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Advances in Applied Science Research, 2012, 3 (6):3482-3490

ISSN: 0976-8610
CODEN (USA): AASRFC

MHD free convection heat and mass transfer flow through a porous medium bounded by a vertical surface in presence of hall current

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ABSTRACT

In this paper the effects of hall current, chemical reaction and radiation on a free convection flow bounded by a vertical surface embedded in porous medium under the influence of uniform magnetic field which is applied normal to the surface is studied. The problem is solved analytically and the expressions for velocity, temperature, concentration, skin friction and rate of heat and mass transfer are derived and the effects of various physical parameters like Magnetic parameter M, radiation parameter F, Grashof number Gr, modified Grashof number Gm, Prandtl number Pr, permeability parameter k and the chemical reaction parameter k_0 *are studied though graphs and tables. It is observed that the velocity and concentration increase during a generative reaction and decrease in a destructive reaction. The same is true for the behavior of the fluid temperature. The presence of magnetic field and radiation diminishes the velocity and also the temperature.*

Key words: Hall current, chemical reaction, MHD, Radiation, Porous medium .

INTRODUCTION

In nature, there exist flows which are caused not only by the temperature differences but also by concentration differences. These mass transfer differences do affect the rate of heat transfer. In industries, many transport process exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion thermo chemical species. The phenomenon of heat and mass transfer frequently exists in chemically processed industries such as food processing and polymer production. Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Magetohydrodynamics is attracting the attention of the many authors due to its applications in geophysics; it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering in MHD pumps, MHD bearings etc. at high temperatures attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic and alter heat transfer and friction characteristic. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. This is of interest because heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications and higher operating temperatures.

Soundalgekar and Takhar [1] first, studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly-Vincentine-Gilles equilibrium model. For the same gas Takhar et al. [2] investigated the effects of radiation on the MHD free convection flow past a semi-infinite vertical plate. Later, Hossain et al. [3] studied the effect of radiation on free convection from a porous vertical plate. Muthucumarswamy and Kumar [4] studied the thermal radiation effects on moving infinite vertical plate in presence of variable temperature and Mass diffusion. An analytical solution for unsteady free convection in porous media has been studied by Magyari et al. [5]. Chamkha et al. [6] studied the effects of Hydro magnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. Mazumdar and Deka [7] studied MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation.

The growing need for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. The presence of a foreign mass in water or air causes some kind of chemical reaction. This may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, for example, polymer production, manufacturing of ceramics or glassware and food processing. A chemical reaction can be codified as either a homogenous or heterogeneous process. This depends on whether it occurs on an interface or a single phase volume reaction. A reaction is said to be of first order if its rate is directly proportional to the concentration itself [8]. The effect of chemical reaction on heat and mass transfer in a laminar boundary layer flow has been studied under different conditions by several authors [9-18]. The effect of a chemical reaction on a moving isothermal vertical surface with suction has been studied by Muthucumarswamy [19]. Recently, Manivannan et al. [20] investigated radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source was investigated by Sharma et al. [21]. Mahapatra et al. [22] studied the effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Rajasekhar et al. [23], Kishan and Srinivas [24], Anjalidevi and David [25], Kishan and Deepa [26] and Gaikwad and Rahuldev [27] studied the effects of various parameters on fluid flow quantities. In all the above studies the combined effect of radiation and chemical reaction on MHD free convective flow in addition to Hall currents have not been considered simultaneously. Here we have made an attempt to study the Hall current effects on a steady flow of viscous fluid through a porous medium bounded by a porous surface subjected to suction with a constant viscosity in the presence of radiation and homogenous chemical reaction of first order.

