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MHD Free convection flow of couple stress fluid in a vertical porous layer

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ABSTRACT

The electrically conducting flow of couple stress fluid in a vertical porous layer is investigated. It is assumed that the fluid possesses constant properties except for density. We assume that the density variation due to temperature differences is used only to express the body force term as buoyancy term. The perturbation method of solution is obtained in terms of buoyancy parameter N valid for small values of N. The flow is analysed for three cases having the parameter l which gives the combined effect of magnetic field and permeability. The velocity and temperature fields are determined. The effects of couple stress parameter K and the parameter l of the velocity and temperature are discussed. It is observed that the velocity decreases with increasing Darcy Number Da and l.

Keywords: MHD free convection flow, Couple stress fluid, vertical channel, Porous layer.

INTRODUCTION

The notation of a polar fluid has been introduced first by Born [1] who associated the resistance to the relative rational motion with the skew symmetric parts of the stress tensor. After Three decades Grad [6], introduced the linear constitutive equations for polar fluids using the concept of statistical mechanics. The constitutive equations relate the skew symmetric part of the usual stress tensor to the relative angular velocity and the couple stress to the gradient of the total angular velocity. Some constitutive equations have been advanced from different view points by Cowin [3], Condiff and Dahler [4] and Eringen [5]. Eringen named a polar fluid as a micro polar fluid because he obtained it by specializing his theory of micro fluids. The theory of dipolar fluids presented by Blusein and Green [2], is a special case of the theory of fluids with deformable substructure obtained by constraining the motion of the director triad to coincide with the local motion of the region. Finally the theory of fluids with couple stresses is proposed if the polar fluid is constrained so that the cosserat triad rotates with the underlying medium but remains rigid. In this way both polar and dipolar fluids reduced to the theory of fluids with couple stresses introduced by Stokes [7].

Several studies of free convection flows in the presence of various geometries have been reported in literature Umavathi [8]. These studies were concerned with Newtonian fluid. But problems in petroleum and chemical industries, geohydrology, extraction of geothermal energy and medicine involve non Newtonian fluid, fluid flow through / past porous media. It will be interesting to study the problems concerning couple stress fluids having the technological importance. The flow in oil reservoirs in the earth can be discussed applying the principles of flow through porous media. Free convection problems arise in the fields of aeronautics, atomic power, chemical Engineering and space research.

Most of these studies deal with the Newtonian fluid flows. So it is considerable importance to make a study on free convective couple stress flows. In this chapter, conducting flow of a couple stress fluid in a vertical porous layer is investigated. Applying perturbation method, the velocity and temperature fields are determined. The effects various parameters on the velocity and temperature are discussed.

Nomenclature

G - *Gravitational acceleration*
K - *Couple stress parameter*
k_0 - *Permeability*
Kc - *Thermal conductivity*
(x,y) - *Space co-ordinates*
U - *Velocity of the fluid along x-direction*
T - *Temperature*
T_0 - *Temperature of the ambient fluid*
B_0 - *Magnetic field intensity*
$2b$ - *Distance between the plates*
α - *Electrical conductivity*
β - *Coefficient of thermal expansion*
η - *Material constant*
μ - *Viscosity*
ρ - *Density*
ρ_0 - *Reference density*

Mathematical formulation of the problem

Consider the flow of electrically conducting couple stress fluid in a vertical porous channel. It is assumed that the fluid possesses constant properties except for density. We assume that the density variation due to temperature differences is used only to express the body force term as buoyancy term (Boussineq approximation). A uniform transverse magnetic field of strength B_0 is applied perpendicular to the porous layer as shown in the figure 1.

