

Mixed symmetry states in platinum isotopes by using the proton and neutron interacting boson model(IBM-2)

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ABSTRACT

In this study the mixed symmetry states (MSS) in even mass Pt isotopes with mass interval $168 \leq A \leq 200$ have been suggested within the framework of the Proton and Neutron Interacting Boson Model (IBM-2). The model calculations predicted a satisfactory fit to the experimental energy levels. It is found that, the energy of states (2_3 , 2_4 , 2_5 , 3_1 and 1_1) were responded rapidly to the variation of Majorana parameters in some isotopes, which indicates the initial property of the mixed symmetry state (MSS). The rest of MSS properties have been applied to those states, $B(M1)$ values and $B(M1)/B(E2)$ ratio, to support those results.

Keywords: IBM-2, Pt. isotopes, mixed symmetry state, Majorana parameters, $B(M1)$

INTRODUCTION

Platinum isotopes have always attracted much attention for physicists up to now, there have already been a lot of experimental works, and abundant data are available. Accordingly, a number of theoretical investigations have been presented to study the properties of these isotopes [1-2]. The excitation energies display the characteristic parabolic pattern as a function of neutron number N , with minimal excitation energy around the $N = 104$ neutron mid-shell nucleuses [3], the intruder structure seems lost in the Pt nuclei. Focusing on the systematics of the energy spectra in the Pt nuclei, one observes a rather sudden drop in the excitation energy of the $2_1^+ 4_1^+$, 6_1^+ , 0_2^+ and 2_2^+ states between $N=110$ and $N=108$. This drop followed by a particularly flat behavior in excitation energy as a function of neutron number until the energy of those states start to move up again around neutron number $N=100$ [4]. The shape of Pt isotopes evolves from spherical to oblate and finally to Prolate shapes when the neutron number decreases from $N=126$ (semi-magic) to $N=104$ mid-shell [5,6], and the dynamic symmetry of Pt isotopes located within $O(6)$ transition region.

In present theoretical work we applied fit experimental energy levels and predict the spectral properties of almost even all Pt isotopes in mass around 200. We also report the predicted excitation energy of the mixed symmetry states. The microscopic calculations have been carried out to support the existence of mixed symmetry states in this mass region.

MATERIALS AND METHODS

The Interacting Proton and neutron Model

The interacting boson approximation (IBA) has been rather successful at describing the collective properties of several medium and heavy nuclei. In the present work we used Proton and Neutron Interacting Boson Model (IBM-2) to study the low lying collective state of even-even Pt isotopes, the Hamiltonian operator can be written as [7-8];

$$H = \varepsilon_d(n_{dv} + n_{d\pi}) + \kappa(Q_v \cdot Q_\pi) + V_{vv} + V_{\pi\pi} + M_{v\pi} \quad (1)$$

where ε_d is the energy difference s and d boson n_ρ is the number of d bosons, where ρ correspond to π (proton) or ν (neutron) bosons, the second term denotes the quadrupole – quadrupole interaction between proton and neutron with strength κ , where the quadrupole operator Q_ρ is define as [9].

$$Q_\rho = [d_\rho^\dagger s_\rho + s_\rho^\dagger d_\rho]^{(2)} + x_\rho [d_\rho^\dagger d_\rho]^{(2)} \tag{2}$$

The term $V_{\pi\pi}$ and $V_{\nu\nu}$ which correspond to the interaction between like – boson, are sometimes included in order to improve the fit to experimental energy spectra and they are expressed as [10-11];

$$V_{\rho\rho} \frac{1}{2} \sum_{L=0,2,4} C_L^\rho ([d_\rho^\dagger d_\rho]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]) \tag{3}$$

And the Majorana term $M_{\nu\pi}$ contains three parameters ξ_1, ξ_2 and ξ_3 can be written as;

$$M_{\nu\pi} = \frac{1}{2} \xi_2 ([s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger]^{(2)} \cdot [s_\nu d_\pi - d_\nu s_\pi]^{(2)}) - \sum_{k=1,3} \xi_k ([d_\nu^\dagger d_\pi^\dagger]^{(k)} \cdot [d_\nu d_\pi]^{(k)}) \tag{4}$$

where the Majorana term plays a great role when to study the mixed symmetry states of the some excited energy level. Mixed symmetry states occur when the motions of the proton and neutrons are not in phase in the quantum state [12]. These states were created by mixture of two wave functions, one for proton and other for neutron [13]. The mixed symmetry states determined by the F - spin formalism, where the F -spin formalism is analogous to the isospin quantum number of the nucleons. Proton bosons and neutron bosons have $F = 1/2$ and z -projection, where $F_z = +1/2, -1/2$ for protons and neutrons respectively. For a system consist of N_π proton boson and N_ν neutron boson, the maximum F -spin is $F = F_{max} = (N_\pi + N_\nu) / 2$, while the mixed symmetry states characterized by decreasing F - spin value ($F = F_{max}, F = F_{max} - 1, F = F_{max} - 2, \dots, F_{min} = |N_\pi - N_\nu| / 2$). In the F -spin space, one can also define the creation and annihilation operators F_+ and F_- by [14-15];

$$F_+ = s_\pi^\dagger s_\nu + (d_\pi^\dagger \tilde{d}_\nu), F_- = s_\nu^\dagger s_\pi + (d_\nu^\dagger \tilde{d}_\pi). \tag{5}$$