Formulation of the problem:

We consider an electrically conducting, radiating, viscous incompressible fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface. The x^* axis is taken along the surface in an upward direction and the y^* axis is normal to it. A uniform magnetic field $B_0$is assumed to be applied in a direction perpendicular to the surface. The fluid properties are assumed to be constant except for the density in the body force term. A chemically reactive species is emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it under goes a homogenous chemical reaction. The reaction is assumed to take place entirely in the stream. Then the fully developed flow under the above assumptions through a highly porous medium is governed by the following set of equations:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$v^*\frac{\partial u^*}{\partial y^*}=\vartheta\frac{\partial^2 u^*}{\partial y*^2}+gB_T(T^*-T_\infty)+gB_C(C^*-C_\infty)-\frac{\sigma B_0^2}{\rho(1+m^2)}u^*-\frac{\vartheta}{k_p}u^* \tag{2}$$

$$v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_P}\frac{\partial^2 T^*}{\partial y*^2}+\frac{\vartheta}{C_P}\left(\frac{\partial u^*}{\partial y^*}\right)^2-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y^*} \tag{3}$$

$$v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y*^2}-k_cC^* \tag{4}$$

It is assumed that the level of species concentration is very low; hence the heat generated due to chemical reaction is neglected. The relevant boundary conditions are given as follows

$$u^* = 0, T^* = T_W, C^* = C_W \text{ at y = 0}$$

$$u^* = 0, T^* = T_\infty, C^* = C_\infty \text{ as y} \to \infty \tag{5}$$

Equation (1) gives that $v^* = constant = -v_0$ (6)

In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self absorption, but it does absorb radiation emitted by the bounadries. Cogle et al. [22] showed that in the optically thick limit for a non gray gas near equilibrium as given below.

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T_w^*)\int_0^\infty K_{?}\ \lambda_w w(\frac{de_{b\lambda}}{dT^*}) / d\lambda = 4I_1(T^* - T_w^*) \tag{7}$$

On introducing the following non dimensional quantities,

$$y = \frac{v_0 y^*}{v}, u = \frac{u^*}{v_0}, \theta = \frac{T^* - T_\infty^*}{T_W{}^* - T_\infty^*}\ C = \frac{C^* - C_\infty^*}{C_W{}^* - C_\infty^*}, \Pr = \frac{\mu c_p}{k}$$

$$S_c = \frac{v}{D}, F = \frac{4I_1\vartheta}{kv_0^2}, Gr = \frac{\vartheta g\beta_T(T_W^* - T_\infty^*)}{v_0^3}, Gm = \frac{\vartheta g\beta_c(C_W^* - C_0^*)}{v_0^3},$$

$$K_C = \frac{\vartheta K_C{}^*}{v_0^2}, E = \frac{v_0^2}{C_P(T_0^* - T_w^*)} M = \frac{\sigma B_0^2\vartheta}{\rho v_0^2}, k = \frac{v_0^2 k_P}{\vartheta^2} \tag{8}$$

The non-dimensional form of the governing equations (2) to (4) reduce to

$$u'' + u' = -Gr\theta - GmC + M_1 u \tag{9}$$

Where $M_1 = \dfrac{M}{1+m^2} + \dfrac{1}{k}$

$$\theta'' + Pr\theta' = -PrEu'^2 + F\theta \tag{10}$$

$$C'' + S_c C' = k_c S_c C \tag{11}$$

The corresponding boundary conditions are given by

$$u = 0, \theta = 1, C = 1 \text{ at y = 0;} \quad u = 0, \theta = 0, C = 0 \text{ as } y \to \infty \tag{12}$$

Solution of the problem:

In order to solve the coupled nonlinear system of equations (9) to (11) with the boundary conditions (12), the following simple perturbation is used. The governing equations (9) to (11) are expanded in powers of Eckert number $E(<<1)$.

$$u = u_0 + Eu_1 + O(E^2), \theta = \theta_0 + E\theta_1 + O(E^2), C = C_0 + EC_1 + O(E^2) \tag{13}$$

Substituting equations (13) into equations (9) to (11) and equating the coefficients at the terms with the same powers of E, and neglecting the terms of higher order, the following equations are obtained.

