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Advances in Applied Science Research, 2012, 3 (3):1589-1597

ISSN: 0976-8610
CODEN (USA): AASRFC

Modified MHD double- diffusive convection coupled with cross-diffusions

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ABSTRACT

The present paper investigates the problem of modified magnetohydrodynamic (MHD) double-diffusive convection coupled with cross-diffusions for the Veronis' and Stern's type configurations. The nasty behaviour of the governing equations of the problem is mollified by the construction of a proper transformation and the relationship between various energies is established. The analysis made brings out that total kinetic energy associated with a disturbance is greater than the sum of its total magnetic and concentration energies in the parameter regime, $Q\sigma_1\pi^{-2} + R_S'\sigma\tau^{-2}\pi^{-4}k_2^{-2} \,/\, \frac{27}{4} \le 1.$ *The results derived herein are valid for quite general nature of boundary conditions.*

Keywords: Modified double-diffusive convection, Chandrasekhar number, Modified Rayleigh numbers, Dufour number, Soret number, Prandtl numbers.
MSC 2000 No: *76E99, 76E06.76E07*

INTRODUCTION

Thermohaline convection or more generally double diffusive convection has matured into a subject possessing fundamental departure from its counterpart, namely single diffusive convection, and is of direct relevance in the fields of oceanography, astrophysics, liminology and chemical engineering etc. For a broad and a recent view of the subject one may be referred to [1]. Two fundamental configurations have been studied in the context of thermohaline instability problem, the first one by [2] wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and the second one by [3] wherein the gradient is destabilizing and the concentration gradient is stabilizing. The main results derived by [2]and [3] for their respective configurations are that both allow the occurrence of a stationary pattern of motions or oscillatory motions of growing amplitude provided the destabilizing concentration gradient or the temperature gradient is sufficiently large. However, stationary pattern of motion is the preferred mode of setting in of instability in case of Stern's configuration whereas oscillatory motions of growing amplitude are preferred in Veronis' configuration. More complicated double-diffusive phenomenon appears if the destabilizing thermal/concentration gradient is opposed by the effect of magnetic field or rotation.

[4] presented a modified analysis of thermal and thermohaline instability of a liquid layer heated underside by emphasizing and utilizing the point that linear theoretical explanation of the phenomenon of gravity dominated thermal instability in a liquid layer heated underside (Be'nard convection) should depend not only upon the Rayliegh number which is proportional to the uniform temperature difference maintained across the layer but also upon other parameter so that a provision could be made in the theory to recognize the fact that a relatively hotter

layer with its heat diffusivity apparently increased/decreased as a consequence of an actual decreased/increased (depending on the fluid) in its specific heat at constant volume must exhibit Be'nard convection at a higher/lower Rayliegh number than a cooler layer under almost identical condition otherwise and further this qualitative effect is not quantitatively insignificant.

The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects [5], [6]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, arises when the mass flux contains a term that depends upon the temperature gradient. The analogous effect that arises from a concentration gradient dependent term in the heat flux is called the Dufour effect. It is now well established fact that the thermosolutal and Soret-Dufour problems are quite closely related, in fact, they are formally identical and identification is done by means of a linear transformation that takes the equations and boundary conditions for the latter problem into those for the former. The analysis of double diffusive convection becomes complicated in case when the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously, they interfere with each other and produce cross diffusion effect (Dufour-Soret effects). The Soret and Dufour coefficients describe the flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient respectively. The coupling of the fluxes of the stratifying agents is a prevalent feature in multicomponent fluid systems. In general, the stability of such systems is also affected by the cross-diffusion terms. Generally, it is assumed that the effect of cross diffusions on the stability criteria is negligible. However, there are liquid mixtures for which cross diffusions are of the same order of magnitude as the diffusivities. There are only few studies available on the effect of cross diffusion on double diffusion convection largely because of the complexity in determining these coefficients. The effect of Soret coefficient on the double–diffusive convection has been studied by [7]. They have reported that the magnitude and sign of the Soret coefficient were changed by varying the composition of the mixture. [8] has mathematically examined the problem of Soret –effect on rotatory thermosolutal convection of the Veronis type and has established a condition under which oscillatory motion of growing amplitude cannot manifest. The problem of Dufour-driven thermosolutal convection has also been considered by [9] and results concerning the linear growth rate and behavior of oscillatory motions have been established. The instability problem of magnetorotatory thermosolutal convection of the Veronis and Stern type has been examined by [10] taking in to account the Dufour effect and semi-circle theorems are derived, that prescribe upper limits for complex growth rate of oscillatory motions of neutral or growing amplitude. [11] has studied the effect of rotation on thermosolutal convection in a compressible couple-stress fluid through porous medium and concluded that the stable salute gradient and rotation introduce oscillatory modes in the system, which were non-existent in their absence. The effects of flow parameters on the velocity field, temperature field and concentration distribution have been studied by [12] and results are presented graphically and discussed quantitatively on the problem of viscous dissipation effects on unsteady free convection and mass transfer flow past an accelerated vertical porous plate with suction. [13] have investigated the problem on hydromagnetic natural convection flow of an incompressible viscoelastic fluid between two infinite vertical moving and oscillating parallel plates.