The projection operator F_z is given by

$$F_z = (N_\pi - N_\nu) / 2 \tag{6}$$

A state composed by N_π proton bosons and N_ν neutron bosons with F -spin quantum number $F = F_{max}$ can be transformed by the successive action of the F -spin raising operator F_+ into a state that consists of proton bosons only. This state has still a total F -spin quantum number $F = F_{max}$ since the raising operator does not change the total F -spin quantum number. This new state has only proton bosons and obviously stays unchanged under a pairwise exchange of proton and neutron labels. Therefore, *IBM-2* states with $F = F_{max}$ are called Full Symmetry States (*FSS*). These states correspond actually to the *IBM-1* states which are all symmetric. All others states with F -spin quantum numbers $F < F_{max}$ contain pairs (at least one) of proton and neutron bosons that are anti-symmetric under a pairwise exchange of protons and neutrons labels. They are called Mixed-Symmetry States (*MSS*) [16]. The general F - spin selection rule is $\Delta F = 0, \pm 1$. Besides, the *M1* transition accrue between two totally symmetric states ($F = F_{max}$) are forbidden, therefor *M1* transition occur between low-lying collective states, these states must be contain components with mixed symmetry state, when ($F < F_{max}$), this allows to use the strength of *M1* transition between low-lying collective state and F - spin mixing [17-18]. The *M1* operator is obtained by making $l = 1$ in the single boson operator of the *IBM-2* and can be written as [19-20];

$$T^{(M1)} = [(d^\dagger \tilde{d})_\pi^{(1)} - (d^\dagger \tilde{d})_\nu^{(1)}] (g_\pi - g_\nu) \tag{7}$$

Where g_π, g_ν are represent the factor of nuclear structure for proton and neutron respectively, the total g - factor is defined by Sambataro's equation [21];

$$g = (g_\pi N_\pi + g_\nu N_\nu) / (N_\pi + N_\nu) \tag{8}$$

RESULTS AND DISCUSSION

1. Energy Spectra

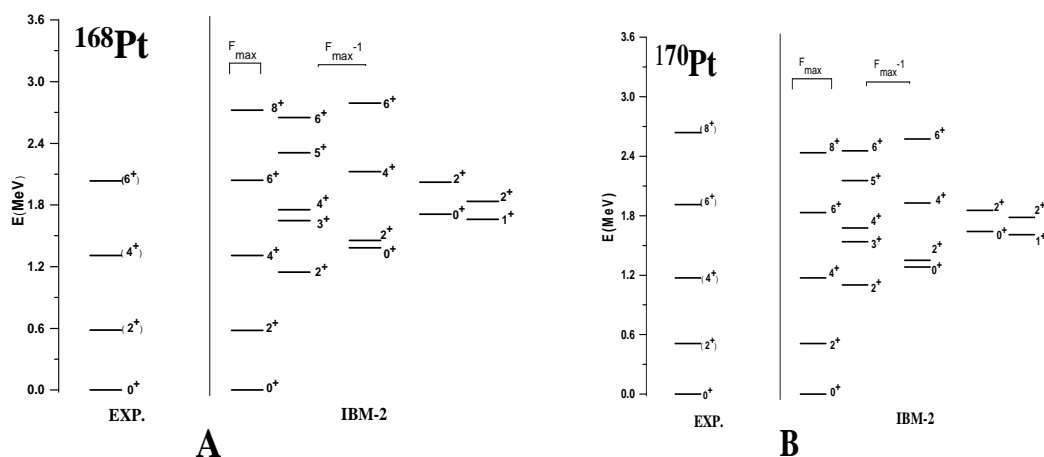
The platinum isotopes are neutron rich, have $Z=78$, the numbers of boson proton $N_\pi = 2$ and boson neutron N_ν varied from 4 in ^{168}Pt isotope to 1^* in ^{202}Pt isotope ($*$ means hole boson). To study the energy levels of Pt isotopes, we need to estimate the parameters used in (*IBM-2*), by applying the *NPBOS* package, the fitted values of

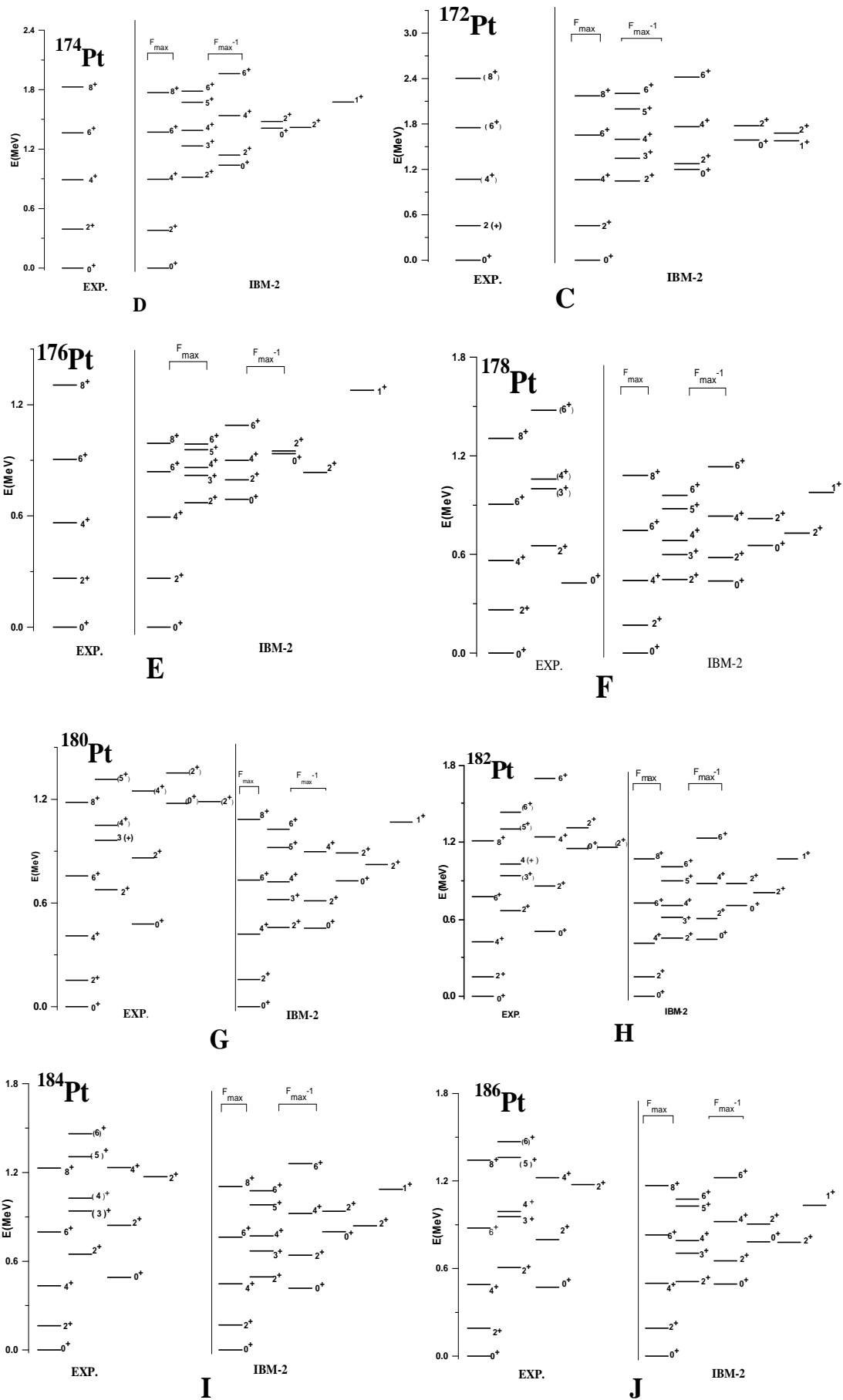
these parameters are listed in Table (1), they were treated as a free parameters and their values were estimated by fitting to the experimental energy level. In general we observed that the (ϵ) parameters decreasing with neutron numbers up to mid shell ($N=104$) and then increases again, the (κ) parameter increases proportionally with the increase the neutrons number and then decreases again after the mid shell, the (χ_v) value always increasing linearly with increasing neutron numbers. The values of Majorana parameters terms ($\zeta_{1,3}$, ζ_2) were selected for a certain values and adjustable with the study of mixed-symmetry states (MSS) of the energy levels. The values of $\chi_{\kappa} = -0.88$ and $\zeta_{1,3} = -0.083$ were fixed for all Pt isotopes.

