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MHD free convection heat and mass transfer flow of viscoelastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime

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ABSTRACT

An analysis has been carried out to obtain the free convective heat and mass transfer characteristics of an incompressible MHD viscoelastic fluid flow immersed in a porous medium of variable permeability bounded by an infinite porous vertical plate in slip flow regime in the presence of a transverse magnetic field with heat source and radiation effects. The permeability of the porous medium decreases exponentially with time about a constant mean. Approximate solutions have been obtained for the velocity, temperature, concentration, skin friction and rate of heat transfer. The numerical results are displayed graphically to study the effect of several pertinent parameters such as viscoelasticity, permeability of the porous medium, magnetic field, Grashof number, modified Grashof number, Schmidt number, Prandtl number, radiation parameter, heat source/sink parameter and rarefaction parameter on the flow, heat and mass transfer characteristics. The numerical results of velocity distribution of viscoelastic fluid are compared with the corresponding flow problems for a viscous fluid.

Key Words: MHD; Free convection; Variable permeability; Viscoelastic fluid; Radiation **2000**
AMS SUBJECT CLASSIFICATION: *76A10, 76R10, 76S05, 76W05*

INTRODUCTION

The theory of non-Newtonian fluids has become a field of very active research for the last few decades as this class of fluids represent, mathematically, many industrially important fluids such as plastic films and artificial fibres in industry. In fact, the increase imergence of non-Newtonian fluids such as molten plastics, pulps, emulsions, aqueous solutions of polyacrylamid and polyisobutylene etc., as important raw materials and chemical products in a large variety of industrial processes has stimulated a considerable attention in recent years to the study of non-Newtonian fluids and their related transport processes. The equations of motion of viscoelastic fluids are one order higher than the Navier Stokes or boundary layer equations. Several authors have considered the viscoelastic fluids and a good list of references on the published work for these fluids can be found in [2, 3, 10, 14, 15, 17, 19, 20, 27].

Free convection phenomenon has been object of extensive research. The importance of this phenomenon is increasing day by day due to the enhanced concern in science and technology about buoyancy induced motions in the atmosphere, the bodies in water and quasi solid bodies such as earth. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices, Nield and Bejan [13]. From technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetosphere, aeronautics, chemical engineering and electronics on account of their varied importance, these flows have been studied by several authors notable amongst them are Shercliff [22] and Cramer [7]. A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman [18]. Singh [23] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion.

The radiative effects have important applications in physics and engineering particularly in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids and power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive fluids. The effects of radiation on heat/mass transfer problems have studied by Mukhopadhyay [12], Taneja [29], Hossain et al. [8] and Hsiao [9].

The study of the flow of a porous medium is of great importance to geophysicists and fluid dynamicists. Yamamoto and Yoshida [32] considered suction and injection flow with convective acceleration through a plane porous wall specifically for the flow outside a vortex layer. The generalization of the above study was presented by Yamamoto and Iwamura [31]. Chawla and Singh [5] studied oscillatory flow past a porous bed. The effects of variable permeability on combined free and forced convection in porous media have studied by Chandrasekhara and Namboodiri [4], Vedhanayagam et al. [30] and Sharma et al. [21]. Singh et al. [26], Singh and Kumar [24], Acharya et al. [1] have studied the effects of permeability variation on free convective flow through a porous medium. Recently, the effects of radiation and magnetic field on the free convective flow along infinite vertical plate have studied by Takhar et al. [28], Maharshi and Tak [11].

In geothermal region, situation may arise when the flow becomes unsteady and slip at the boundary take place as well. In such situation of slip flow, ordinary continuum approach fails to yield satisfactory results. Many authors have solved problems taking slip conditions at the boundary (Singh [25]).

In the present work, we make an attempt to investigate the problem of free convective heat and mass transfer of unsteady, incompressible MHD viscoelastic fluid embedded in a porous medium of variable permeability bounded by an infinite porous vertical plate in slip flow regime with heat source and radiation effects. The permeability of the porous medium decreases exponentially with time about a constant mean. The effects of various governing parameters entering into the problem like viscoelasticity, permeability or porosity parameter, magnetic parameter, Grashof number, modified Grashof number, Schmidt number, heat source/sink parameter, rarefaction parameter, the influence of Prandtl number and radiation parameter on the velocity, temperature, skin friction, rate of heat transfer are investigated and analyzed with the help of their graphical representations.