In his investigation of magneto hydrodynamic simple Be'nard convection problem [14] has sought unsuccessfully the regime in terms of the parameters of the system alone, in which the total kinetic energy associated with a disturbance exceeds the total magnetic energy associated with it, since these considerations are of decisive significance in deciding the validity of the principle of exchange of stabilities. However, the solution for w $(= cons\tan t(\sin\pi\, z))$ is not correct mathematically (and Chandrasekhar was aware of it).Banerjee et. al. until 1985 did not pursue their investigation in this direction and consequently did not see this connection. This gap in the literature on magnetoconvection has been completed by [15] who presented a simple mathematical proof to establish that Chandrasekhar's conjecture is valid in the regime $Q\,\sigma_1 \le \pi^2$ and further this result is uniformly applicable for any combination of a dynamically free or rigid boundary when the region outside the liquid are perfectly conducting or insulating. They showed that in the parameter regime $\frac{Q\sigma_1}{\pi^2} \le 1$ the total kinetic energy associated with a disturbance is greater than the total magnetic energy associated with it.

The present analysis extends this energy consideration to the problem of hydromagnetic modified double diffusive convection coupled with cross-diffusions to the type described by [2]. It is establish that in the parameter regime

$Q\sigma_1\pi^{-2} + R'_S\sigma\tau^{-2}\pi^{-4}k_2^{-2} \;/\frac{27}{4} \le 1$, the total kinetic energy associated with a disturbance is greater than the sum of its total magnetic and concentration energies. A similar characterization theorem for hydromagnetic double diffusive convection problem coupled with cross- diffusions of the type described by [3] is also established.

2. Mathematical formulation of the Problem

The relevant governing equations and boundary conditions for the modified hydromagnetic double-diffusive instability problem coupled with cross-diffusions in their non-dimensional linearized form are easily seen to be given by [4, 8, 9]

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T\, a^2\theta - R_s a^2\phi - QD\left(D^2 - a^2\right)h_z \qquad (2.1)$$

$$\left[D^2 - a^2 - \langle 1 - T_O\alpha_2\rangle p\right]\theta + \left[\, D_T(D^2 - a^2) - T_O\hat{\alpha}_2 R_3 p\right]\phi = -\left[\langle 1 - T_O\alpha_2\rangle + T_O\hat{\alpha}_2 R_3\right]w \;, \qquad (2.2)$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi + S_T(D^2 - a^2)\,\theta = -\frac{w}{\tau} \;, \qquad (2.3)$$

and

$$\left(\mathrm{D}^2 - \mathrm{a}^2 - \frac{\mathrm{p}\sigma_1}{\sigma}\right)\mathrm{h}_z = -\mathrm{D}\mathrm{w} \,, \qquad (2.4)$$

with

$w = 0 = \theta = \phi$ on both the boundaries,

$D^2 w = 0$ on a tangent stress–free boundary everywhere,

$Dw = 0$ on a rigid boundary,

$h_z = 0$ on both the boundaries if the regions outside

the fluid are perfectly conducting,

$$\left.\begin{aligned} Dh_z &= -ah_z \;\; at\;\; z = 1 \\ Dh_z &= ah_z \;\; at\;\; z = 0 \end{aligned}\right\} \quad \text{if the regions outside the fluid are insulating.} \qquad (2.5)$$