Zero order terms: $u_0'' + u_0' = -Gr\theta_0 - GmC_0 + M_1 u_0$ (14)

$$\theta_0'' + Pr\theta_0' - F\theta_0 = 0 \tag{15}$$

$$C_0'' + S_C C_0' = S_C k_C C_0 \tag{16}$$

First order terms: $u_1'' + u_1' = -Gr\theta_1 - GmC_1 + M_1 u_1$ (17)

$$\theta_1'' + Pr\theta_1' - F\theta_1 = -\Pr u_0'^2 \tag{18}$$

$$C_1'' + S_C C_1' = S_C k_C C_1 \tag{19}$$

The corresponding boundary conditions are

$$u_0 = 0, u_1 = o, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0 \quad \text{at } y = 0$$

$$u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \quad \text{as } y \to \infty \tag{20}$$

Solving equations (14) to (19) under the boundary conditions (20), the following solutions are obtained.

$$C_0 = C_1 = e^{-k_1 y} \tag{21}$$

$$\theta_0 = e^{-k_2 y} \tag{22}$$

$$u_0 = (-k_3 - k_4)e^{-l_1 y} + k_3 e^{-k_2 y} + k_4 e^{-k_1 y} \tag{23}$$

$$\begin{aligned} u_1 &= k_{18}e^{-2k_2 y} + k_{19}e^{-2k_1 y} + k_{20}e^{-2l_1 y} + k_{21}e^{-l_2 y} + k_{22}e^{-l_3 y} \\ &+ k_{23}e^{-l_4 y} + k_{24}e^{-l_6 y} + k_{25}e^{-k_1 y} - k_{26}e^{-l_7 y} \end{aligned} \tag{24}$$

$$\theta_1 = k_{11}e^{-2k_2 y} + k_{12}e^{-2k_1 y} + k_{13}e^{-2l_1 y} + k_{14}e^{-l_2 y} + k_{15}e^{-l_3 y} + k_{16}e^{-l_4 y} - k_{17}e^{-l_6 y} \tag{25}$$

The expressions for the constants involved in equations (21) to (25) are given in the appendix.

RESULTS AND DISCUSSION

In order to point out the effects of various parameters on flow characteristic, the following discussion is set out. The values of Prandtl number are chosen Pr=7 (water) and Pr=0.71 (air). The value of the Schmidt number is chosen to represent the presence of species by hydrogen (0.22). Velocity profiles are presented in figures 1 to 8. Figure 1 depicts the velocity profile at the absence of radiation and MHD. From this figure it is observed that fluid velocity increases and reaches its maximum over a very short distance from the plate and then gradually to zero for various values of k. These results are in good agreement with the results of those Mahapatra et al. [21]. Figure 2 shows the effect of magnetic parameter M on the velocity. From this figure it is observed that velocity decreases, in both the cases of air and water, as the value of M is increased. It is true as the magnetic force retards the flow, velocity decreases. Velocity increases as the Hall parameter increase as shown in Figure 3. Figure 4 depicts the velocity profiles for different values of chemical reaction parameter k_0. A generative reaction ($k_0<0$) increases the fluid flow velocity, whereas a destructive reaction ($k_0>0$) reduces it. In the case of water its magnitude is less than that of air. From figure 5 it is clear that an increase in the permeability parameter results in an increase in the velocity for both the cases of generative reaction and destructive reaction. A radiation effect on velocity is shown in figure 6, from this figure it is observed that velocity decreases as the radiation parameter F increases. Velocity profiles for different values of Gr and Gm, shown in figures 7 and 8. From these figures it is noticed that velocity increases with the increase Gr and Gm.

