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MHD free convection flow of a dissipative fluid over a vertical porous plate in porous media

R. Alizadeh[1] and K. Rahmdel[2]

[1]Department of Mechanical Engineering, Quchan Branch Islamic Azad University, Iran, Quchan
[2]Department of Mechanical Engineering, Engineering Faculty, The University of Guilan, Iran, Rasht

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ABSTRACT

Numerical solutions are obtained for the MHD free convection flow of a dissipative fluid along a vertical porous plate in porous media with mass transfer, the surface of which is exposed to a constant heat flux. The non-linear system of partial differential equations is numerically solved by the implicit finite-difference method. Velocity, temperature and concentration profiles, local skin-friction, local Nusselt and local Sherwood numbers are plotted for air. The influence of the Suction rate parameter, buoyancy ratio parameter, porous media parameter , dissipation number, Schmidt number and magnetic parameter on heat and mass transfer are discussed.

Keywords: MHD, free convection, dissipative fluid, porous media, porous plate

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INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its application in MHD pumps, MHD bearings etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomena of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy.

Numerous works have studied this problem, the first of which, Pohlhausen [1], did not consider viscous dissipation but obtained a solution employing the integral method. Harris et al. [2] investigated the transient free convection from a vertical plate when the plate temperature is suddenly changed, obtaining an analytical solution (for small time values) and a numerical solution until the steady-state is reached. Polidori et al. [3] proposed a theoretical approach to the transient dynamic behaviour of a natural convection boundary-layer flow when a step variation of the uniform heat flux is applied, using the Karman–Pohlhausen integral method. Other authors studied the effect of the surface temperature oscillation [4,5]. Kassem [6] solved the problem for unsteady free-convection flow from a vertical moving plate subjected to constant heat flux. Gebhart [7] was the first to study who studied the problem taking viscous dissipation into account and this author defined the non-dimensional dissipation parameter. Takhar and Soundalgekar [8] studied the effect of a harmonic oscillation in the plate temperature in the form of a travelling wave convected in the direction of the free-stream of viscous incompressible fluids. Pantokratoras [9] solved the problem in a stationary situation using the finite-difference method, with isothermal and uniform flux boundary conditions in the wall, taking into account viscous dissipation. Soundalgekar et al. [10] solved the transient problem with an isothermal vertical wall. When heat and mass transfer occurs simultaneously, it leads to a complex fluid motion (the combination of temperature and concentration gradients in the fluid will lead to buoyancy-driven flows). This problem arises in numerous engineering processes, for example, biology and chemical processes, nuclear waste repositories and the extraction of geothermal energy. Soundalgekar and Ganesan [11] solved the problem of transient free convection with mass transfer on an isothermal vertical flat plate. Gokhale and Samman [12] studied the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. They obtained many conclusions concerning the effect of the variations of the different non-dimensional parameters that defined the problem on the time required to reach the steady-state. When the presence of a uniform magnetic field is considered, a new problem can be studied, "Unsteady free convection MHD with coupled heat and mass transfer". This problem has attracted the interest of many researchers in view of its application in astrophysics, geophysics fluid dynamics and engineering. Shanker and Kishan [13] studied the effects of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. Ganesan and Rani [14] solved the unsteady free convection flow over a vertical cylinder under the influence of a magnetic field problem, without taking into account the viscous dissipation. Hossain et al. [15] considered surface temperature oscillations, using three different methods, including perturbation and asymptotic methods, the local non-similarity method and an implicit finite-difference method. Aboeldahad and Elbarbary [16] employed a numerical solution using a fourth-order Runge–Kutta scheme to study the Hall effects on the heat and mass transfer. the natural convection flow of a conducting visco-elastic liquid between two heated vertical plates under the influence of transverse magnetic field has been studied by Sreehari Reddy et al [17]. Suneetha et al.[18] have analyzed the thermal radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration is analyzed by taking into account of the heat due to viscous dissipation. Recently Suneetha et al. [19] studied the effects of thermal radiation on the natural conductive heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hiteesh [20] studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

The object of the present paper is to study the transient free convection flow of an incompressible dissipative viscous fluid past a vertical porous plate in porous media, under the influence of a uniform transverse magnetic field in the presence of variable constant heat flux. The dimensionless governing equations are solved by using an implicit finite difference method .