Table 1. IBM-2 Parameter for even-even Pt isotopes

Isotope	N_v	ϵ	κ	χ_v	$CL_{v(0,2,4)}$	$CL_{\pi(0,2,4)}$	ζ_2
¹⁶⁸ Pt	4	0.999	-0.184	-0.224	-0.310,-0.192,-0.198	000,000,000	+0.040
¹⁷⁰ Pt	5	0.975	-0.184	-0.128	-0.310,-0.188,-0.185	000,000,000	+0.035
¹⁷² Pt	6	0.961	-0.184	-0.021	-0.310,-0.188,-0.182	000,000,000	+0.030
¹⁷⁴ Pt	7	0.903	-0.183	+0.106	-0.310,-0.188,-0.180	000,000,000	+0.025
¹⁷⁶ Pt	8	0.806	-0.182	+0.122	-0.270,-0.198,-0.188	000,000,000	+0.020
¹⁷⁸ Pt	9	0.594	-0.177	+0.263	-0.295,-0.140,-0.090	000,000,000	+0.015
¹⁸⁰ Pt	10	0.588	-0.179	+0.290	-0.294,-0.115,-0.081	000,000,000	+0.010
¹⁸² Pt	11	0.603	-0.172	+0.330	-0.289,-0.115,-0.078	000,000,000	+0.005
¹⁸⁴ Pt	10*	0.570	-0.172	+0.337	-0.283,-0.090,-0.078	000,000,000	0.000
¹⁸⁶ Pt	9*	0.560	-0.164	+0.355	-0.288,-0.090,-0.078	000,000,000	-0.005
¹⁸⁸ Pt	8*	0.594	-0.161	+0.490	-0.200,-0.113,-0.050	000,000,000	-0.010
¹⁹⁰ Pt	7*	0.570	-0.156	+0.500	0.000, 0.000, 0.000	000,000,000	-0.015
¹⁹² Pt	6*	0.570	-0.170	+0.510	0.000, 0.000, 0.000	000,000,000	-0.020
¹⁹⁴ Pt	5*	0.574	-0.174	+0.530	0.000, 0.000, 0.000	000,000,000	-0.025
¹⁹⁶ Pt	4*	0.576	-0.174	+0.559	0.000, 0.000, -0.050	000,000,000	-0.030
¹⁹⁸ Pt	3*	0.600	-0.198	+0.910	-0.100, -0.100,-0.050	000,000,000	-0.035
²⁰⁰ Pt	2*	0.631	-0.198	+0.924	0.000, -0.050, -0.050	000,000,000	-0.040
²⁰² Pt	1*	0.651	-0.199	+0.967	0.000, -0.000, 0.000	000,000,000	-0.045

The Hamiltonian operator in equation (1) was diagonalized to the binding energy and then excited energy level, the best fit to excited energy levels have been observed specially for the ground band (i.e. the states $2^+_{1,4}, 4^+_{1,6}$ and $8^+_{1,1}$), while the second and third bands it was found to have acceptable agreement, where these bands have some properties of the β and γ band so called (quasi beta and gamma band). The appearance $0^+_{2,3}$ states between ($4^+_{1,4}$) for (¹⁸⁸Pt, ¹⁹⁰Pt, ¹⁹⁶Pt and ²⁰⁰Pt) isotopes have been observed, and between ($4^+_{2,6}$) for (¹⁹⁴Pt) isotope and between ($2^+_{2,4}$) for (¹⁹⁴Pt) isotope. As a result for this appearance, this state consider as the band head of considered quasi beta bands. Figure (1) shows a comparison between theoretical and available experimental energy levels for all studied platinum isotopes [22].