NOMENCLATURE

B_0	:	magnetic flux density
C	:	concentration of the fluid
C_p	:	specific heat at constant pressure
D	:	coefficient of chemical molecular diffusivity
G_r	:	Grashof number
G_m	:	modified Grashof number
g	:	acceleration due to gravity
h_1	:	rarefaction parameter
K	:	permeability of the porous medium
k	:	thermal conductivity
K_0	:	permeability or porosity parameter
M	:	magnetic field parameter
N_u	:	Nusselt number
P_r	:	Prandtl number
q_r	:	radiative heat flux
R	:	radiation parameter
S^*	:	source/sink coefficient
S	:	heat source/sink parameter
S_c	:	Schmidt number
T	:	temperature
u, v	:	component of velocities along x and y directions, respectively

GREEK SYMBOLS

□	:	coefficient of volumetric expansion
□*	:	volumetric coefficient of expansion with concentration
□	:	electrical conductivity
□	:	fluid density
□	:	fluid dynamic viscosity
□	:	fluid kinematic viscosity
□	:	dimensionless temperature
□	:	skin friction
□	:	dimensionless concentration
□$_0$	:	viscoelastic coefficient
□$_1$	:	viscoelastic parameter
w	:	wall condition
∞	:	free stream condition

Formulation and solution

We consider two-dimensional, unsteady, hydromagnetic, free convection with radiation and heat and mass transfer flow, of a viscous incompressible and electrically conducting viscoelastic fluid, via a porous medium of variable permeability, occupying a semi-infinite region of the space bounded by an infinite vertical porous plate with constant suction and heat source in slip flow regime. A magnetic field of uniform strength is applied transversely to the direction of the flow. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. We take the $\tilde{x}$axis along the plate and $\tilde{y}$axis normal to it and the flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the fluid. The viscous dissipation and Darcy's dissipation terms are neglected for small velocities [16].

Under these assumptions, the governing boundary layer equations of continuity, momentum, energy and diffusion could be written as follows:

continuity: $$\frac{\partial v}{\partial y} = 0 \tag{1}$$

linear momentum: $$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \upsilon\left(1 + \lambda_0 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K(t)}u - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

energy: $$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{S^*}{\rho C_p}(T - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

diffusion: $$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The permeability of the porous medium is assumed to be of the form
$$K(t) = K_0\left(1 + \varepsilon e^{-nt}\right) \tag{5}$$
where K_0 is the mean permeability of the medium, n the real constant, t the time and ε (< < 1) is a constant quantity.

The radiative heat flux q_r is given by [4]:
$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I \tag{6}$$
where $I = \int_0^\infty k_{\lambda n}\frac{\partial e_{b\lambda}}{\partial T}d\lambda$, $k_{\lambda n}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Plank function.

The boundary conditions are:
$$\begin{aligned} &u = L_1\frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w \quad &\text{at } y = 0 \\ &u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad &\text{as } y \to \infty \end{aligned} \tag{7}$$
where $L_1 = \left(\frac{2 - m_1}{m_1}\right)L$, L being the mean free path and m_1 the Maxwell's reflection coefficient.

The continuity equation (1) gives
$$v = -v_0 \tag{8}$$
where $v_0 > 0$ is constant suction velocity at the plate.
We introduce the following dimensionless quantities:

$$y^* = \frac{y v_0}{\upsilon},\ t^* = \frac{v_0^2 t}{4\upsilon},\ n^* = \frac{4\upsilon n}{v_0^2},\ u^* = \frac{u}{v_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},$$
$$G_r = \frac{\upsilon g \beta (T_w - T_\infty)}{v_0^3},\quad G_m = \frac{\upsilon g \beta^* (C_w - C_\infty)}{v_0^3},\ K_0^* = \frac{K_0 v_0^2}{\upsilon^2},\ P_r = \frac{\mu C_p}{k},$$
$$R = \frac{4\upsilon I}{\rho C_p v_0^2},\quad S_c = \frac{\upsilon}{D},\ M^2 = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2},\ h_1 = \frac{L_1 v_0}{\upsilon},\ S = \frac{S^* \upsilon}{\rho C_p v_0^2},\ \lambda_1 = \frac{\lambda_0 v_0^2}{4\upsilon}$$

The equations (2) to (4) in view of (5) to (6) in non-dimensional form after dropping the asterisks (□) over them reduce to:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r\theta + G_m\phi + \left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \left[M^2 + \frac{1}{K_0(1 + \varepsilon e^{-nt})}\right]u \tag{9}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + (S - R)\theta \tag{10}$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} \tag{11}$$

with corresponding boundary conditions

$$\begin{aligned} &u = h_1 \frac{\partial u}{\partial y},\quad \theta = 1,\quad \phi = 1 \qquad \text{at } y = 0 \\ &u \to 0,\quad \theta \to 0,\quad \phi \to 0 \qquad \text{as } y \to \infty \end{aligned} \tag{12}$$

The partial differential equations (9) to (11) are reduced to ordinary one by assuming the following expressions for velocity, temperature and concentration

$$u(y,t) = u_0(y) + \varepsilon e^{-nt} u_1(y) \tag{13}$$
$$\theta(y,t) = \theta_0(y) + \varepsilon e^{-nt} \theta_1(y) \tag{14}$$
$$\phi(y,t) = \phi_0(y) + \varepsilon e^{-nt} \phi_1(y) \tag{15}$$