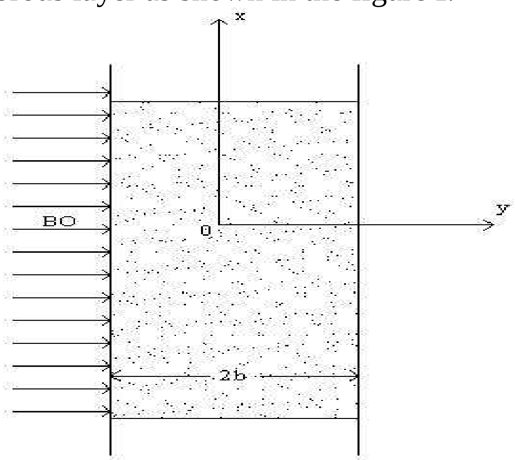

Fig. 1 Physical Model

The governing equations are

$$\frac{\eta}{\mu}\frac{d^4u}{dy^2}-\frac{d^2u}{dy^2}-\frac{g\beta(T-T_0)}{v}+\left(\frac{\sigma B_0^2}{\mu}+\frac{1}{K_0}\right)u=0 \quad (1)$$

$$\frac{d^2T}{dy^2}+\frac{\rho_0 v}{K_C}\left(\frac{du}{dy}\right)^2=0 \quad (2)$$

$$\rho=\rho_0\left[1-\beta(T-T_0)\right] \quad (3)$$

The quantities in the above equations are explained in the nomenclature. The boundary conditions are

$$u=\frac{d^2u}{dy^2}=0 \qquad at \qquad y=\pm b \quad (4)$$

$$T=T_1 \qquad at \qquad y=\pm b \quad (5)$$

Boundary conditions on velocity represent the no slip conditions, disappearance of the couple stress at the solid boundaries. The conditions on temperature states that the plates are isothermally maintained at the same temperature T_1.

Non-dimensionalization of the flow quantities

We introduce the following nondimensional quantities to make basic equations and boundary conditions dimensionless.

$$y=\frac{y^*}{b},\quad \theta=\frac{T-T_0}{T_1-T_0},\quad u^*=\frac{v}{g\beta b^2(T_1-T_0)}u \quad (6)$$

In the view of the above nondimensional quantities the basic equations (1) and (2) take the following form after dropping asterisks.

$$k\frac{d^4u}{dy^4}-\frac{d^2u}{dy^2}+l^2u=0 \quad (7)$$

$$\frac{d^2\theta}{dy^2}+N\left(\frac{du}{dy}\right)^2=0 \quad (8)$$

where

$K=\frac{\eta}{\mu b^2}$ is the couple stress parameter

$Da=\frac{k_0}{b^2},\quad M^2=\frac{\sigma B_0^2 b^2}{\rho\upsilon},\quad l^2=M^2+\frac{1}{Da}$ is the Hartmann number

$N=\frac{\rho_0 g^2\beta^2 b^4(T_1-T_0)}{vK_c}$ is the buoyancy parameter

The boundary conditions are

$$u=\frac{d^2u}{dy^2}=0 \qquad at \qquad y=\pm 1 \quad (9)$$

$$\theta=1; \qquad at \qquad y=\pm 1 \quad (10)$$

To find the closed form solutions of equations (7) and (8) is a difficult task because of coupled nonlinear nature of differential equations. However for vanishing N, equations become linear and can be solved exactly. But vanishing N will lead to neglecting viscous dissipation and N also indicates the state of the plates. Hence neglecting N is not feasible. We apply perturbation technique for solving the problem. Using perturbation technique, solutions of (1) and (2) is written in the form

$$(u,\theta)=(u_0,\theta_0)+N(u_1,\theta_1)+N^2(u_2,\theta_2)+..... \quad (11)$$

where u_0 and θ_0 as the solutions for vanishing N and first and higher order terms gives a correction to u_0 and θ_0, which accounts for the dissipative effects. Using (11) in (7) and (8) and equating the like power of N we get