Mathematical model

Consider the free convection flow of a dissipative fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux (Fig. 1) under the action of a transverse magnetic field. Under these assumptions and Boussinesq's approximation, the flow is governed by the following system of equations:

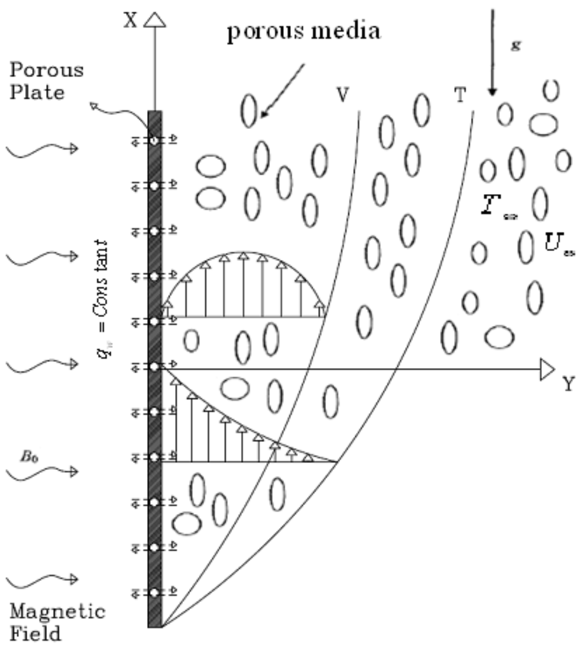

Figure. 1. Sketch of the physical model

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\beta g\left(T-T_\infty\right)+\beta^* g\left(c-c_\infty\right)-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{K}u \quad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

Mass equation:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \quad (4)$$

where u and v are components of the velocity in x and y directions, respectively, υ is the kinematic viscosity , β is the volumetric coefficient of thermal expansion , β^* is the volumetric expansion

coefficient for mass transfer , g is the acceleration due to gravity, ρ is the density , K is the permeability coefficient , σ fluid electrical conductivity , B_0 is magnetic induction , α is fluid thermal diffusivity , c_p is specific heat at constant pressure , T is the temperature, T_∞ is the temperature of the fluid far away from the plate , C is the concentration, C_∞ is the concentration far away from the plate and D is the molecular diffusivity.

The necessary boundary conditions are:

$$\begin{aligned}
&u=0 \quad ,v=0 \quad ,T=T_\infty \quad ,c=c_\infty \quad \text{at} \quad x=0\\
&u=0 \quad ,v=-V_0 \quad ,-k\frac{\partial T}{\partial y}=q \quad ,c=c_\infty \quad \text{at} \quad y=0\\
&u=0 \quad ,T\to T_\infty \quad ,c\to c_\infty \quad ,c=c_\infty \quad \text{at} \quad y\to\infty
\end{aligned} \quad (5)$$

Now introduce the following non dimensional quantities:

$$\begin{aligned}
&U=\frac{uL}{\upsilon Gr^{\frac{1}{2}}} \quad ,V=\frac{vL}{\upsilon Gr^{\frac{1}{4}}} \quad ,X=\frac{x}{L} \quad ,Y=\frac{y}{LGr^{-\frac{1}{4}}} \quad ,\theta=\frac{\left(T-T_\infty\right)kGr^{\frac{1}{4}}}{qL}\\
&C=\frac{c-c_\infty}{c_w-c_\infty} \;,Gr=\frac{g\beta L^4 q}{k\upsilon^2} \;,Sc=\frac{\upsilon}{D} \;,N=\frac{\beta^*\left(c_w-c_\infty\right)k}{\beta qL} \;,M=\frac{\sigma B_0^2 L}{\mu Gr^{\frac{1}{2}}}\\
&K_p=\frac{L}{KGr^{\frac{1}{2}}} \quad ,S=-\frac{\upsilon Gr^{\frac{1}{4}}}{L}V_0 \quad ,\varepsilon=\frac{g\beta L}{c_p} \quad ,\Pr=\frac{\upsilon}{\alpha}
\end{aligned} \quad (6)$$

