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Minimizing Rental Cost for Specially Structured Two Stage Flow Shop Scheduling, Processing Time, Setup Time Associated With Probabilities Including Weightage of Jobs

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ABSTRACT

This paper is an attempt to develop a new heuristic algorithm, an alternative to the traditional algorithm proposed by Johnson (1954) to find the optimal schedule of jobs to minimize the utilization time of the machines and hence, their rental cost for two stage specially structured flow shop scheduling under specified rental policy in which processing times, set up times are associated with their respective probabilities including weightage of jobs. In most of literature the processing times are always considered to be random, but there are significant situations in which processing times are not merely random but bear a well defined structural relationship to one another. A numerical illustration is given to support the algorithm.

Keywords: Specially structured flow shop scheduling, weightage of jobs, setup time, rental cost.

INTRODUCTION

In flow shop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machines will optimize some well define criteria. Every job will go on these machines in a fixed order of machines. The research into flow shop problems has drawn a great attention in the last decades with the aim to minimize the cost and to maximize the effectiveness of industrial production. Various techniques have been search out to deal with flow shop scheduling problem such as critical path method, branch and bound algorithm, heuristic method, Gants Charts, method of adjacent pair wise job interchange, tabbu search method, idle operator method etc. The optimization algorithm for two, three stage flow shop problem in order to minimize the processing times have been developed by Johnson [10]. Smith [16] considered minimize of mean flow time and maximum tardiness. Some of the note worthy heuristic approaches are due to Sen et al [14], Chandersekhran [3], Bagga and Bhambani [2], Gupta Deepak [7], Narain [12,13], Chakarvarthy [4], Maggu & Das [11] etc. Yoshida & Hitomi [19] considered the problem with setup time separated from processing time. Setup includes work to prepare the machine for processing. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant. Maggu [11] gave the algorithm to minimization of weighted mean flow time of jobs. The weight of a job shows the relative priority over some other job in a schedule of jobs.

In the sense of providing relative importance in the process Chandermouli [5] associated weight with jobs.

Gupta, Sharma & Bala Shashi [9] studied specially structured n×2 flowshop scheduling under specified rental policy in which processing times are associated with probabilities. This paper is an attempt to extend the study made by Gupta, Sharma & Bala Shashi by introducing the setups and weightage in jobs. Thus the problem discussed here become wider and more practical in process industry. We have obtained an algorithm which gives minimum utilization time & hence minimum rental cost.

Practical Situation:
In our day to day working in factories and industries, many applied and experimental situations occur. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various quality of papers sugar, oil are produced with relative importance i.e. weight in jobs hence weightage of jobs is significant. Various practical situations occur in real life when one has got assignment but does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting to establish an industry or factory, an industrialist does not have enough money or does not want to take risk of investing huge money to purchase machines. So he prefers to take the machines on rent. Renting enables saving working capital, gives option for having the equipment and allows up gradation to new technology.

Notations:
S: Sequence of jobs 1, 2, 3, ….., n.
S_k: Sequence obtained by applying Johnson's procedure.
M_j: Machine j, j= 1,2
a_{ij}: Processing time of i^{th} job on machine M_j.
p_{ij}: Probability associated to the processing time a_{ij}.
A_{ij}: Expected processing time of job on machine M_j
s_{ij}: Set up time of i^{th} job on machine M_j.
q_{ij}: Probability associated to the set up time s_{ij}.
S_{ij}: Expected set up time of i^{th} job on machine M_j.
w_i: Weight of i^{th} job
C_j : Rental cost of i^{th} machine.
t_{ij} (S_k): Completion time of i^{th} job of sequence S_k on machine M_j
$T_{ij}(S_k)$: Idle time of machine M_j for job i of the sequence s_k.
$U_j(S_k)$: Utilization time for which machine M_j is required.
$R(S_k)$: Total rental cost for the sequence S_k of all machine.

Definition:
Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + a_{ij} \times p_{ij} + s_{i-1,j} \times q_{i-1,j}$
$= \max(t_{i-1,j}, t_{i,j-1}) + A_{ij} + S_{i-1,j}$ for $j \geq 2$.
Where
A_{ij} = Expected time of i^{th} job on j^{th} machine.
S_{ij} = Expected setup time of i^{th} job on j^{th} machine.

Rental Policy:
The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing of job s, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on 1^{st} machine.