Substituting equations (13) to (15) in equations (9) to (11) and equating the coefficients of like powers of ε, the following equations are obtained:

$$u_0'' + u_0' - \left(M^2 + \frac{1}{K_0}\right)u_0 = -G_r\theta_0 - G_m\phi_0 \tag{16}$$

$$(1 - n\lambda_1)u_1'' + u_1' - \left(M^2 + \frac{1}{K_0} - \frac{n}{4}\right)u_1 = -G_r\theta_1 - G_m\phi_1 - \frac{u_0}{K_0} \tag{17}$$

$$\theta_0'' + P_r\theta_0' + (S - R)P_r\,\theta_0 = 0 \tag{18}$$

$$\theta_1'' + P_r\theta_1' + P_r\left(S - R + \frac{n}{4}\right)\theta_1 = 0 \tag{19}$$

$$\phi_0'' + S_c\phi_0' = 0 \tag{20}$$

$$\phi_1'' + S_c\phi_1' + \frac{n}{4}S_c\phi_1 = 0 \tag{21}$$

with corresponding boundary conditions

$$\left.\begin{array}{l} u_0 = h_1u_0',\ u_1 = h_1u_1',\ \theta_0 = 1,\ \theta_1 = 0,\ \phi_0 = 1,\ \phi_1 = 0 \quad \text{at } y = 0 \\ u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0,\ \phi_0 \to 0,\ \phi_1 \to 0 \quad \text{as } y \to \infty \end{array}\right\} \tag{22}$$

where primes denote differentiation with respect to *y*.
The solutions of (16)̃(21), satisfying the boundary conditions (22), are substituted in equations (13)̃(15), then

$$u(y,t) = A_1e^{-\alpha_1 y} + A_2e^{-\alpha_2 y} + A_3e^{-S_c y} + \varepsilon e^{-nt}\left[B_1e^{-\alpha_1 y} + B_2e^{-\alpha_2 y} + B_3e^{-\alpha_3 y} + B_4e^{-S_c y}\right] \tag{23}$$

$$\theta(y,t) = e^{-\alpha_2 y} \tag{24}$$

$$\phi(y,t) = e^{-S_c y} \tag{25}$$

where

$$\alpha_1 = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M^2 + \frac{1}{K_0}\right)}\right], \quad \alpha_2 = \frac{\left[P_r + \sqrt{P_r^2 - 4(S-R)P_r}\right]}{2},$$

$$\alpha_3 = \frac{1}{2(1-n\lambda_1)}\left[1 + \sqrt{1 + 4(1-n\lambda_1)\left(M^2 + \frac{1}{K_0} - \frac{n}{4}\right)}\right], \quad A_1 = \frac{-\left[(1+h_1\alpha_2)A_2 + (1+h_1S_c)A_3\right]}{(1+h_1\alpha_1)},$$

$$A_2 = \frac{-G_r}{\alpha_2^2 - \alpha_2 - \left(M^2 + \frac{1}{K_0}\right)}, \quad A_3 = \frac{-G_m}{S_c^2 - S_c - \left(M^2 + \frac{1}{K_0}\right)},$$

$$B_1 = \frac{-\left(A_1/K_0\right)}{(1-n\lambda_1)\alpha_1^2 - \alpha_1 - \left(M^2 + \frac{1}{K_0} - \frac{n}{4}\right)}, \quad B_2 = \frac{-\left(A_2/K_0\right)}{(1-n\lambda_1)\alpha_2^2 - \alpha_2 - \left(M^2 + \frac{1}{K_0} - \frac{n}{4}\right)},$$

$$B_3 = \frac{-\left[B_1(1+h_1\alpha_1) + B_2(1+h_1\alpha_2) + B_4(1+h_1S_c)\right]}{(1+h_1\alpha_3)},$$

$$B_4 = \frac{-\left(A_3/K_0\right)}{(1-n\lambda_1)S_c^2 - S_c - \left(M^2 + \frac{1}{K_0} - \frac{n}{4}\right)},$$

From equation (23), we calculate the skin friction

$$\tau = -A_1\alpha_1 - A_2\alpha_2 - A_3 S_c - \varepsilon e^{-nt}\left[B_1\alpha_1 + B_2\alpha_2 + B_3\alpha_3 + B_4 S_c\right] \tag{26}$$

From equation (24), we calculate the rate of heat transfer in terms of Nusselt number

$$Nu = \alpha_2 = \frac{\left[P_r + \sqrt{P_r^2 - 4(S-R)P_r}\right]}{2} \tag{27}$$

RESULTS AND DISCUSSION

In order to get a clear insight of the physical problem, the numerical computations have been carried out for various values of material parameters such as viscoelastic parameter $\square_1$, porosity parameter K_0, magnetic parameter M, Schmidt number S_c, radiation parameter R, heat source/sink parameter S, Grashof number G_r, modified Grashof number G_m, Prandtl number P_r and rarefaction parameter h_1 which are of physical and engineering interest. The numerical results are displayed with the help of graphical illustrations in figures 1–10. These figures depict the velocity profiles, skin friction profiles, temperature profiles and rate of heat transfer. In figures 1–4 and 9, the variations of velocity and temperature are compared in air (P_r =0.71) and water (P_r =7.0) while in figures 5–6, the results of velocity distribution of viscoelastic fluid ($\square_1$ =1) are compared with the corresponding flow problems for a viscous fluid ($\square_1$=0). From figures 1–4 and 9, it is clear that both velocity and temperature increase in air compared with water.