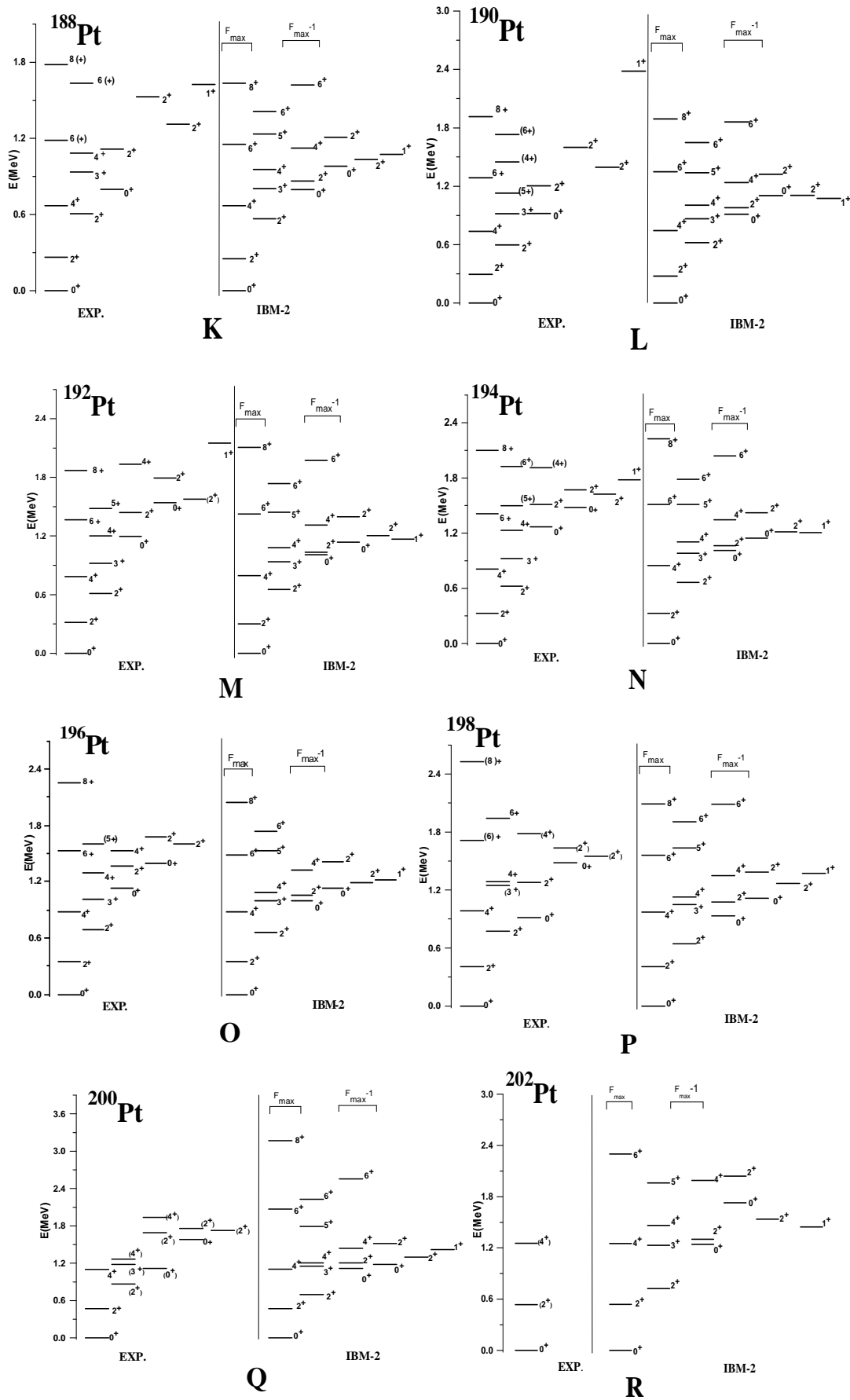
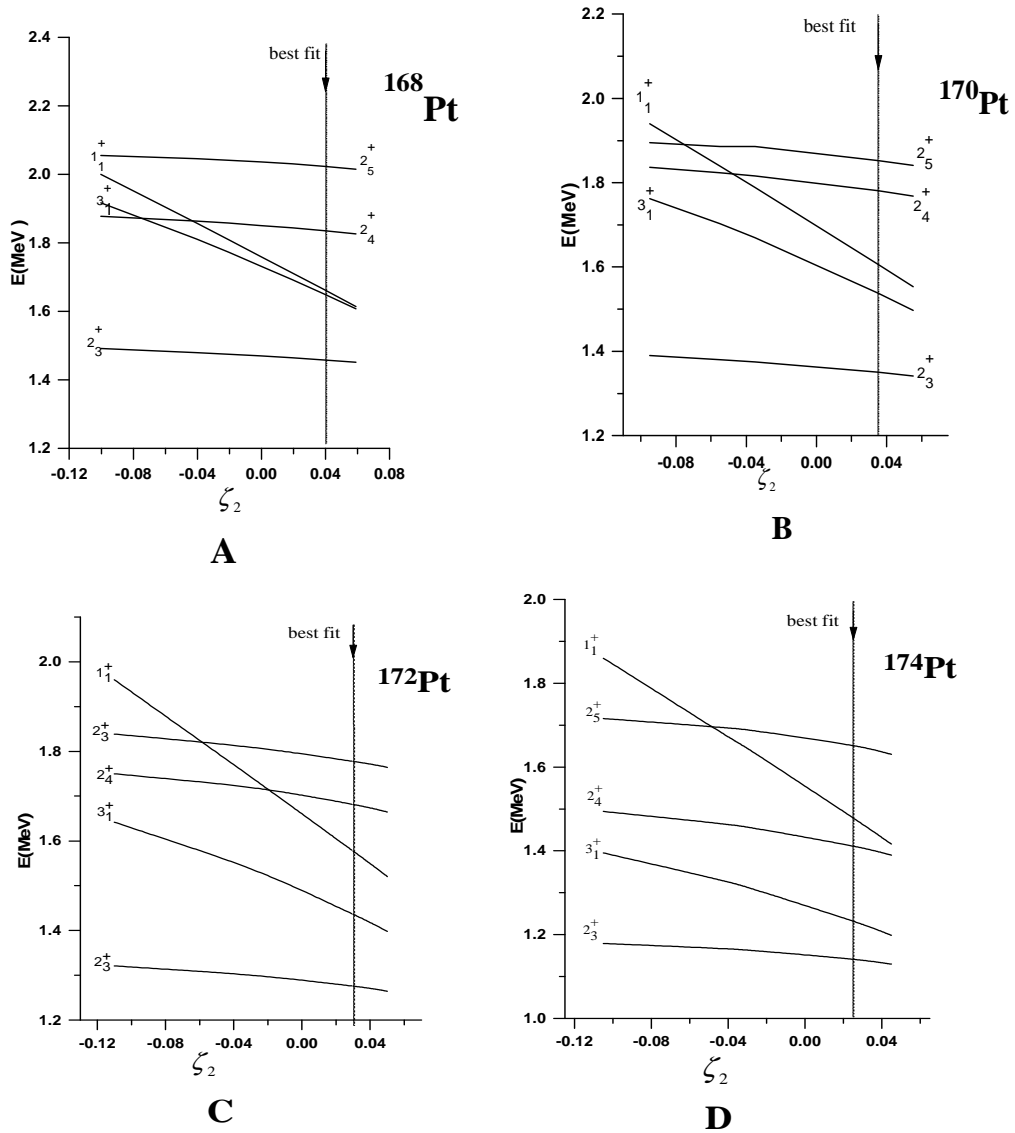
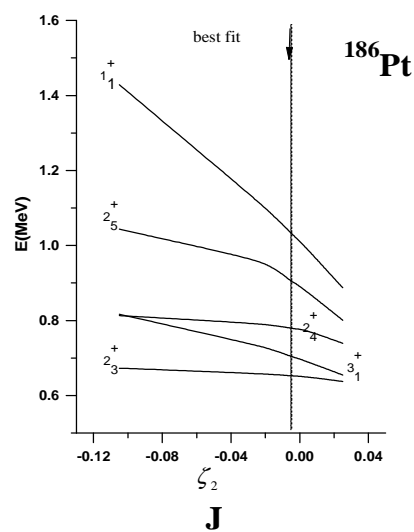
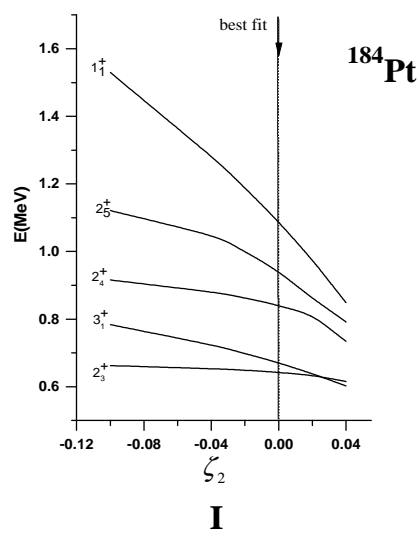