Problem formulation:
Let n jobs 1, 2, 3, ….., n be processed on two machines M_1 and M_2 in a way such that no passing in allowed. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the set up time of i^{th} job on j^{th} machine with probability q_{ij}. Let A_{ij} and S_{ij} be the expected processing time and set up time respectively of i^{th} job on j^{th} machine; w_i be the weight of i^{th} job. A''_{ij} be the weighted flow time of the i^{th} job on j^{th} machine such that either

$A''_{i1} \geq A''_{i2}$

or $A''_{i1} \leq A''_{i2}$ for all values of i,
Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machine.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1				Machine M_2				Weight
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	w_i
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}	w_1
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}	w_2
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}	w_3
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	a_{n2}	p_{n2}	s_{n2}	q_{n2}	w_n

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum A_{i1} \times C_1 + U_2(S_k) \times C_2$

Subject to constraint: Rental policy P.
Our objective is to minimize rental cost of machines while minimizing the utilization time.

Assumptions:

1. Jobs are independent to each other. Let n-jobs be processed through two machines M_1, M_2 in the order M_1M_2.
2. Machine break down is not considered.
3. Pre-emption is not allowed.
4. Either the weighted flow time of i^{th} job of machine M_1 is longer than the weighted flow time of i^{th} job on machine M_2 or the weighted flow time of i^{th} job on machine M_1 is shorter than the weighted flow time of i^{th} job on machine M_2 for all i.
   Either $A''_{i1} \geq A''_{i2}$ Or $A''_{i1} \leq A''_{i2}$
5. $\sum p_{i1} = 1$, $\sum q_{i1} = 1$, $\sum p_{i2}=1$, $\sum q_{i2}=1$
6. $0 \leq p_{i1} \leq 1$, $0 \leq p_{i2} \leq 1$, $0 \leq q_{i1} \leq 1$ and $0 \leq q_{i2} \leq 1$

Algorithm:

Step 1: Calculate the expected processing time and expected set up time as follow:
$A_{ij} = a_{ij} \text{ x } p_{ij}$ and $S_{ij} = s_{ij} \text{ x } q_{ij}$ $\forall$ i, j

Step 2: Calculate expected flow time for two machines M_1 and M_2 as follow:
$A'_{i1} = A_{i1} - S_{i2}$ and $A'_{12} = A_{i2} - S_{i1}$ $\forall$ i

Step 3: If $\min(A'_{i1}, A'_{12}) = A'_{i1}$

Then $G_i = A'_{i1} + w_i$

And $H_i = A'_{12}$

If $\min(A'_{i1}, A'_{12}) = A'_{12}$

$G_i = A'_{12}$

Then $H_i = A'_{12} + w_i$

Step 4: Find weighted flow time for two machines M_1 and M_2 as follows:

$$A''_{i1} = \frac{G_i}{w_i} \text{ and } A''_{i2} = \frac{H_i}{w_i}$$

Step 5: Define a new reduced problem with the processing times A''_{i1} and A''_{i2}. As defined in step 4.

Step 6: Check the structural relationship:
Either $A''_{i1} \geq A''_{i2}$ $\forall$ i, j
Or $A''_{i1} \leq A''_{i2}$ $\forall$ i, j
If the above relation hold good then go to step 7 else modify the data.

Step 7: Obtain the job J_1 (say) having maximum processing time on 1^{st} machine obtain the job J_n (say) having minimum processing time on 2^{nd} machine.

Step 8: If $J_1 \neq J_n$ then put J_1 on the 1^{st} position and J_n on the last position and go to step 11 otherwise go to step 9.

Step 9: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1. Also take the difference of processing time of Job J_n on M_2 from job J_{n-1} (say) having next minimum processing time on M_2. Call this difference as G_2.

Step 10: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on the first position and J_{n-1} on the last position.

Step 11: Arrange the remaining (n-2) jobs between 1^{st} job and last job in any order, there by we get the sequences S_1, S_2, …… S_r.

Step 12: Compute the total completion time CT (S_k), k=1, 2, …, r. By computing in-out table for sequence S_k, K= 1, 2, ….. r

Step13: Calculate utilization time $U_2(S_k)$ of 2^{nd} machine.

$U_2(S_k) = CT(S_k) - A_{i1}(S_k)$; K=1, 2, … , r

Step14: Find rental cost $R(S_i) = A_{i1}(S_k) \times C_1 + U_2(S_k) \times C_2$ where C_1 & C_2 are the rental cost per unit time of 1^{st} and 2^{nd} machine respectively.