The velocity profiles for different values of magnetic field parameter M are plotted against y in Fig. 1. In air and water, it is observed that the velocity decreases with the increasing values of M which shows that the velocity decreases in the presence of magnetic field, as compared to its absence. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

For different values of porosity parameter K_0, the velocity profiles are demonstrated in Fig. 2. It is clear that the velocity increases due to increasing values of porosity parameter both in air and water. It is also observed that the magnitude of velocity distribution across the boundary layer increases and then approaching to zero as y increases.

Fig. 3 illustrates the effect of heat source ($S > 0$) or heat sink ($S < 0$) on the velocity. In air, it is obvious that an increase in S leads to a rise in the values of velocity. It is also observed that at some fixed value of S, the velocity graph in water is lower than the respective velocity graph in air.

Fig. 4 exhibits the velocity distribution for several values of radiation parameter R. Both in air and water, the velocity profiles show a decrease with the increase of radiation parameter.

The variations of velocity field for different values of Grashof number G_r and modified Grashof number G_m are plotted against y in Figs. 5 and 6. The results are displayed for viscous fluid ($\square_1$=0) as well as viscoelastic fluid ($\square_1$ =1). For both fluids, the velocity increases with the increasing values of G_r and G_m. It is also obvious that at some fixed value of G_r or G_m, the velocity graph for viscous fluid is lower than the respective velocity graph for viscoelastic fluid which indicates that the velocity increases for viscoelastic fluid compared with viscous fluid. Physically, $G_r > 0$ means heating of the fluid or cooling of the boundary surface, $G_r < 0$ means cooling of the fluid or heating of the boundary surface and $G_r = 0$ corresponds to the absence of free convection current.

The skin friction profiles are displayed against M in Figs. 7 and 8 for various values of material parameters which are listed in the figure captions. From Fig. 7, it is clear that the skin friction increases with the increase in K_0, G_r, G_m, S and decreases with the increase in h_1. Fig. 8 shows that the skin friction is more in viscoelastic fluid as compared to that in viscous fluid. It is also clear that the skin friction increases in air and decreases in water. Also, the effect of increasing values of S_c and R results in a decreasing skin friction. Furthermore, the skin friction increases in the absence of magnetic field, as compared to its presence.

Typical variations of dimensionless temperature profiles in air and water for different values of radiation parameter and heat source/sink parameter are plotted against y in Fig. 9. The effect of radiation parameter on the temperature is depicted in Fig. 9(a). It is noticed from the figure that the temperature decreases with the increasing values of radiation parameter R. The effect of radiation parameter is to reduce the temperature significantly in the flow region. The increase of radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases as the thermal boundary layer thickness becomes thinner. Fig. 9(b) displays the effect of S on the temperature. It is clear that the temperature increases with the increase in S. From Figs. 9(a) and 9(b), it has been observed that the temperature profiles are lower in water as compared to that in air. The reason is that smaller values of P_r are equivalent to increasing thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of P_r.

Variations of rate of heat transfer in terms of Nusselt number with R for different values of P_r are presented in Fig. 10. From the figure, it is noticed that the rate of heat transfer increases with the increasing values of Prandtl number. Furthermore, the effect of increasing values of R results in the increasing rate of heat transfer.

CONCLUSION

The main goal of this article is to study the unsteady MHD heat and mass transfer free convection flow of an incompressible electrically conducting viscoelastic fluid immersed in a porous medium of variable permeability bounded by an infinite porous vertical plate in slip flow regime with heat source and radiation effects. The permeability of the porous medium decreases exponentially with time about a constant mean. The fundamental parameters found to affect the problem under consideration are the viscoelastic parameter, magnetic parameter, permeability parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number, heat source parameter, radiation parameter and rarefaction parameter. It is found that the velocity and temperature decreases in water compared with air. It is also observed that the velocity as well as skin friction increases with increasing values of K_0, S, G_r, G_m whereas reverse trend is seen with R and M. Additionally, the rate of heat transfer is increased with increment in P_r.