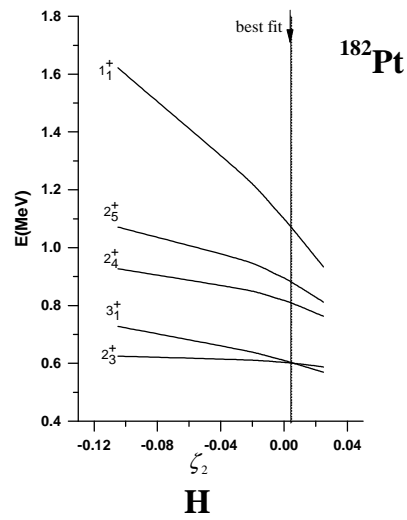
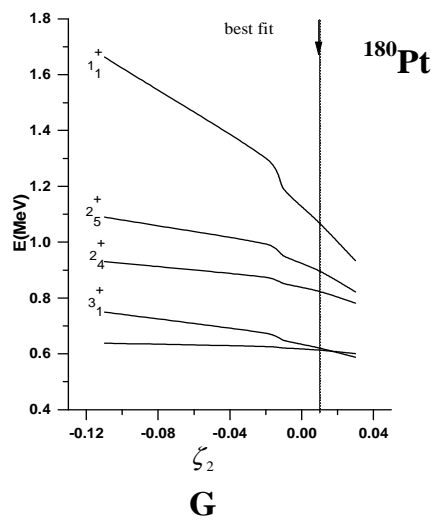
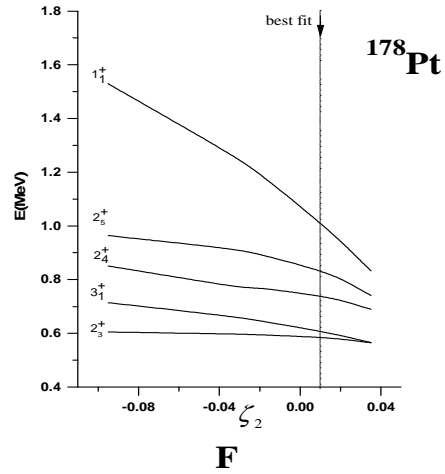
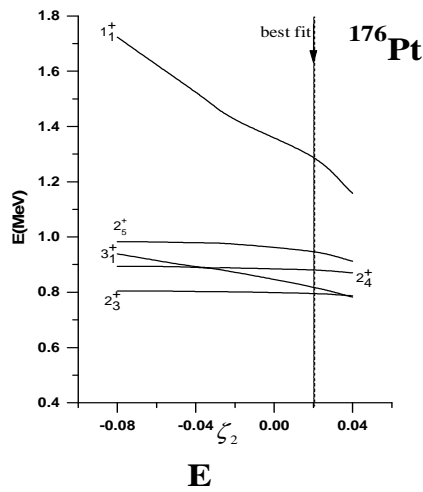