where L is the wall height , X is the dimensionless axial coordinate , Y is the dimensionless axial coordinate perpendicular to X , U,V is the dimensionless velocities , θ is the dimensionless temperature , C is the non-dimensional species concentration , q is the heat flux at the plate, Sc is the Schmidt number, N is the buoyancy ratio parameter, Gr is the Grashof number , K_p is porous media parameter , S is the Suction rate parameter , M is the magnetic parameter , $\Pr$ is the prandtl number and ε is the dissipation number .

Then the governing equations reduce to the following non-dimensional boundary-layer equations:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+\theta Gr^{-\frac{1}{4}}+NC-Mu-K_p u \tag{8}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2}+\varepsilon Gr^{\frac{1}{4}}\left(\frac{\partial U}{\partial Y}\right)^2 \tag{9}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{10}$$

The dimensionless boundary conditions become:

$$\begin{aligned}
&U(0,Y)=0\ ,V(0,Y)=0\ ,\ \theta(0,Y)=0\ ,\ C(0,Y)=0 \ \text{ at }\ X=0\\
&U(X,0)=0\ ,V(X,0)=S\ ,\ \frac{\partial\theta(X,0)}{\partial Y}=-1\ ,\ C(X,0)=1\ \text{ at }\ Y=0\\
&U(X,\infty)=0\ ,\ \theta(X,\infty)=0\ ,\ C(X,\infty)=0\ \text{ at }\ Y\to\infty
\end{aligned} \tag{11}$$

For practical applications, the major physical quantities of interest in heat transfer include the local skinfriction coefficient C_{fX} , the local Nusselt number Nu_X and the local Sherwood number Sh_X . They can been expressed as follows:

$$C_{f_X} Gr^{\frac{1}{4}}=\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{12}$$

$$\frac{Nu_X}{Gr^{\frac{1}{4}}}=\frac{X}{\theta_W} \tag{13}$$

$$\frac{Sh_X}{Gr^{\frac{1}{4}}}=-X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{14}$$

3- Numerical Solution of the problem

The governing equations (7-10) are steady, coupled and non-linear with boundary conditions. An implicit finite - difference method has been employed to solve the nonlinear coupled equations, as described (Thomas algorithm) in Carnahan et al [21]. The finite difference equations corresponding to equations (7 – 10) are as follows :

$$\frac{U_{i,j}^{n+1}-U_{i-1,j}^{n+1}}{\Delta X}+\frac{U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}}{2\Delta Y}=0 \tag{15}$$

$$U_{i,j}^{n}\frac{U_{i,j}^{n+1}-U_{i-1,j}^{n+1}}{\Delta X}+v_{i,j}^{n}\frac{U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}}{2\Delta Y} \tag{16}$$

$$=\frac{U_{i,j+1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j-1}^{n+1}}{\Delta Y^2}+Gr^{-\frac{1}{4}}\theta_{i,j}^{n}+NC_{i,j}^{n}-MU_{i,j}^{n}-K_p U_{i,j}^{n}$$

$$U_{i,j}^{n}\frac{\theta_{i,j}^{n+1}-\theta_{i-1,j}^{n+1}}{\Delta X}+v_{i,j}^{n}\frac{\theta_{i,j+1}^{n+1}-\theta_{i,j-1}^{n+1}}{2\Delta Y} \tag{17}$$

$$=\frac{1}{\Pr}\frac{\theta_{i,j+1}^{n+1}-2\theta_{i,j}^{n+1}+\theta_{i,j-1}^{n+1}}{\Delta Y^{2}}+\varepsilon Gr^{\frac{1}{4}}\left(\frac{U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}}{2\Delta Y}\right)^{2}$$

$$U_{i,j}^{n}\frac{C_{i,j}^{n+1}-C_{i-1,j}^{n+1}}{\Delta X}+v_{i,j}^{n}\frac{C_{i,j+1}^{n+1}-C_{i,j-1}^{n+1}}{2\Delta Y}=\frac{1}{Sc}\frac{C_{i,j+1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j-1}^{n+1}}{\Delta Y^{2}} \tag{18}$$