Zeroth – order equation

$$\frac{d^2\theta_0}{dy^2}=0 \quad (12)$$

$$K\frac{d^4u_0}{dy^4}-\frac{d^2u_0}{dy^2}+l^2u_0=\theta_0 \quad (13)$$

First order equations

$$\frac{d^2\theta_1}{dy^2}=-\left[\frac{du_0}{dy}\right]^2 \quad (14)$$

$$K\frac{d^4u_1}{dy^4}-\frac{d^2u_1}{dy^2}+l^4u_1=\theta_1 \quad (15)$$

and so on

The corresponding boundary conditions are

$$u_0=\frac{d^2u_0}{dy^2}=0 \qquad at \qquad y=\pm1 \quad (16)$$

$$\theta_0=1 \qquad at \qquad y=\pm1 \quad (17)$$

$$u=\frac{d^2u_1}{dy^2}=\theta_1=0 \qquad at \qquad y=\pm1 \quad (18)$$

Solutions of equations (12) to (14) subject to boundary conditions (16) to (17) are derived and are shown below for different choices of l. Assuming the effect of dissipation on velocity is negligible; we find its effect only on temperature filed.

Solution of the problem

Case I: Suppose that $4l^2k<1$

$$\theta_0=1$$

$$u_0=A_1\cosh ay+A_3\cosh by+\frac{1}{l^2}$$

$$\theta_1=\frac{r_1}{2}(1-y^2)+\frac{r_1}{4\alpha^2}(\cosh 2\alpha y-\cosh 2\alpha)+\frac{r_2}{2}(1-y^2)$$

$$+\frac{r_2}{4\beta^2}(\cosh 2\beta y-\cosh 2\beta)+\frac{r_3}{4\beta^2}(\cosh 2\beta y-\cosh 2\beta)$$

$$+\frac{r_3}{(\alpha+\beta)^2}(\cosh(\alpha+\beta)y-\cosh(\alpha+\beta))-\frac{r_3}{(\alpha-\beta)^2}\left[\cosh(\alpha-\beta)y-\cosh(\alpha-\beta)\right]$$

Case II: Suppose that $4l^2k = 1$

$$u_0 = \frac{1}{l^2}\left[1 - \frac{G\cosh by}{2\cosh b} + \frac{by\sinh by}{2\cosh b}\right]$$

$$\theta_1 = r_1(\cosh 2by - \cosh 2b) + r_2(y\sinh 2by - \sinh 2b) + r_3(y^2\cosh 2by - \cosh 2b) + r_4(y^4 - 1) + r_5(y^2 - 1)$$

Case III: Suppose that $4l^2k > 1$

$$u_0 = \frac{1}{l^2}\left[1 - \frac{c}{F}\cosh\gamma y\cos\delta y + \frac{D}{F}\sinh\gamma y\sinh\delta y\right] \theta_1 = I_1[\cosh 2\gamma y - \cosh 2\gamma] + I_2(\cosh 2\delta y - \cos 2\delta)$$

$$+I_3(\sinh 2\gamma y\sin 2\delta y - \sinh 2\gamma\sin 2\delta)$$

$$+I_4(\cosh 2\gamma y\cos 2\delta y - \cosh 2\gamma\cos 2\delta) + I_5(y^2 - 1)$$ where

$$\alpha = \frac{1}{2k}\left(1 + \sqrt{1 - 4kl^2}\right)^{1/2}, \beta = \frac{1}{2k}\left(1 - \sqrt{1 - 4kl^2}\right)^{1/2}, A_1 = \frac{\beta^2}{2l^2(\alpha^2 - \beta^2)\cosh\alpha}$$

$$A_3 = \frac{-\alpha^2}{2l^2(\alpha^2 - \beta^2)\cosh\beta}, A_1 = A_2, \quad A_2 = A_3, b = \frac{1}{\sqrt{2k}}, G = 2 + b\tanh b, \ g_1 = \frac{-b^2(1 - G)}{4l^2\cosh^2 b}$$

$$g_2 = \frac{-b^4}{4l^2\cosh^2 b}, g_3 = g_1 b, r_1 = \frac{g_1}{8b^2} + \frac{3g^2}{32b^4} - \frac{g_3}{4b^3}, r_2 = \frac{g_3}{4b^2} - \frac{g_2}{16b^3}$$