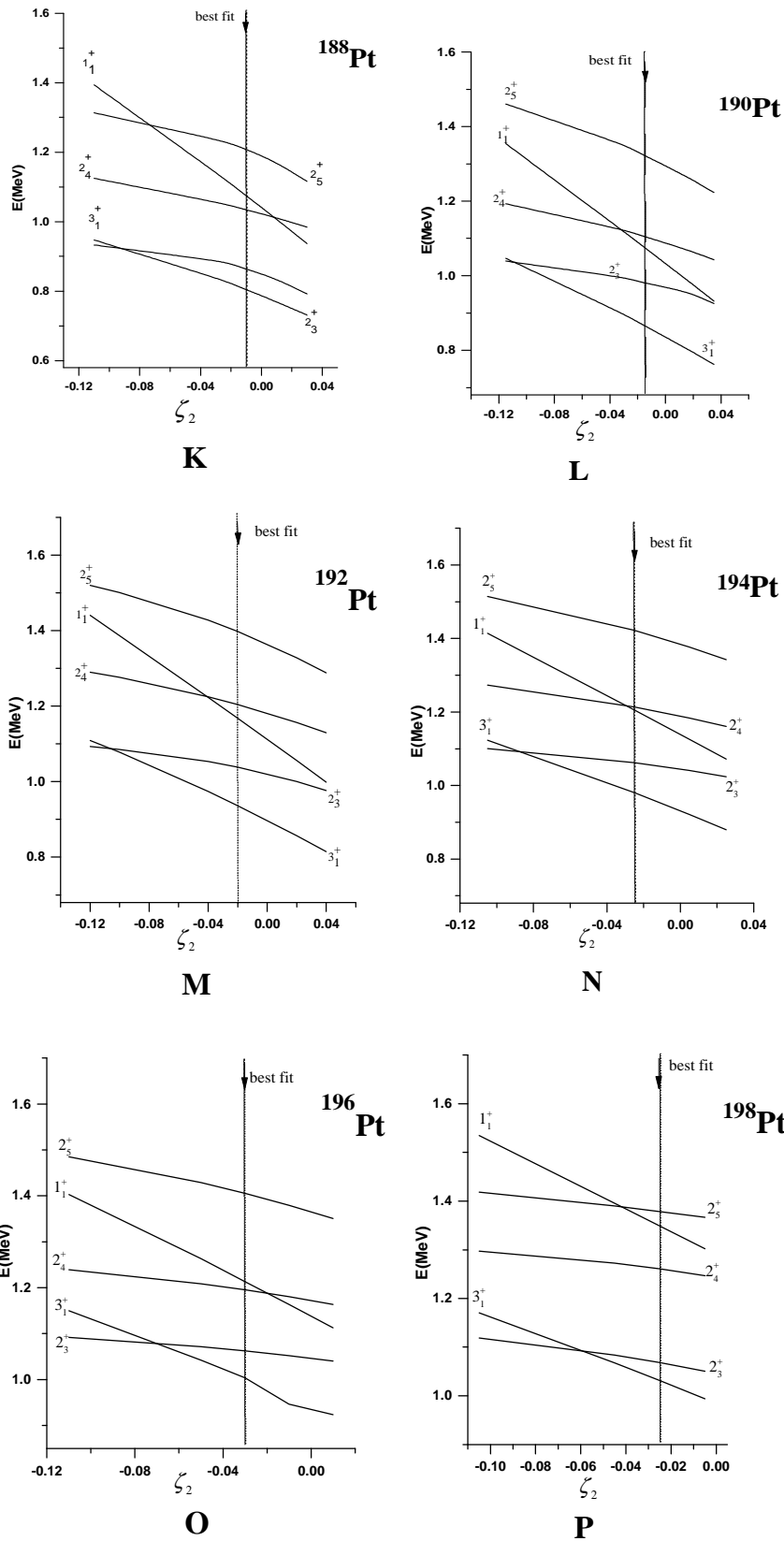
Figure 1. Comparison between calculated energy level and the available experimental data A= ^{168}Pt , B= ^{170}Pt , C= ^{172}Pt , D= ^{174}Pt , E= ^{176}Pt , F= ^{178}Pt , G= ^{180}Pt , H= ^{182}Pt , I= ^{184}Pt , J= ^{186}Pt , K= ^{188}Pt , L= ^{190}Pt , M= ^{192}Pt , N= ^{194}Pt , O= ^{196}Pt , P= ^{198}Pt , Q= ^{200}Pt , R= ^{202}Pt