The region of integration is considered as a rectangle with sides $X_{\max}(=1)$ and $Y_{\max}(=10)$, where corresponding to $Y\rightarrow\infty$ which lies far from the momentum , energy and concentration boundary layers. An appropriate mesh sizes considered for the calculation are $\Delta X=0.01$, $\Delta Y=0.05$. The local truncation error is $o(\Delta Y^{2},\Delta X)$ and it tends to zero as ΔY and ΔX tend to zero. Hence the scheme is compatible. Stability and compatibility ensures convergence.

RESULTS AND DISCUSSION

The velocity, temperature and concentration profiles have been computed by using the an implicit finite - difference method. The numerical calculations are carried out for the effect of the flow parameters such as Suction rate parameter (S), Prandtl number (Pr), Schmidth number (Sc), Grashof number (Gr), magnetic parameter (M), dissipation number (ε), buoyancy ratio parameter (N), porous media parameter (K_p) on the velocity, temperature and concentration distribution of the flow fields are presented graphically in figure 2-18.

The effects of magnetic parameter on the velocity, temperature, and concentration profiles are shown in Figs. 2-4. These presented profiles are those at X = 0.5. It is seen that, velocity decreases with increase in magnetic parameter. temperature and concentration increases with increase in the magnetic parameter.

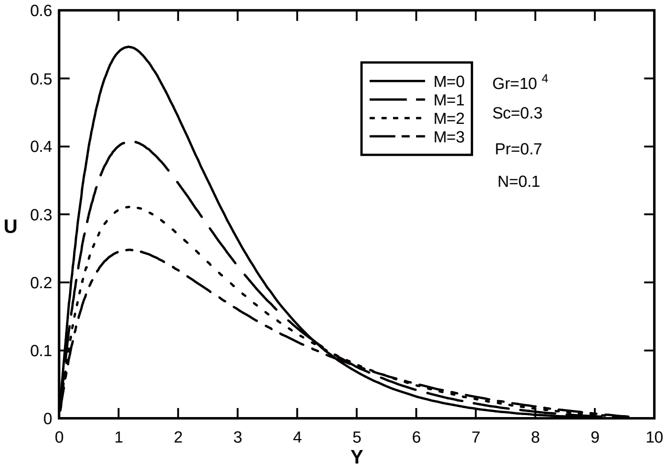

Figure 2. Effect of magnetic parameter on dimensionless velocity Profiles at dissipation number ($\varepsilon=1.0$)

The effects of buoyancy ratio parameter (N) on the velocity, temperature, and concentration profiles are shown in Figs. 5-7. It is observed that the velocity increases with increase in buoyancy ratio parameter. temperature and concentration decreases with increase in the buoyancy ratio parameter.

The effects of Suction rate parameter (S) on the velocity, temperature, and concentration profiles are shown in Figs. 8-10. It is observed that the velocity, concentration and temperature increases with increase in the Suction rate parameter .

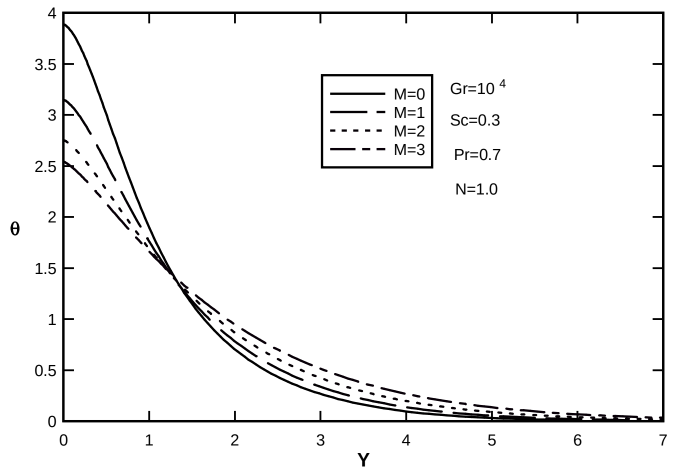

Figure 3. Effect of magnetic parameter on dimensionless temperature distributions at dissipation number ($\varepsilon = 1.0$)