Now using equation (2.3) in equation (2.2), we have

$$\left[G_1(D^2 - a^2) - p\right]\theta + G_2(D^2 - a^2)\,\phi = -w \;, \qquad (2.6)$$

where

$$G_1 = \frac{1 - \hat{\alpha}_2 T_0 R_3\tau\, S_T}{1 - T_0\alpha_2} \quad \text{and} \quad G_2 = \frac{D_T - \hat{\alpha}_2 T_0 R_3\tau}{1 - T_0\alpha_2}\,.$$

The meanings of symbols from physical point of view are as follows;

z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $\sigma_1 = \frac{\upsilon}{\eta}$ is the magnetic Prandtl number, $\tau = \frac{\eta_1}{\kappa}$ is the Lewis number, $R_T = \frac{g\alpha\beta_1 d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $R_S = \frac{g\alpha\beta_2 d^4}{\kappa\upsilon}$ is the concentration Rayleigh number, Q = $\frac{\mu_e{}^2 H^2 d^2}{4\pi\rho_o \upsilon\eta}$ is the Chandrasekhar number , $D_T = \frac{\beta_2 D_f}{\beta_1 \kappa}$ is the Dufour number, $S_T = \frac{\beta_1 S_f}{\beta_2 \eta_1}$ is the Soret number, α_2 is the coefficient of specific heat due to variation in temperature, $\hat{\alpha}_2$ is the analogous coefficient due to variation in concentration, $R_3 = \frac{\beta_2}{\beta_1}$ is the ratio of concentration gradient to thermal gradient, T_0 is the temperature at the lower boundary, ϕ is the concentration, θ is the temperature, p is the complex growth rate, w is the vertical velocity, h_z is the vertical magnetic field.

In equations (1)–(6), z is real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , a^2 is a constant, $\sigma > 0$ is a constant, $\sigma_1 > 0$ is a constant, $\tau > 0$ is a constant, R_T and R_S are positive constants for the Veronis' configuration and negative constant for Stern's configuration, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi(z) = \phi_r(z) + i\phi_i(z)$ and $h_z = h_{z_r} + i\, h_{z_i}$ are complex valued functions(and their real and imaginary parts are real valued).

3. Linear Transformation and Mathematical Analysis

The nature of the system (2.1)-(2.6) is clearly qualitatively different from those of double-diffusive convection problems ($D_T = 0 = S_T$) as now we have coupling between all the three eigen- functions $w, \theta, and\ \phi$ in all the three equations. Consequently, they behave nastily and obstruct any attempt for the elegant extension of the earlier results for the double-diffusive convection problems to the present generalized set up. The nasty behavior of these equations is mollified by the linear transformations given by:

$$\tilde{w} = (S_T + B)\, w,$$

$$\tilde{\theta} = E\theta + F\phi,$$

$$\tilde{\phi} = S_T\theta + B\phi\ ,$$

$$\tilde{h}_z = (\ S_T + B)\, h_z, \qquad (3.1)$$

where

B $= -\frac{1}{\tau}A,$ E $= \frac{S_T + B}{G_2 + A}A,$ F $= \frac{S_T + B}{G_2 + A}G_2,$ $A = G_1 - \tau$

and A is a positive root of the equation

$$A^2 - (G_1 - \tau)A - \tau\, S_T G_2 = 0\,.$$

The system of equations (2.1), (2.3), (2.4) and (2.6) together with boundary conditions (2.5), upon using the transformations (3.1) and omitting the tilde sign for simplicity, assumes the following form:

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T{}' \, a^2\theta - R_S{}' \ a^2\phi - QD\left(D^2 - a^2\right)h_z\ , \qquad (3.2)$$

$$\left[k_1\left(D^2 - a^2\right) - p\right]\theta = -w\,, \qquad (3.3)$$

$$\left[k_2(D^2 - a^2) - \frac{p}{\tau}\right]\phi = -\frac{w}{\tau}\ , \qquad (3.4)$$