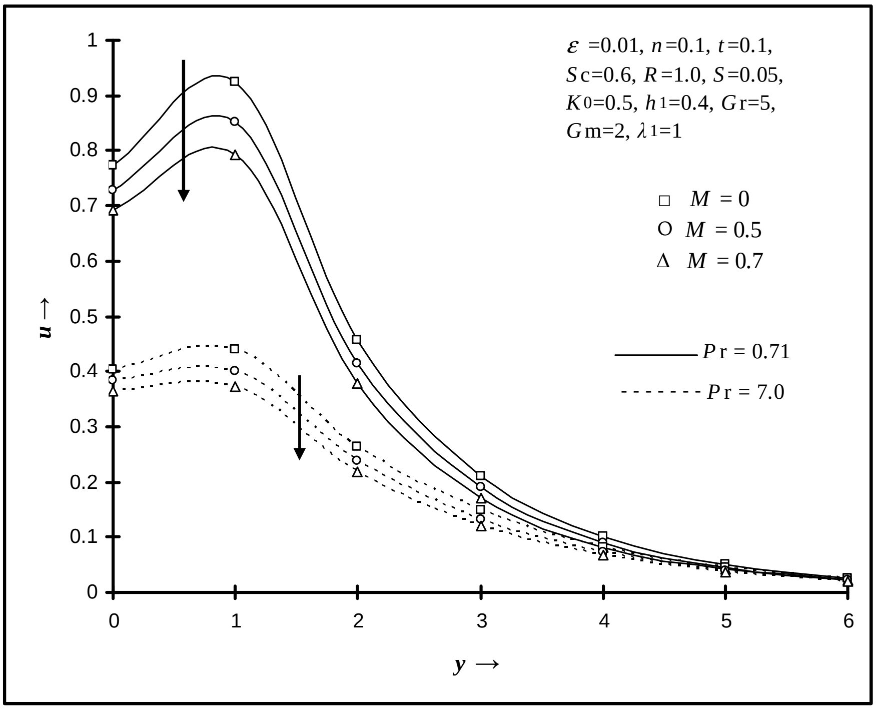

Fig.1. Variation of velocity *u* with *y* for several values of *M*.

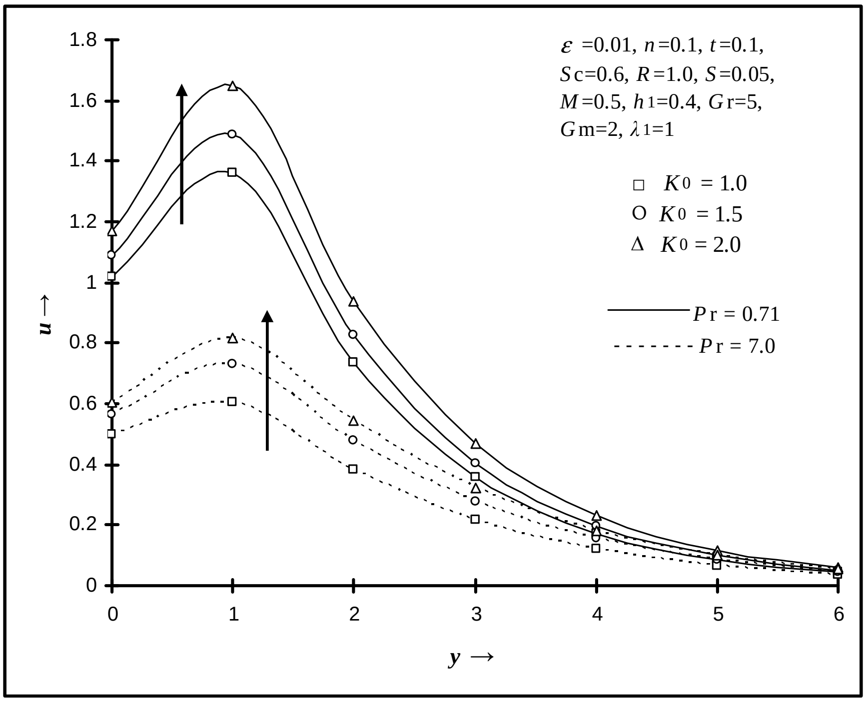

Fig.2. Variation of velocity *u* with *y* for several values of K_0.

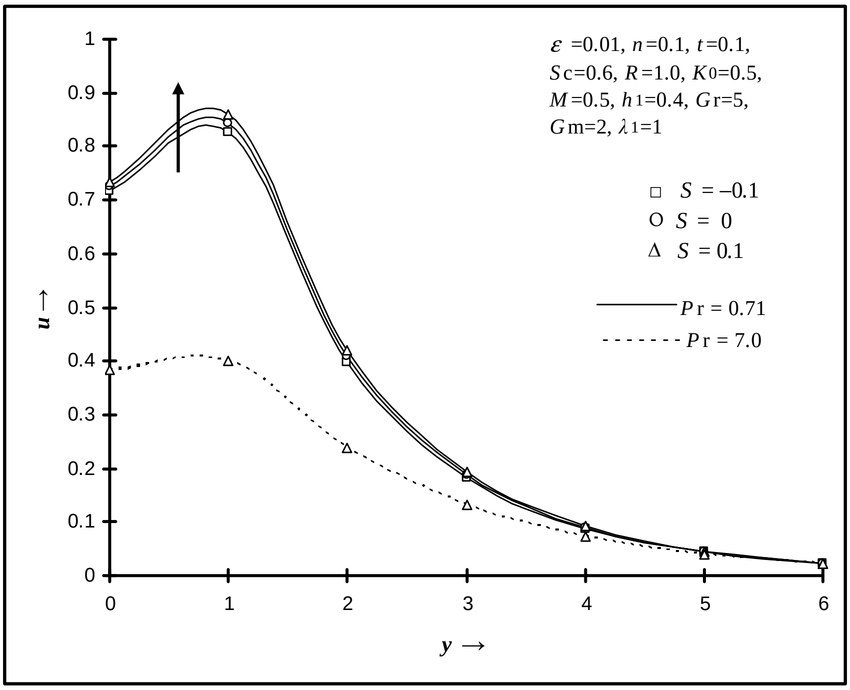

Fig.3. Variation of velocity *u* with *y* for several values of *S*.