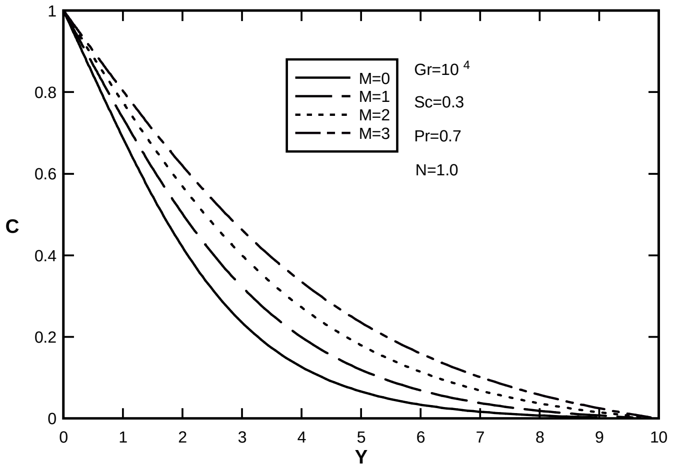

Figure 4. Effect of magnetic parameter on dimensionless concentration distributions dissipation number ($\varepsilon = 1.0$)

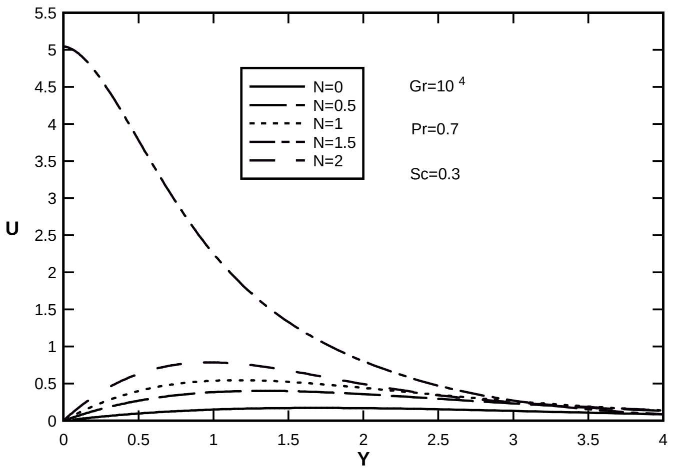

Figure 5. Effect of buoyancy ratio parameter (N) on dimensionless velocity Profiles at dissipation number ($\varepsilon = 1.0$)

The effects of porous media parameter (K_p) on the velocity, temperature, and concentration profiles are shown in Figs. 11-13. It is observed that the velocity decreases with increase in porous media parameter . temperature and concentration increases with increase in the porous media parameter .

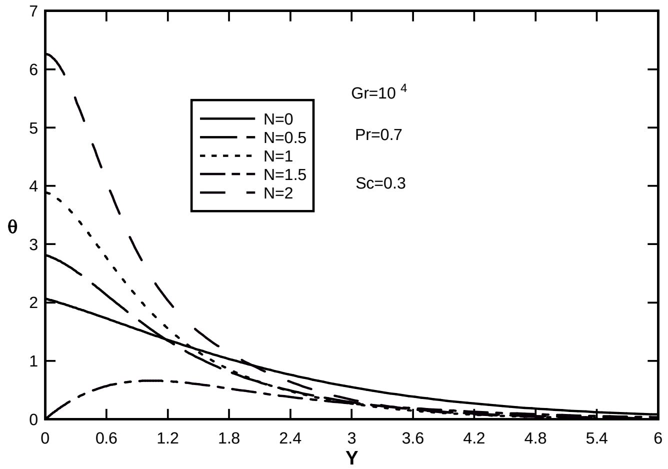

Figure 6. Effect of buoyancy ratio parameter (N) on dimensionless temperature distributions at dissipation number ($\varepsilon = 1.0$)