Temperature profiles are displayed through figures 10- 12. Effect of magnetic parameter M in the case of water and air is observed on the temperature, it is observed that temperature decreases with the increase in M; in the case of water the magnitude of the decrease of temperature is very low. From figures 10 ,11and 12 it is clear that temperature decreases with the increase in radiation parameter F, the Schmidt number Sc and it shows the reverse effect in case of permeability parameter k. figure 13 depicts the concentration profiles with the variations in chemical reaction parameter k_0, from this figure it is observed that a destructive reaction reduces the concentration. This is due to the fact that for $k_0>0$ the last term in the mass diffusion equations (16) and (19) becomes positive and it contributes to the concentration reduction. At the same time, the same term mentioned an equation becomes negative for a generative reaction$k_0<0$and, as a result, it leads to a concentration increase. Concentration profiles for different values of Schmidt number Scare shown in Figure 14, it is noticed that concentration decreases with an increase in Sc.

The **rate of heat transfer** in terms of the Nusselt number is given by

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=k_2+E(2k_2k_{11}+2k_1k_{12}+2l_1k_{13}+l_2k_{14}+l_3k_{15}+l_4k_{16}-l_6k_{17} \qquad (26)$$

The Non-dimensional **skin friction** at the surface is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=(k_3+k_4)l_1-k_3k_2-k_4k_1+E(-2k_2k_{18}-2k_1k_{19}-2l_1k_{20}-l_2k_{21}-l_3k_{22}$$

$$-l_4k_{23}-l_6k_{24}-k_1k_{25}e^{-k_1y}+l_7k_{26} \qquad (27)$$

Another important physical quantity of interest is the **Sherwood number** which is in non-dimensional form is given by $S_h=-\left(\frac{\partial C}{\partial y}\right)_{y=0}=k_1(1+E)$ (28)

A variation in the heat transfer rate expressed in terms of the Nusselt number is shown table 1, from this table it is observed that Nu increases with the increase in magnetic parameter M and radiation parameter F, where as it shows reverse effect within the case of permeability parameter k ,Hall parameter m and chemical reaction parameter k_0.A variation in skin friction τ is shown in table 2, from this table it is noticed that skin friction decreases with an increase in M and F, where as it shows reverse effect in the case of k, m and k_0. Similarly a variation in the Sherwood number is shown in table 3, it is observed that Sherwood number (Sh) increases with an increase in Eckert number E and Schmidt number Sc and shows the reverse effect in the case of k_0.

Table 1. Rate of heat transfer Nu for different values of M, m, F, k_0, k With fixed values for Pr=0.71, Gr=5, Gm=5.

M	m	F	K_0	K	**Nu**
1	1	0.5	0.04	0.1	**1.0836**
2	1	0.5	0.04	0.1	**1.0837**
3	1	0.5	0.04	0.1	**1.0906**
2	1	1.0	0.04	0.1	**1.3635**
2	1	2.0	0.04	0.1	**1.7674**
2	1	0.5	0.00	0.1	**1.0863**
2	1	0.5	-0.04	0.1	**1.0847**
2	1	0.5	0.04	0.7	**0.7643**
2	1	0.5	0.04	1.0	**0.6678**
2	1	0.5	0.04	2.0	**0.4833**
2	3	0.5	0.04	0.1	**1.0812**
2	5	0.5	0.04	0.1	**1.0802**
2	10	0.5	0.04	0.1	**1.0797**

Table 2. Skin friction for different values of M, m, F, k_0, k with fixed values for Pr=0.71, Gr=5, Gm=5.

E	Sc	K_0	**Sh**
0.01	0.22	0.04	**0.2571**
0.05	0.22	0.04	**0.2673**
0.1	0.22	0.04	**0.2800**
0.2	0.22	0.04	**0.3055**
0.05	0.60	0.04	**0.6695**
0.05	0.78	0.04	**0.8590**
0.05	0.22	0.00	**0.2310**
0.55	0.22	-0.04	**0.1758**

Table 3. Variations of Sherwood number Sh for Various values of E, Sc, k_0 with the fixed values of M=2, Gr=5, Gm=5, Pr=0.71, k=0.1