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)h_z = -Dw\,, \tag{3.5}$$

with

$w = 0 = \theta = \phi$ on both the boundaries,

$D^2 w = 0$ on a tangent stress–free boundary everywhere,

$Dw = 0$ on a rigid boundary,

$h_z = 0$ on both the boundaries if the regions outside

the fluid are perfectly conducting,

$$\left.\begin{array}{l} Dh_z = -ah_z \;\; at \;\; z = 1 \\ Dh_z = ah_z \;\; at \;\; z = 0 \end{array}\right\}$$ if the regions outside the fluid are insulating,

(3.6)

where

$k_1 = G_1 + \dfrac{\tau\, G_2 S_T}{A}$, $k_2 = 1 + \dfrac{S_T G_2}{\tau\, B}$ are positive constants and

$R_T^{/} = \dfrac{(G_2 + A)(R_T G_1 + R_S S_T)}{BA - S_T G_2}$, $R_S^{/} = \dfrac{(S_T + B)(R_S A + R_T G_2)}{BA - S_T G_2}$ are respectively the modified thermal Rayliegh number and the modified concentration Rayliegh number.

We now prove the following theorems:

Theorem 1: If (p, w, θ, ϕ, h_z), $p = p_r + ip_i$, $p_r \geq 0$ is a solution of equations (3.2) – (3.5) together with boundary conditions (3.6) with, $R_T > 0$ $R_S > 0$ and $Q\sigma_1\pi^{-2} + R_S'\sigma\tau^{-2}\pi^{-4}k_2^{-2} \,/\, \dfrac{27}{4} \leq 1$ then

$$\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz > Q\sigma_1\int_0^1\left(|Dh_z|^2 + a^2|h_z|^2\right)dz + R_s{}^{/}a^2\sigma\int_0^1|\phi|^2 dz\,. \tag{3.7}$$

Proof: Multiplying equation (3.5) by h_z^* (the complex conjugate of h_z), integrating the resulting equation over the range of z by parts a suitable number of times, and making use of the boundary conditions (3.6) we get

$$aM + \int_0^1\left(|Dh_z|^2 + a^2|h_z|^2\right)dz + \frac{p\sigma_1}{\sigma}\int_0^1|h_z|^2dz = -\int_0^1 w\; Dh_z^*\,, \tag{3.8}$$

where $M = \left\{\left(|h_z|^2\right)_0 + \left(|h_z|^2\right)_1\right\} \geq 0$.

Equating the real part of equation (3.8), we get

$$aM + \int_0^1\left(|Dh_z|^2 + a^2|h_z|^2\right)dz + \frac{p_r\sigma_1}{\sigma}\int_0^1|h_z|^2dz$$

$$= \text{Real part of}\left(-\int_0^1 w\; Dh_z^*\; dz\right)$$

$$\leq \left| \int_0^1 w \; Dh_z^* \; dz \right|$$

$$\leq \int_0^1 |w||Dh_z| dz$$

$$\leq \left\{ \int_0^1 |w|^2 dz \right\}^{1/2} \left\{ \int_0^1 |Dh_z|^2 dz \right\}^{1/2} \quad . \tag{3.9}$$

(using Schwartz inequality)

Since $p_r \geq 0$, therefore from the inequality (3.9), we get

$$\int_0^1 |Dh_z|^2 dz < \left\{ \int_0^1 |w|^2 dz \right\}^{1/2} \left\{ \int_0^1 |Dh_z|^2 dz \right\}^{1/2}$$

or

$$\int_0^1 |Dh_z|^2 dz < \int_0^1 |w|^2 dz \quad . \tag{3.10}$$

Using inequality (3.10), it follows from inequality (3.9) that

$$\int_0^1 \left(|Dh_z|^2 + a^2 |h_z|^2 \right) dz < \int_0^1 |w|^2 dz. \tag{3.11}$$