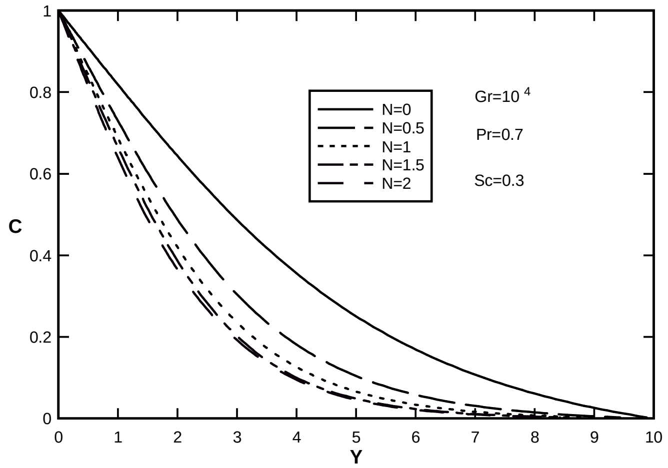

Figure 7. Effect of buoyancy ratio parameter (N) on dimensionless concentration distributions dissipation number ($\varepsilon = 1.0$)

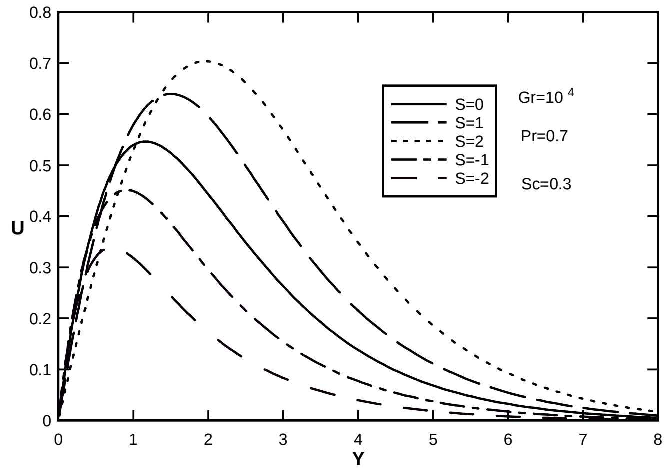

Figure 8. Effect of Suction rate parameter (S) on dimensionless velocity Profiles at dissipation number ($\varepsilon = 1.0$)

The effect of Schmidth number (Sc) on the concentration profile is shown in Fig. 14. It is observed that the concentration decreases with increase in Schmidth number .

The effect of Prandtl number (Pr) on the temperature profile is shown in Fig. 15. It is observed that the temperature decreases with increase in the Prandtl number.

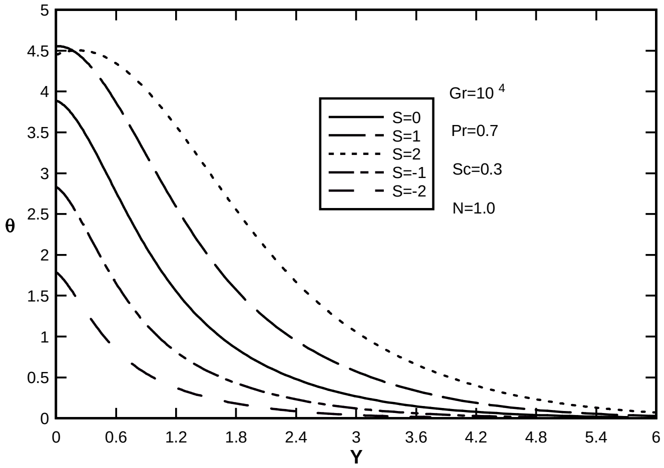

Figure 9. Effect of Suction rate parameter (S) on dimensionless temperature distributions at dissipation number ($\varepsilon = 1.0$)