M	m	F	k_0	k	**τ**
1	1	0.5	0.04	1	**7.9577**
2	1	0.5	0.04	1	**6.9002**
3	1	0.5	0.04	1	**6.1855**
2	1	1.0	0.04	1	**6.5858**
2	1	2.0	0.04	1	**6.2507**
2	1	3.0	0.04	1	**6.0548**
2	1	0.5	0.00	1	**7.0315**
2	1	0.5	-0.04	1	**7.2462**
2	1	0.5	0.04	0.7	**6.2737**
2	1	0.5	0.04	0.1	**3.0254**
2	3	0.5	0.04	0.1	**8.9022**
2	5	0.5	0.04	0.1	**9.4051**
2	10	0.5	0.04	0.1	**9.6704**

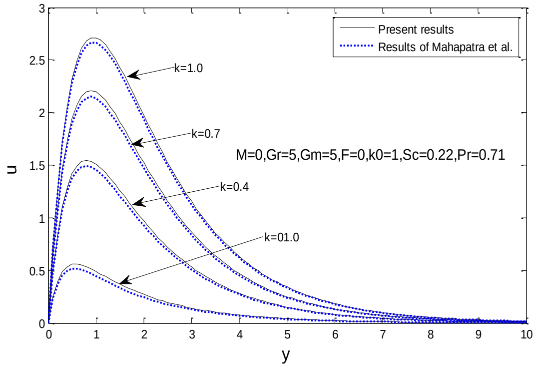


Fig.1. Velocity profiles for different values of k when M=0 and F=0.

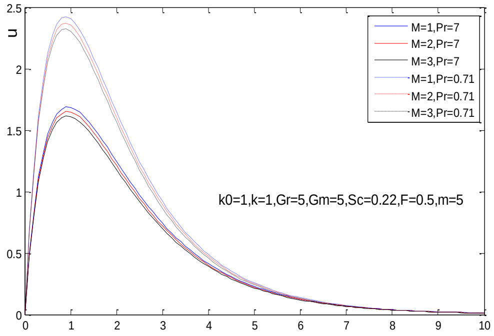


Fig.2. Velocity profiles for different values of M

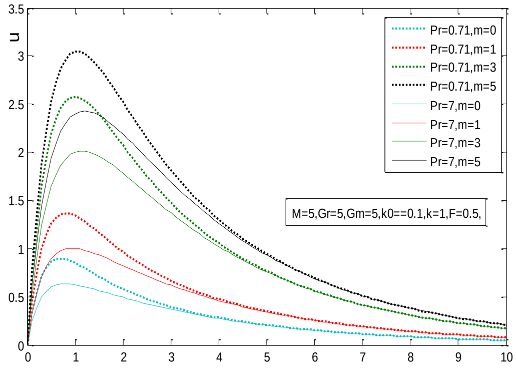


Fig.3. Velocity profiles for different values of m

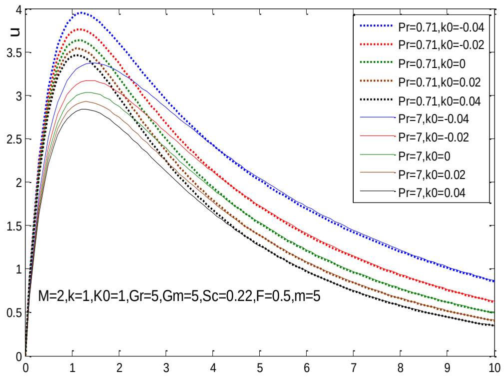
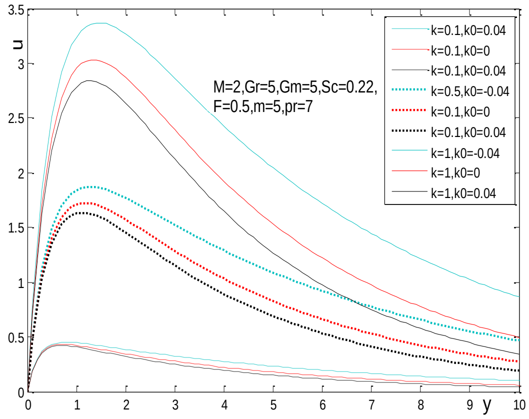