2- The mixed symmetry state

When study the effect of Majorana parameter ($\zeta_{1,3}, \zeta_2$) on the calculated excitation energy level, we fixed the value of $\zeta_{1,3} = -0.083$ for all isotopes and vary the ζ_2 between (+0.040 \rightarrow -0.045) around the best-fitted. It is found that energy values of the state; $2_3, 2_4, 2_5, 3_1, 1_1$ are responded rapidly to the changes of the ζ_2 parameters in some isotopes only, therefore these states are verified the first property of the Mixed symmetry state (MSS). Figure (2) explain the energy variation of these state as a function of the Majorana parameter ζ_2 .







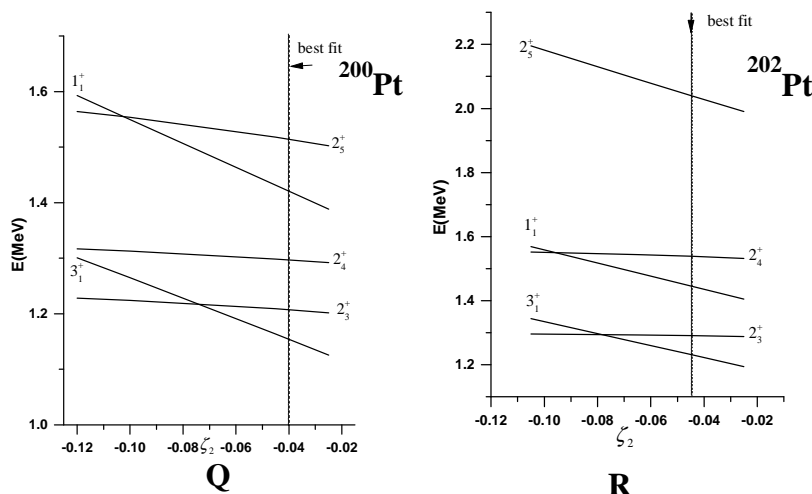


Figure 2.variation of mixed symmetry state as a function of the Majorana parameter ζ_2 . A=¹⁶⁸Pt, B=¹⁷⁰Pt, C=¹⁷²Pt, D=¹⁷⁴Pt, E=¹⁷⁶Pt, F=¹⁷⁸Pt, G=¹⁸⁰Pt, H=¹⁸²Pt, I=¹⁸⁴Pt, J=¹⁸⁶Pt, K=¹⁸⁸Pt, L=¹⁹⁰Pt, M=¹⁹²Pt, N=¹⁹⁴Pt, O=¹⁹⁶Pt, P=¹⁹⁸Pt, Q=²⁰⁰Pt, R=²⁰²Pt

Table2.theB(M1) transition of Pt. isotopes , units (μ_N)²

Transition	¹⁶⁸ Pt		¹⁷⁰ Pt		¹⁷² Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.00111	----	0.00142	----	0.00170
2 ₂ ⁺ →2 ₁ ⁺	----	0.00254	----	0.00221	----	0.00167
2 ₃ ⁺ →2 ₁ ⁺	----	0.00034	----	0.00076	----	0.00129
2 ₄ ⁺ →2 ₁ ⁺	----	0.00001	----	0.00003	----	0.00018
2 ₅ ⁺ →2 ₁ ⁺	----	0.00004	----	0.000075	----	0.000011
Transition	¹⁷⁴ Pt		¹⁷⁶ Pt		¹⁷⁸ Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.00201	----	0.00164	----	0.00131
2 ₂ ⁺ →2 ₁ ⁺	----	0.00092	----	0.00010	----	0.00005
2 ₃ ⁺ →2 ₁ ⁺	----	0.00175	----	0.00051	----	0.00008
2 ₄ ⁺ →2 ₁ ⁺	----	0.00041	----	0.00002	----	0.00029
2 ₅ ⁺ →2 ₁ ⁺	----	0.000038	----	0.00013	----	0.00029
Transition	¹⁸⁰ Pt		¹⁸² Pt		¹⁸⁴ Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.001850	----	0.00192	----	0.00237
2 ₂ ⁺ →2 ₁ ⁺	----	0.00005	----	0.000047	----	0.00012
2 ₃ ⁺ →2 ₁ ⁺	----	0.00008	----	0.00010	----	0.00011
2 ₄ ⁺ →2 ₁ ⁺	----	0.00029	----	0.00042	----	0.00065
2 ₅ ⁺ →2 ₁ ⁺	----	0.00029	----	0.00023	----	0.00073
Transition	¹⁸⁶ Pt		¹⁸⁸ Pt		¹⁹⁰ Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.00253	----	0.00249	----	0.00245
2 ₂ ⁺ →2 ₁ ⁺	----	0.00030	----	0.00083	----	0.00167
2 ₃ ⁺ →2 ₁ ⁺	----	0.00010	----	0.00194	----	0.00143
2 ₄ ⁺ →2 ₁ ⁺	----	0.00100	----	0.00003	----	0.00001
2 ₅ ⁺ →2 ₁ ⁺	----	0.00102	----	0.000029	----	0.00047
Transition	¹⁹² Pt		¹⁹⁴ Pt		¹⁹⁶ Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.00227	----	0.00199	----	0.00167
2 ₂ ⁺ →2 ₁ ⁺	0.000245	0.00153	0.000104	0.00163	0.01587	0.00185
2 ₃ ⁺ →2 ₁ ⁺	----	0.00145	----	0.00120	0.0013	0.00089
2 ₄ ⁺ →2 ₁ ⁺	----	0.00001	----	0.000015	----	0.00001
2 ₅ ⁺ →2 ₁ ⁺	----	0.00046	----	0.00048	----	0.00050
Transition	¹⁹⁸ Pt		²⁰⁰ Pt		²⁰² Pt	
	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
1 ₁ ⁺ →0 ₁ ⁺	----	0.00131	----	0.00088	----	0.00044
2 ₂ ⁺ →2 ₁ ⁺	0.00165	0.00167	----	0.00192	----	0.00202
2 ₃ ⁺ →2 ₁ ⁺	----	0.00034	----	0.00026	----	0.00013
2 ₄ ⁺ →2 ₁ ⁺	----	0.00010	----	0.00004	----	0.00036
2 ₅ ⁺ →2 ₁ ⁺	----	0.00037	----	0.00059	----	0.000033

3- The M1 Transitions

In order to calculate M1 transition one should estimate the value of the g-factor for proton and neutron in equation (8). It was found that the predicted values for $g_v = 0.315(\mu_N)$ and $g_\pi = 0.398(\mu_N)$, where $(g_\pi - g_v) = 0.083(\mu_N)$ were the best fitted values. The $B(M1)$ reduced transition probabilities were calculated by using eq.(7) and listed in Table (2) for all Pt isotopes. The experimental values of $B(M1)$ were taken from reference [22].