Since w (0) = 0 = w (1), therefore using [16], we get

$$\int_0^1 |w|^2 dz < \frac{1}{\pi^2} \int_0^1 |Dw|^2 dz \quad . \tag{3.12}$$

It follows from inequalities (3.11) and (3.12) that

$$\int_0^1 \left(|Dh_z|^2 + a^2 |h_z|^2 \right) dz < \frac{1}{\pi^2} \int_0^1 |Dw|^2 dz$$

$$< \frac{1}{\pi^2} \int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz$$

or

$$Q\sigma_1 \int_0^1 \left(|Dh_z|^2 + a^2 |h_z|^2 \right) dz + R_s^{/} \sigma \, a^2 \int_0^1 |\phi|^2 dz$$

$$< \frac{Q\sigma_1}{\pi^2} \int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz + R_s^{/} \; \sigma \, a^2 \int_0^1 |\phi|^2 dz \quad . \tag{3.13}$$

Multiplying equation (3.4) by the complex conjugate of the equation (3.4) and integrating by parts over the vertical range of z for an appropriate number of times and making use of the boundary conditions (3.6) we get

$$k_2^2 \int_0^1 \left(\left| D^2 \phi \right|^2 + 2a^2 |D\phi|^2 + a^4 |\phi|^2 \right) dz + 2 p_r k_2^2 \int_0^1 \left(|D\phi|^2 + a^2 |\phi|^2 \right) dz$$

$$+\frac{|p|^2}{\tau^2}\int_0^1 |\phi|^2 dz = \frac{1}{\tau^2}\int_0^1 |w|^2 dz \ . \tag{3.14}$$

Since, $p_r \geq 0$, therefore from equation (3.14), we get

$$\int_0^1 \left(|D^2\phi|^2 + 2a^2 |D\phi|^2 + a^4 |\phi|^2 \right) dz < \frac{1}{\tau^2 k_2^2}\int_0^1 |w|^2 dz \ . \tag{3.15}$$

Since $\phi(0) = 0 = \phi(1)$, therefore using [16], we get

$$\pi^2 \int_0^1 |\phi|^2 dz < \int_0^1 |D\phi|^2 dz$$

and also

$$\pi^4 \int_0^1 |\phi|^2 dz \leq \int_0^1 |D^2\phi|^2 dz \ . \ \text{(using Schwartz inequality)} \tag{3.16}$$

It follows from inequalities (3.15) and (3.16) that

$$(\pi^2 + a^2)^2 \int_0^1 |\phi|^2 dz < \frac{1}{\tau^2 k_2^2}\int_0^1 |w|^2 dz$$

or

$$\frac{(\pi^2 + a^2)^2}{a^2}\int_0^1 |\phi|^2 dz < \frac{1}{a^2\tau^2 k_2^2}\int_0^1 |w|^2 dz < \frac{1}{a^2\tau^2 k_2^2(\pi^2 + a^2)}\int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz$$

or

$$a^2 \int_0^1 |\phi|^2 dz < \frac{1}{\frac{27}{4}\pi^4 \tau^2 k_2^2}\int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz \ ,$$

since the minimum value of $\frac{(\pi^2 + a^2)^3}{a^2}$ for $a^2 > 0$ is $\frac{27\pi^4}{4}$.

or $$R_s' a^2 \sigma \int_0^1 |\phi|^2 dz < \frac{R_s' \sigma}{\frac{27}{4}\pi^4 \tau^2 k_2^2}\int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz \ . \tag{3.17}$$

Now from inequalities (3.13) and (3.17), we get

$$Q\sigma_1 \int_0^1 \left(|Dh_z|^2 + a^2 |h_z|^2 \right) dz + R_s' a^2 \sigma \int_0^1 |\phi|^2 dz$$

$$< \left(\frac{Q\sigma_1}{\pi^2} + \frac{R_s' \sigma}{\frac{27}{4}\tau^2 \pi^4 k_2^2} \right) \int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz \ . \tag{3.18}$$

Therefore, if $\frac{Q\sigma_1}{\pi^2}+\frac{R_s{}'\sigma}{\frac{27}{4}\tau^2\pi^4k_2^2}\le 1$, then from inequality (3.18), we get

$$\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz > Q\sigma_1\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz + R_s{}'a^2\sigma\int_0^1|\phi|^2\,dz\ , \tag{3.19}$$

and this completes the proof of the theorem.