Fig.4. Velocity profiles for different values of k_0



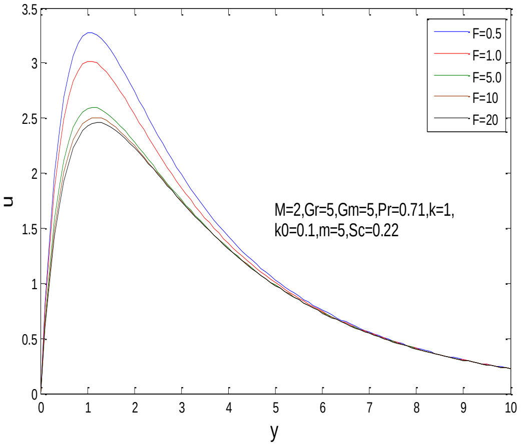
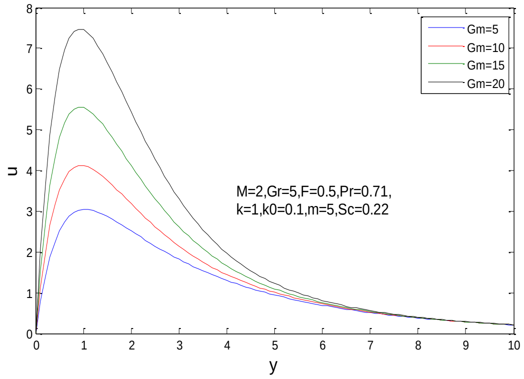


Fig.5. Velocity profiles for different values of k **Fig.6. Velocity profiles for different values of F**



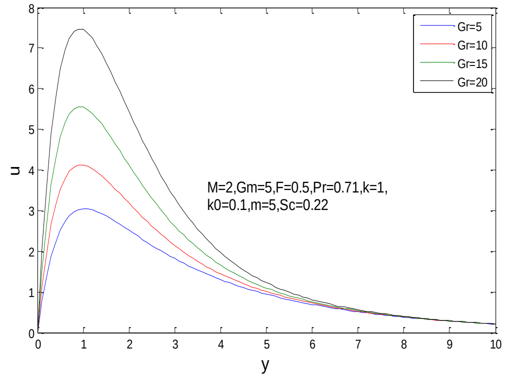
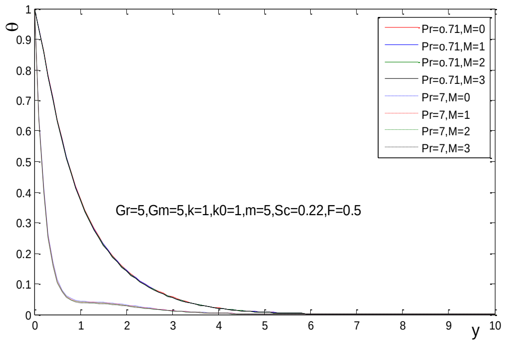


Fig.7. Velocity profiles for different values of Gm **Fig.8. Velocity profiles for different values of Gr.**



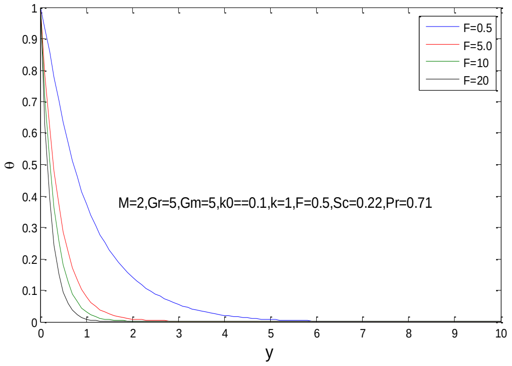


Fig.9. Temperature profiles for different values of M **Fig.10. Temperature profiles for different values of F.**