Numerical Illustration

Consider 5 jobs, 2 machines problem to minimize the rental cost with weights of jobs, processing time and set up time associated with their respective probabilities are given in following table. The rental cost per unit time for machines M_1 and M_2 are 10 units and 5 units respectively. Our objective is to obtain optimal schedule to minimize the utilization time and hence the rental cost of machines under the rental policy (P).

Jobs	Machine M_1				Machine M_2				Weight
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	w_i
1	30	0.2	1	0.1	15	0.2	3	0.1	2
2	45	0.3	3	0.2	10	0.3	4	0.1	3
3	50	0.1	4	0.2	20	0.2	2	0.3	1
4	60	0.2	3	0.3	18	0.2	3	0.2	1
5	40	0.2	2	0.2	12	0.1	1	0.3	2

Solution:

As per Step 1: Expected processing time and setup time for machines M_1 and M_2 is as follow:

Jobs	Machine M_1		Machine M_2		Weight
i	A_{i1}	S_{i1}	A_{i2}	S_{i2}	w_1
1	6.0	0.1	3.0	0.3	2
2	13.5	0.6	3.0	0.4	3
3	5.0	0.8	4.0	0.6	1
4	12.0	0.9	3.6	0.6	1
5	8.0	0.4	1.2	0.3	2

As per step 2, 3 & 4: reduced problem with weighted flow time for two machine M_1 and M_2 is as follow:

Jobs i	Machine M_1 A''_{i1}	Machine M_2 A''_{i2}
1	2.85	2.45
2	4.37	1.8
3	4.4	4.2
4	11.4	3.7
5	3.85	1.4

Data is as per step 6 i.e. the condition $A''_{i1} \geq A''_{i2}$ for all i hold good.

Also max A''_{i1} = 11.4 which is for job 4 i.e. $J_1 = 4$.

And min A''_{i2} = 1.4 which is for job 5 i.e. $J_n = 5$.

Since $J_1 \neq J_n$
Therefore, we put $J_1 = 4$ on the first position
And $J_n = 5$ on the last position.
Therefore as per step 11 the optimal sequences are:
S_1 : 4 – 1 – 2 – 3 – 5
S_2 : 4 – 2 – 1 – 3 – 5
S_3 : 4 – 1 – 2 – 3 – 5
S_4 : 4 – 2 – 3 – 1 – 5
S_5 : 4 – 3 – 1 – 2 – 5
S_6 : 4 – 3 – 2 – 1 – 5

Due our conditions, the total elapsed time is same for all the six sequences.

The in-out table for any one of these say for S_1: 4 – 1 – 2 – 3 – 5 is as follow:

Job	Machine M_1	Machine M_2
i	in - out	in - out
4	0 – 12.0	12.0 – 15.6
1	12.9 – 18.9	18.9 – 21.9
2	19.0 – 32.5	32.5 – 35.5
3	33.1 – 38.1	38.1 – 42.1
5	38.9 – 46.9	46.9 – 48.1

Total elapsed time = $CT(S_1)$ = 48.1 units

And utilization time for M_2 = 48.1 – 12.0 = 36.1 units

Also $\sum_{i=1}^{n} A_{i1}$ = 49.6 units

Therefore total rental cost for S_1 is $R(S_1)$ = 649.5 units

Hence the total cost rental cost for each of the sequence $\{S_k\}$, k == 1, 2,…6 is
$R(S_k)$ = 649.5 units

Remarks: If we solve the above problem by Johnson's rule [1] we get the optimal sequence as S = 3 – 4 – 1 – 2 – 5.

Jobs	Machine M_1	Machine M_2
i	In Out	In Out
3	0 – 5.0	5.0 – 9.0
4	5.8 – 17.8	17.8 – 21.4
1	18.7 – 24.7	24.7 – 27.3
2	24.8 – 38.3	38.3 – 41.3
5	38.9 – 46.9	46.9 – 48.1

Therefore the total elapsed time = CT(S) = 48.1
And utilization time for M_2 = U_2 (S) = 43.1 units
Therefore rental cost is R(S) = 684.5 units

CONCLUSION

1. The algorithm proposed here for specially structured two stage flow shop scheduling problem processing time, setup times associated with probabilities with weightage of jobs is more efficient as compared to the algorithm proposed by Johnson [10] to find an optimal sequence to minimize the utilization time of the machine and hence their rental cost.
2. The study on n×2 specially structured flow shop scheduling may be further extended by including various parameters, such as break down intervals, weightage of jobs etc.
3. The study may further be extend for n job 3 machines flow shop problem.

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