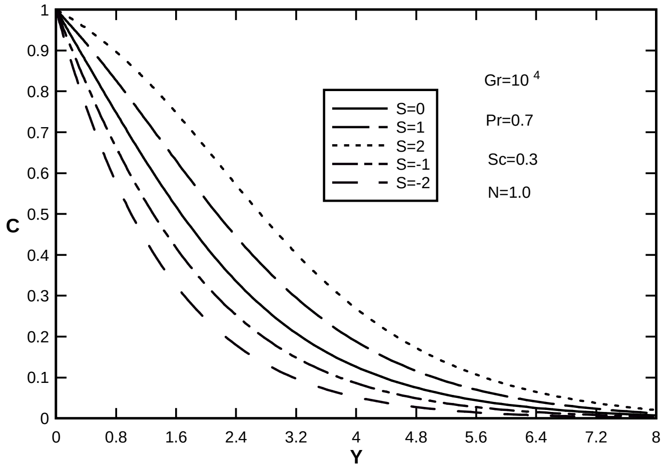

Figure 10. Effect of Suction rate parameter (S) on dimensionless concentration distributions dissipation number ($\varepsilon = 1.0$)

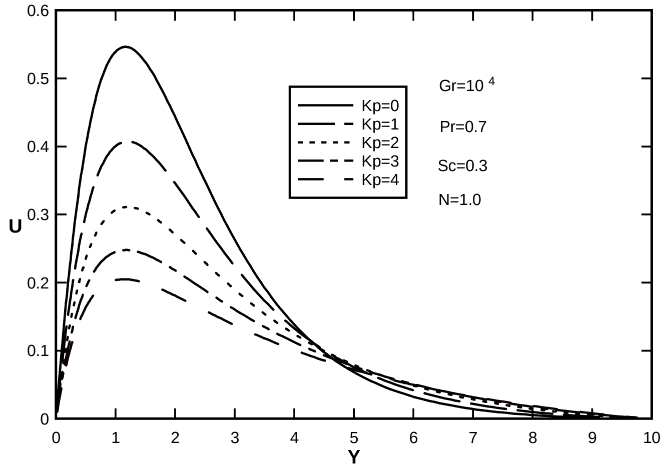

Figure 11. Effect of porous media parameter (K_p) on dimensionless velocity Profiles at dissipation number ($\varepsilon = 1.0$)

Fig. 16 present the axial evolution of the local skin-friction $C_{fx}Gr^{\frac{1}{4}}$ in steady-state situation. From this figure it is observed that an increase in the magnetic parameter M leads to a decrease in C_{fx} .

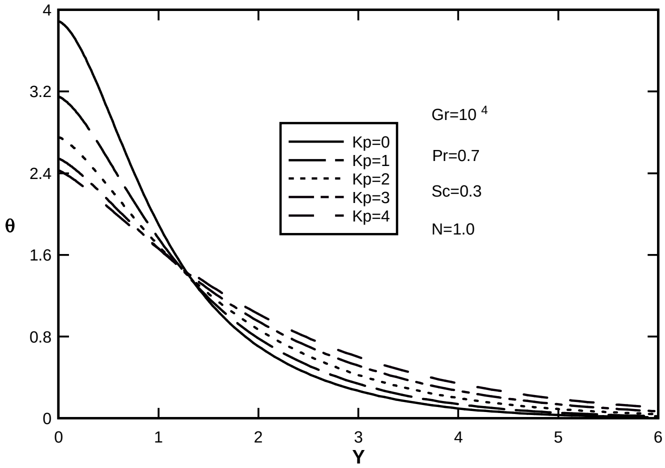

Figure 12. Effect of porous media parameter (K_P) on dimensionless temperature distributions at dissipation number ($\varepsilon = 1.0$)

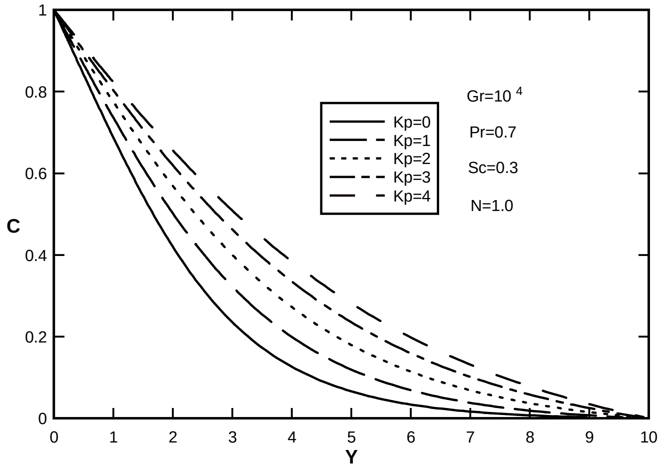

Figure 13. Effect of porous media parameter (K_P) on dimensionless concentration distributions dissipation number ($\varepsilon = 1.0$)