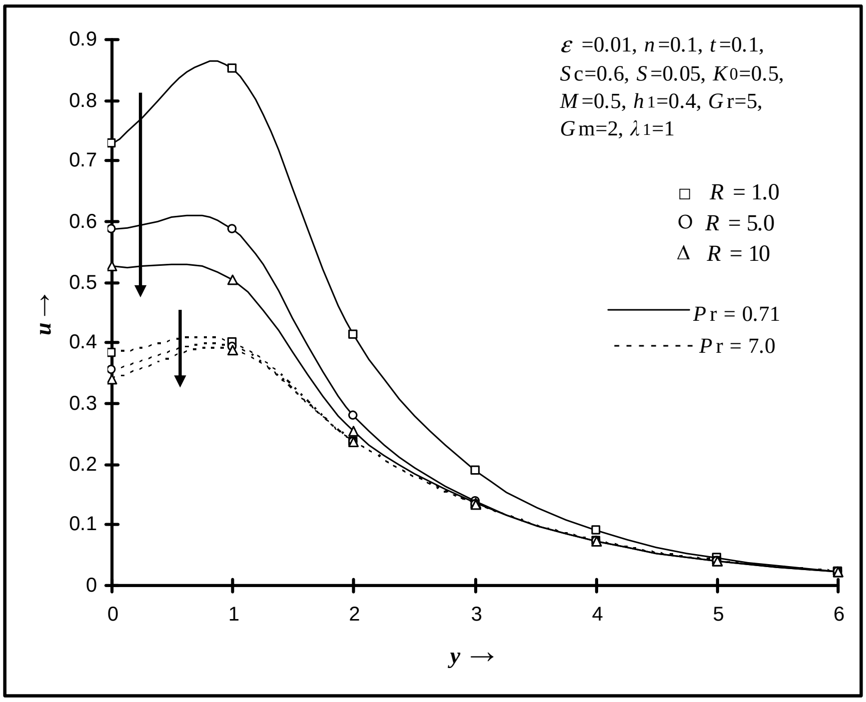

Fig.4. Variation of velocity *u* with *y* for several values of *R*.

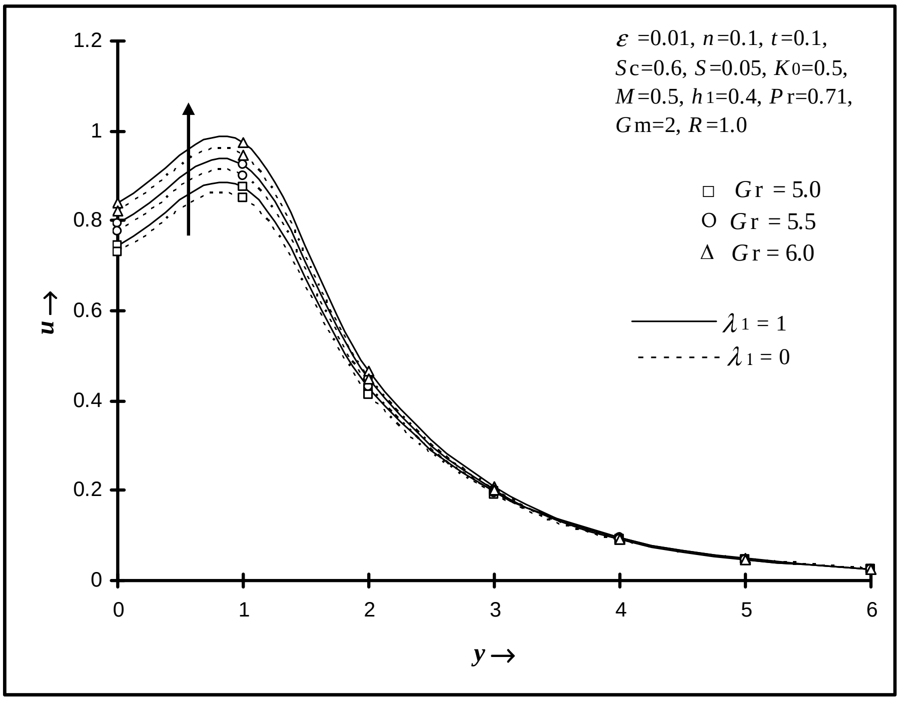

Fig.5. Variation of velocity *u* with *y* for several values of G_r.