We note that the left hand side of equation (3.18) represents the total kinetic energy associated with a disturbance while the right hand side represents the sum of its total magnetic and concentration energies, and Theorem 1 may be stated in the following equivalent form:

At the neutral or unstable state in the hydromagnetic double diffusive convection problem of the Veronis' type coupled with cross-diffusions, the total kinetic energy associated with a disturbance is greater than the sum of its total magnetic and concentration energies in the parameter regime $Q\sigma_1\pi^{-2}+R_S'\sigma\tau^{-2}\pi^{-4}k_2{}^{-2}\ /\frac{27}{4}\le 1$ and this result is uniformly valid for any combination of dynamically free or rigid boundaries that are either perfectly conducting or insulating.

Theorem 2: If $(p, w, \theta, \phi, h_z)$, $p = p_r + ip_i$, $p_r \ge 0$ is a solution of equation (3.2) –(3.5) together with boundary conditions (3.6) with $R_T < 0$, $R_S < 0$, and $\frac{Q\sigma_1}{\pi^2}+\frac{|R_T'|\sigma}{\frac{27}{4}\pi^4k_1{}^2}\le 1$, then

$$\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz > Q\sigma_1\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz + |R_T{}'|\,a^2\sigma\int_0^1|\theta|^2\,dz\ . \tag{3.20}$$

Proof: Similar to that of Theorem1.

We note that the left hand side of equation (3.20) represents the total kinetic energy associated with a disturbance while the right hand side represents the sum of its total magnetic and thermal energies, and Theorem 2 may be stated in the following equivalent form:

At the neutral or unstable state in the hydromagnetic double diffusive convection problem of the Stern's type coupled with cross-diffusions, the total kinetic energy associated with a disturbance is greater than sum of its total magnetic and thermal energies in the parameter regime $\frac{Q\sigma_1}{\pi^2}+\frac{|R_T'|\sigma}{\frac{27}{4}\pi^4k_1^2}\le 1$ and this result is uniformly valid for any combination of dynamically free or rigid boundaries that are either perfectly conducting or insulating.

REFERENCES

[1] Brandt A., Fernando H.J.S., *Double–diffusive convection*, **1996,** American Geophysical Union.
[2] Stern M.E, *Tellus*, **1960,** 12**,** 172.
[3] Veronis G, *J. Mar. Res.*, **1965**, 3, 1.
[4] Banerjee M.B., Gupta J.R., Shandil R.G., Sharma K.C., KatochD.C. J.Math.Phy.Sci. **1983,** 17,603.
[5] de Groot S.R., Mazur P, *Nonequilibrium Thermodynamics*, **1962,** North Holland, Amsterdam.
[6] Fitts D.D., *Nonequlibrium Thermodynamics*, **1962,** McGraw Hill, New York.
[7] Hurle D.T.J, Jakeman., E., *Soret-Driven Thermosolutal Convection*, **1971,** J. Fluid Mech. ,47, 668.

[8] Mohan, H., *The Soret Effect on the Rotatory Themosolutal Convection of the Veronis Type,* **1996,** Indian J. Pure Appl. Math, 27,609.
[9] Mohan H. *J. Math. Anal. Appl*., **1998**, 218, 569.
[10] Mohan H., *Advances in Applied Science Research,* **2012**, 3(2), 1052.
[11] Kumar P., *Advances in Applied Science Research,* **2012**, 3(2), 871.
[12] Malga Bala Siddulu, Kishan Naikoti, *Advances in Applied Science Research,* **2011,** 2(6), 460.
[13] Sreekanth S, Vankatraman S, Rao Sreedhar G, Saravana R, *Advances in Applied Science Research,* **2011,** 2(5), 185.
[14] Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability,* **1961,** Oxford: Clarendon Press.
[15] Banerjee M.B., Katyal S.P., J. *Math. Anal. Appl*., **1988,**129**,** 383.
[16] Schultz M.H., *Spline Analysis,* **1973,** Prentice-Hall, Englewood Cliffs, N.J.