$$r_3 = \frac{g_2}{4b^2}, r_4 = \frac{-g_2}{2b^3}, r_5 = \frac{g_2}{2b^4}, r_6 = \frac{g_2}{24}, r_7 = \frac{-g_1}{4}, I_1 = -\frac{(k_1^2 + k_2^2)}{16\gamma^2}, I_2 = -\frac{(k_1^2 + k_2^2)}{16\delta^2},$$

$$I_3 = -\frac{(k_1^2 - k_2^2)}{\delta(\gamma^2 + \delta^2)}\gamma\delta + \frac{k_1k_2(\delta^2 - \gamma^2)}{\delta(\gamma^2 + \delta^2)}, I_4 = \frac{k_1k_2\gamma\delta}{4(\gamma^2 + \delta^2)} + \frac{(k_1^2 - k_2^2)(\delta^2 - \gamma^2)}{16(\gamma^2 + \delta^2)},$$

$$I_5 = \frac{k_1^2 - k_2^2}{8}, \gamma = \frac{1}{2\sqrt{k}}\left(\frac{2}{l}\sqrt{k} + 1\right)^{1/2}, \delta = \frac{1}{2\sqrt{k}}\left(\frac{2}{l}\sqrt{k} - 1\right)^{1/2},$$

$$F = \cos^2\delta + \cosh^2\gamma - 1, C = R\sinh\gamma\sin\delta + \cosh\gamma\cos\delta, D = \sinh\gamma\sin\delta - R\cosh\gamma\cos\delta$$

$$R = \frac{\gamma^2 - \delta^2}{2\gamma\delta}, \quad k_1 = \frac{-1}{lF}(C\gamma + D\delta), \ k_2 = \frac{-1}{lF}(D\gamma - C\delta)$$

RESULTS AND DISCUSSION

The physical quantities of interest include rate of heat transfer q wall shear stress τ and mass flow rate Q. Heat flux entering at $y = -1$ and leaving at $y = +1$ can be calculated from the temperature profile. Knowing the velocity filed, one can easily obtain the expressions for shear stress and flow rate. These are given by

$$q = \left(\frac{d\theta}{dy}\right)_{y=\pm 1} \quad (19)$$

$$\tau = \left(\frac{du_0}{dy} - k\frac{d^2u_0}{dy^2}\right)_{y=\pm 1} \quad (20)$$

and

$$Q = \int_{-1}^{1} u_0 \, dy \quad (21)$$

From the present formulation of the problem of free convection flow of couple stress fluid in a vertical porous layer, the nature of flow and heat transfer are discussed. Perturbation technique valid for small values of N is used for analytic solutions and combined effect of magnetic field and porous medium are considered (weak, comparable and strong in comparison with couple stress parameter K) and the following conclusions are drawn. We observed that the results obtained for this problem reduced to the corresponding areas of Umavathi [8]. We note that results are shown graphically for only one half of the channel using to the symmetry condition.

Three cases are analysed (i) $4l^2k < 1$ (ii) $4l^2k = 1$ and (iii) $4l^2k > 1$.

The results for first case are shown in figures (2) – (3). The velocity profiles are drawn in fig (2) and (3) for different values of l, and Da, with fixed N=0.01, It is observed that the velocity decreases with the increment in y for a fixed y. The velocity decreases with the increasing l. The velocity decreases with the increasing Darcy Number Da.

The results for second case are shown in figures (4) – (7). The velocity profiles are drawn in fig (4) – (5) with fixed N = 0.01, K=2, and Da = 2. The velocity profiles are flat near, the centre line of the porous channel. The velocity decreases with the increasing y for a given value N. The velocity is observed to be decreasing trend owning to an increase in the buoyancy parameter N.

The results for third case are shown in figures (8) - (11). The velocity profiles are drawn in fig (8) – (9) for different values of l, Da, K with fixed N = 0.01. It is observed that the velocity decreases with the increment in y. For a fixed y, the velocity decrees with the increasing l.