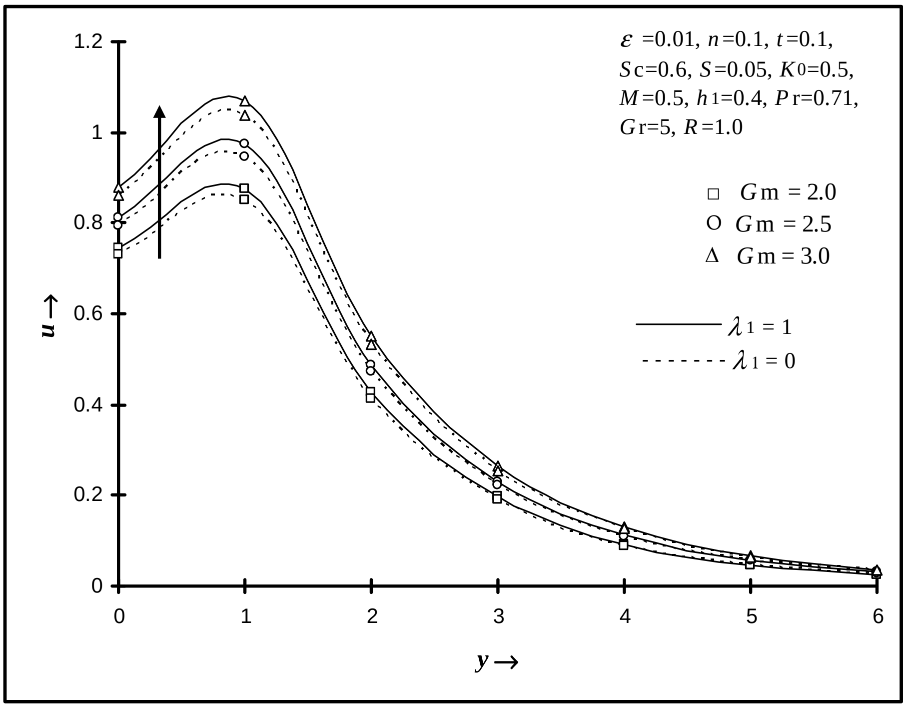

Fig.6. Variation of velocity *u* with *y* for several values of G_m.

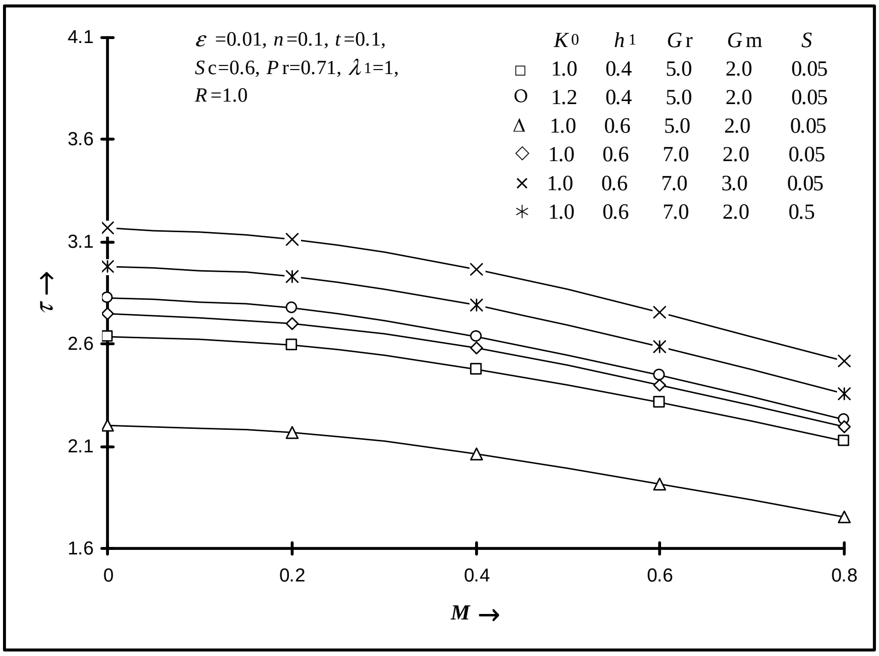

Fig.7. Skin friction τ plotted against M for different values of K_0, h_1, G_r , G_m and S.

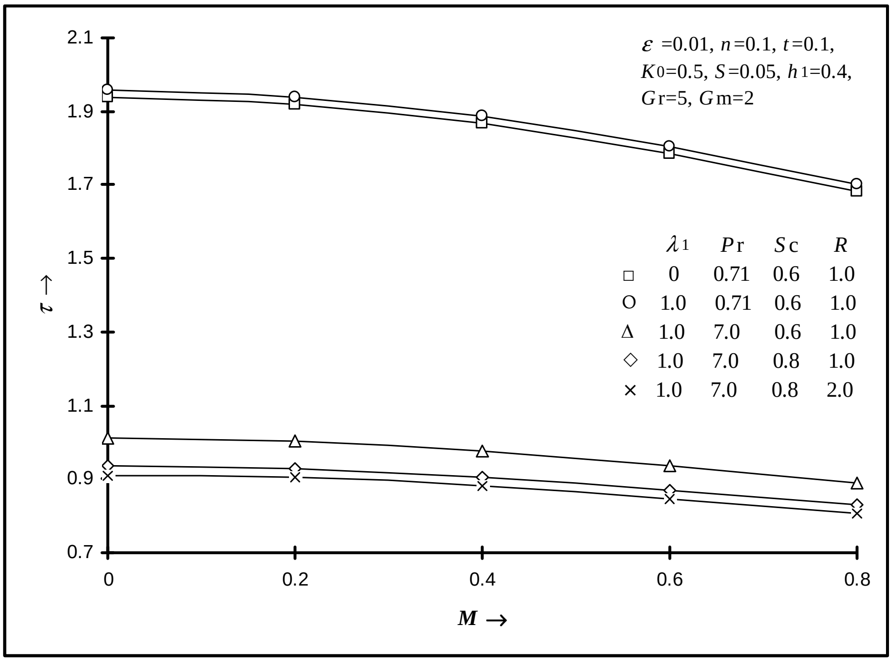

Fig.8. Skin friction τ plotted against M for different values of □$_1$, P_r , S_c and R.