In most cases in table (2) it has been recognized that the value of $B(M1; 1^+_1 \rightarrow 0^+_1)$ has a large value in comparison with any other transition probability. So one should compare and normalize them by the ratio defined as [23];

$$R = \frac{B(M; 1^+_1 \rightarrow 0^+_1)}{B(M; I^+_i \rightarrow I^+_f)} \text{ where } i=2,3,4,5 \text{ and } f=1, I=2 \tag{9}$$

where R value determined by the value of $B(M; I^+_i \rightarrow I^+_f)$ transition, if the R value is large, this means that a small value of $B(M; I^+_i \rightarrow I^+_f)$ transition and state (I^+_i) is a fully symmetric state (FSS). However, if the R value is small, this means that a large value of $B(M; I^+_i \rightarrow I^+_f)$ transition and state (I^+_i) is a mixed symmetry state (MSS). Figure(3) shows the value of R for different M1 transitions, as a function of mass number in all Pt isotopes.

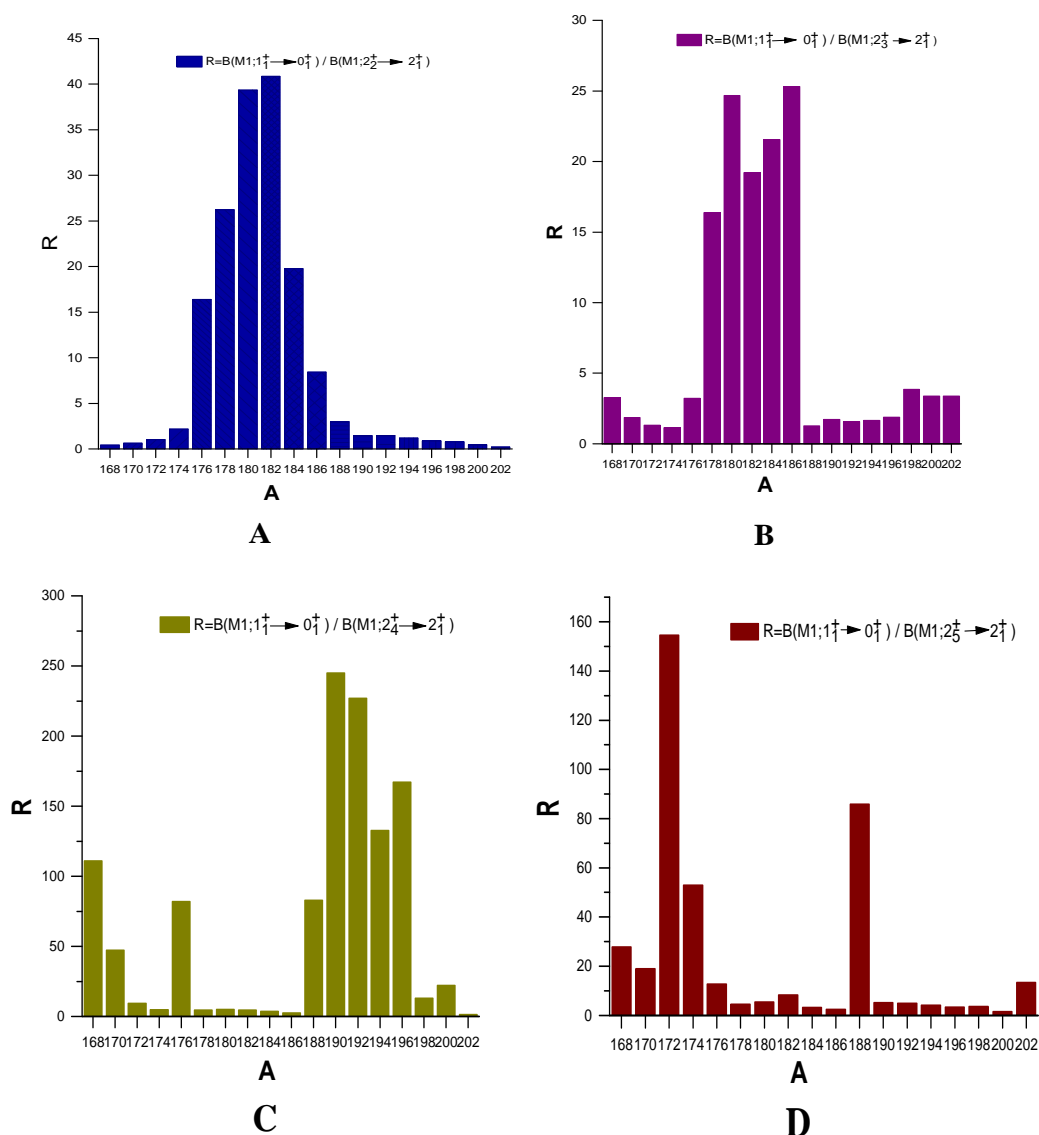


Figure 3. The ratio $R = \frac{B(M; 1^+_1 \rightarrow 0^+_1)}{B(M; I^+_i \rightarrow I^+_f)}$, where $i=2,3,4,5, f=1, I=2$ as a function of mass number A

In figure (A and B), we note the R ratio for 2^+_2 and 2^+_3 states are small for $(N_v = 4, 5, 6, 7, 8^*, 7^*, 6^*, 5^*, 4^*, 3^*, 2^*, 1^*)$. The ratio in figure (C and D) shows a small value of the ratio R for 2_4 and 2_5 states.

for($N_v=6,7,8,9,10,11,10^*,9^*,3^*,2^*,1^*$) and ($8,9,10,11,10^*,9^*,7^*,6^*,5^*,4^*,3^*,2^*,1^*$) respectively. This because of that all of these isotopes have large value of $B(M1)$, so these states are mixed symmetry states.

CONCLUSION

We have presented the predicted excitation spectra and $B(M1)$ for even mass Pt isotopes in the framework of IBM-2. A suggestion of spin and parity for some highly excited state has been made. We have examined the properties of mixed symmetry states on excited states around 2 MeV and found that, the states ($2_3, 2_4, 2_5, 3_1$ and 1_1) are good candidate for mixed symmetry states. The excitation energy of these states found to be very sensitive to variation of Majorana parameter specially ξ_2 parameter. It is also found that, there is a clear effect of $B(M1)$ transition on the characteristic of the mixed symmetry state in particular 1^+_1 state.

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