The variation of temperature with y is calculated for different values of l with fixed N = 0.01, m = 0.5, as shown in figure (6) - (7). It is observed that the temperature decreases with an increase in the parameter l. Further for a given y, the increase in the Darcy number leads to decrease in the temperature of the fluid in the velocity porous channel.

The variation of temperature with y is calculated for different values of 'l' with fixed N = 0.01, K = 2, Da = 10 and is shown in figure (10) and (11). It is observed that the temperature decreases with an increase in the parameter 'l'. Further for a given y, the increase in the Darcy number leads to decrease in the temperature of the fluid in the vertical porous channel.

In the above results, the absence of buoyancy i.e., for N = 0, temperature profiles are linear which can be justified through basic differential equations. As N increases both velocity and temperature increases. This is due to the fact that as N increases there will be onset of convection. As K increases mass flow rate decreases.

From the observations and investigations done one may conclude that applied magnetic field, porous medium couple stress and buoyancy force play important role to control the flow nature. Since the applied magnetic field tries to maintain the asymmetry of the stress tensor, this effect of magnetic field can be exploited in deciding to have either Newtonian or couple stress fluid for investigation. Further the increase in Da give rise to decrease the temperature and these effects can be exploited in refrigeration systems. If the magnetic field reduces the velocity gradients if may also be used effectively in shielding the shear fragile red cells from "Hemolysis" (Umavathi, 1999).

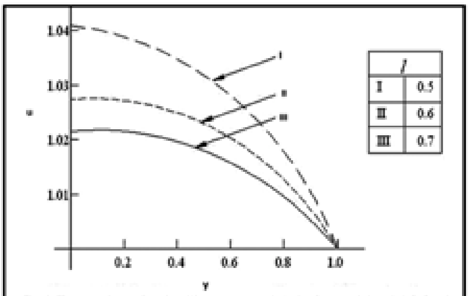

Fig. 2. The velocity profiles for different values of 'l' with fixed N=0.01, K=0.5, Da=2

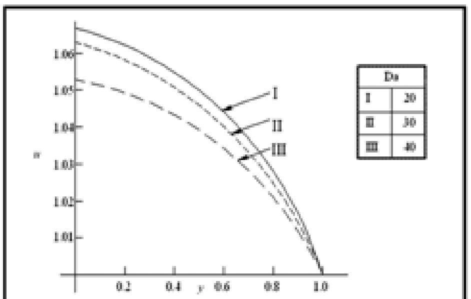

Fig.3 The velocity profiles for different values of Da with fixed N=0.01, K=0.5,l=0.5

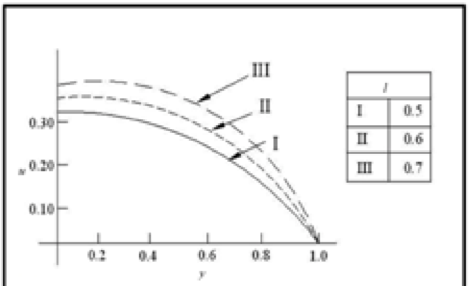

Fig. 4 The Velocity profiles for different values of with fixed N=0.01, Da=0.0001, K=0.2

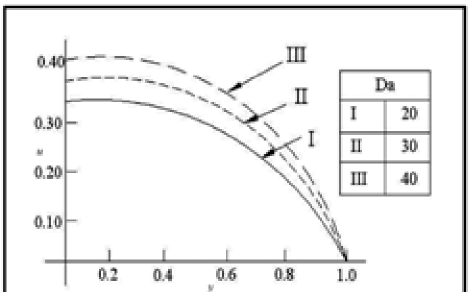

Fig.5 The Velocity profiles for different values of Da with fixed N=0.01, l=0.5, K=0.2

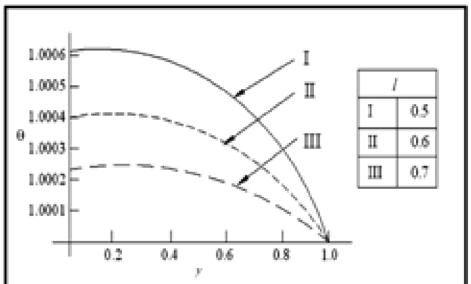

Fig.6 The Temperature profiles for different values of with fixed N=0.01, Da=10, K=0.1