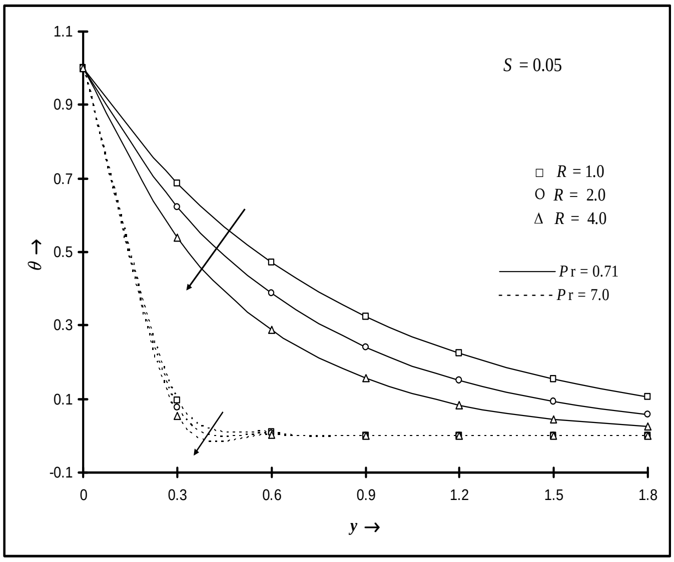

Fig.9 (a)

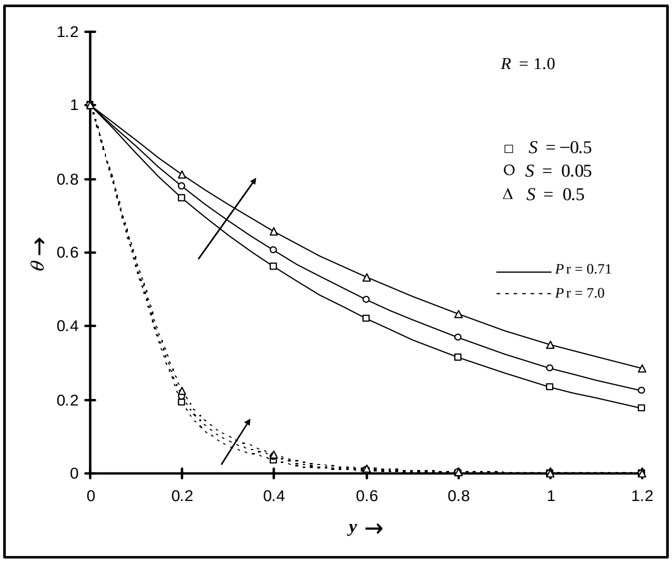

Fig.9 (b)

Fig.9. Effects of P_r , S and R on the temperature □.

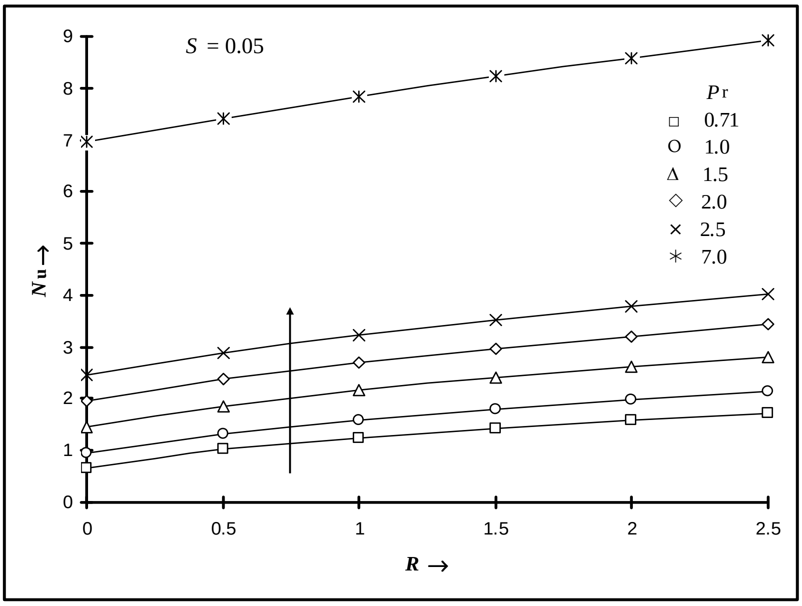

Fig.10. Variation of rate of heat transfer N_u with R for several values of P_r .

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