was able to formulate expression which is found handy in determining fundamental natural frequency of the box- type satellite structure. The formulated expression, i.e. Eqn. (40) enables the fundamental frequency of the box satellite structure to be effectively determined without using commercial finite element software. Therefore the formulated equation can be used to verify other free vibration analyses and to evaluate the precision of commercial software.

REFERENCES

[1] Bezukhov, N; Luzhim, O; and Kolkunov, N. (**1990**): Stability and Dynamics of Structures Learnt by Example; Mir Publishers, Moscow.
[2] Dickson, S.M and Warburton, G.B (**1967**): "Vibration of Box-Type Structures"; J. Mech. Eng. Sc. Vol.9 No.4, 325-335.
[3] Lin, Tian Ran and Pan, Jie (**2009**): "Vibration Characteristic of a Box-Type Structure; J. Vibration and Acoustics, vol. 131, No.3. 1 -45.
[4] Nwaogazie, Ify .L. (**2004**): Finite Element Modelling of Engineering Systems; Special Publishers limited Enugu.
[5] Petyt, Maurice (**1990**); Introduction to Finite Element Vibration Analysis; Cambridge University Press, Cambridge.
[6] Popplewell, N. (**1971**): "The Vibration of a Box-Type Structure 1.Natural Frequencies and Normal Modes" J. Sound and Vibration vol. 14,No.3, 357-365.
[7] Rao, S.S. (**1992**): The Finite Element Method in Engineering Vibrations, 2nd edition, Pergamum press, Oxford.
[8] Smith, J.W(**1988**):Vibration of Structures, Application in Civil Engineering; Chapman and Hall, London.
[9] Szilard, Rudolph (**2004**): Theories and Applications of Plate Analysis: Classical, Numerical, and Engineering Method; John Wiley and Sons, New Y

Appendix A

List of Symbols

a,b,c	*Dimension of box*
D	*Flexural Rigidity* = $Eh^3/12(1-\mu^2)$
h	*Thickness of plate and uniform box*
E	*Young modulus*
μ	*Poison ratio*
ω	*Natural frequency*
ω_{11}	*Fundamental natural frequency*
λ	*Frequency parameter* = $\rho h\omega^2/D$
$dw/dx = \theta_y$	*Slope in the y- direction*
$dw/dx = \theta_x$	*Slope in the x- direction*
ρ	*Mass density*
$w_{(x,y)}$	*Shape function/displacement function*
π	*22/7*
Sin π	*Sines of* 180^o