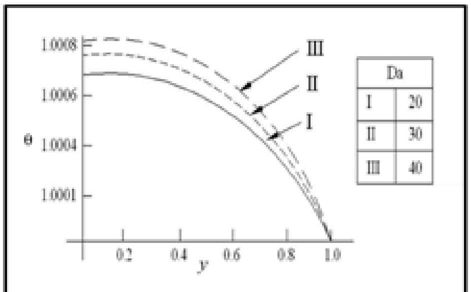

Fig.7 The Temperature profiles for different values of Da with fixed l=0.5, N=0.01, K=0.1

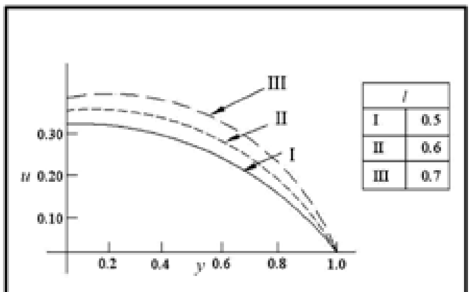

Fig. 8 The Velocity profiles for different values of l with fixed N=0.01, Da=0.0001, K=0.2

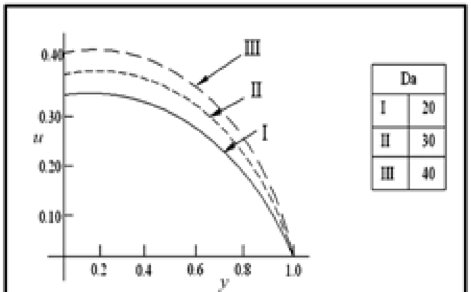

Fig.9 The Velocity profiles for different values of Da with fixed N=0.01, l=0.5, K=0.2

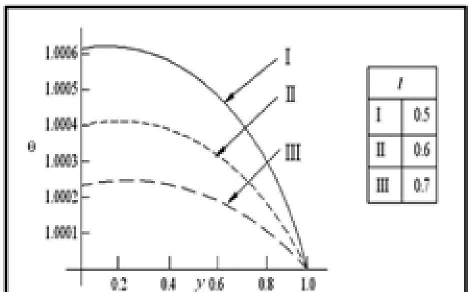

Fig. 10 The Temperature profiles for different values of 'l' with fixed N=0.01, Da=10, K=0.1

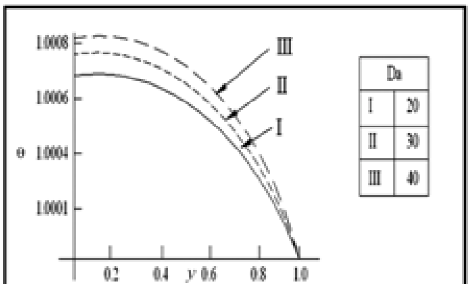

Fig.11 The Temperature profiles for different values of Da with fixed l =0.5, N=0.01, K=0.1

REFERENCES

[1]Born. M, *Z.Physics*, (**1920**), 1,221.
[2]Bleustein. J.L, AND Green. A.E, *Int. J. Eng. Sci*., (**1967**), 5,323.
[3]Cowin. S.C, Ph.D, thesis. The Pennsylvania state University, University park, Pennsylvania, (**1962**).
[4]Condiff. D.W, and Dahler. J.S, *Phys. of Fluids*., (**1964**), 7, 842
[5]Eringen, A.C. *J. Math. Mech*. 16, 1 (**1966**).
[6]Grad. M, *Commn. Pure. Appl. Math*., (**1952**). 5, 455.
[7]Stokes. V.K, *Phys. Fluids*., (**1966**), 9, 1709.
[8]J.C Umavathi, *A.M.S.E, (***1999**). vol.68. No 1. pp.1-24.