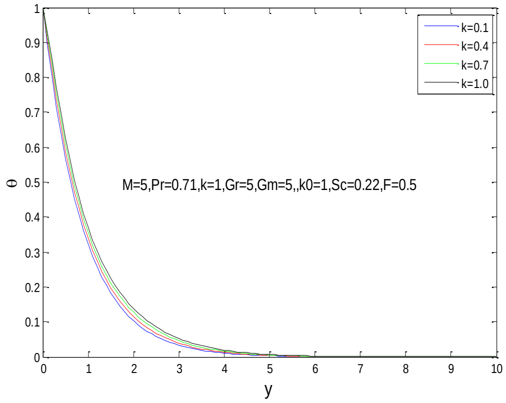


Fig.11. Temperature profiles for different values of k.

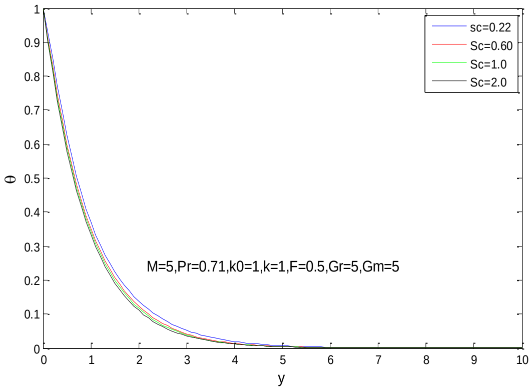


Fig.12. Temperature profiles for different values of Sc

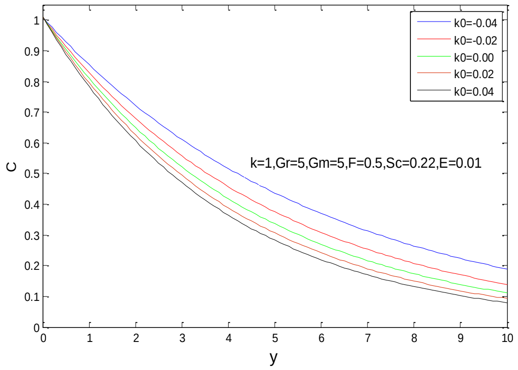


Fig.13. Concentration profiles for different values of k_0

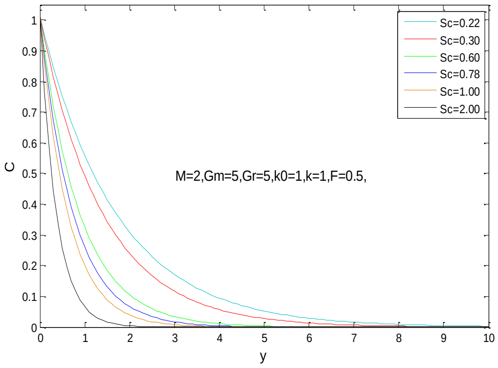


Fig.14. Concentration profiles for different values of Sc

CONCLUSION

In this paper we have studied the effects of hall current, chemical reaction and radiation on MHD free convection flow through a porous medium bounded by a vertical surface. In the analysis of the flow the following conclusions are made

i. The velocity of a fluid increases with the permeability parameter k, Hall parameter m and decreases with the increase in Magnetic parameter M, Radiation parameter F.

ii. A generative reaction ($k_0<0$) increases the fluid flow velocity, whereas a destructive reaction ($k_0>0$) reduces it.

iii. In most cases the velocity attains a maximum near the surface and there after decreases.

iv. Temperature decreases with the increase Magnetic parameter, radiation parameter but it shows the reverse effect in the case of chemical reaction parameter.

v. A destructive reaction reduces the concentration whereas the generative reaction increases it.

vi. Nusselt number increases with the increase in magnetic parameter M and radiation parameter F, where as it shows reverse effect in the case of permeability parameter k and concentration parameter k_0.

vii. Skin friction decreases with an increase in M and F, where as it shows reverse effect in the case of k ,mand k_0

viii. Sherwood number (Sh) increases with an increase in Eckert number E and Schmidt number Sc and shows the reverse effect in the case of k_0.

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