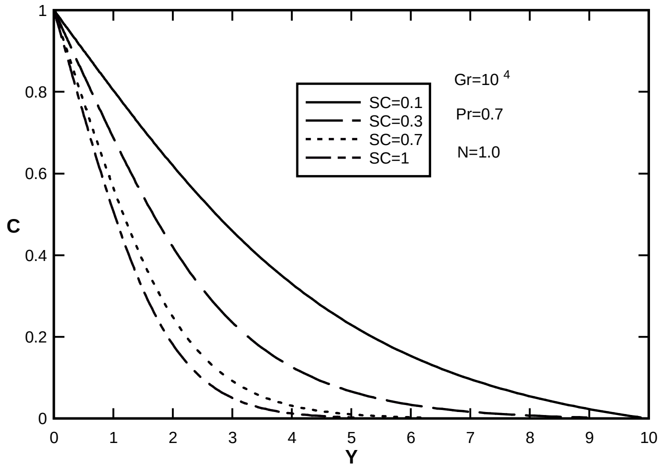

Figure 14. Effect of Schmidth number (Sc) on dimensionless concentration distributions dissipation number ($\varepsilon = 1.0$)

Fig. 17 depict the axial evolution of the local Nusselt number $Nu_x / Gr^{\frac{1}{4}}$ in a steady-state situation. From this figure it is observed that an increase in the magnetic parameter M leads to a increase in Nu_x .

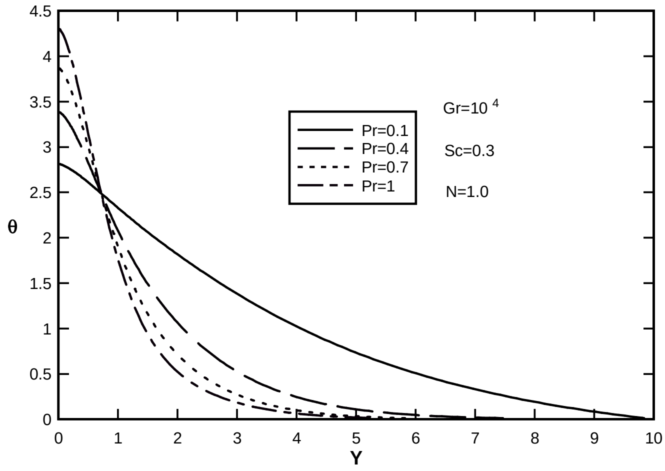

Figure 15. Effect of Prandtl number (Pr) on dimensionless temperature distributions at dissipation number ($\varepsilon = 1.0$)

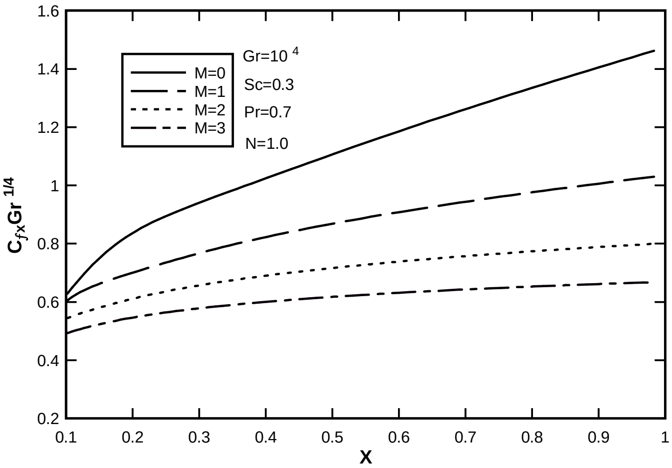

Figure 16. Effect of magnetic parameter on Local skin-friction

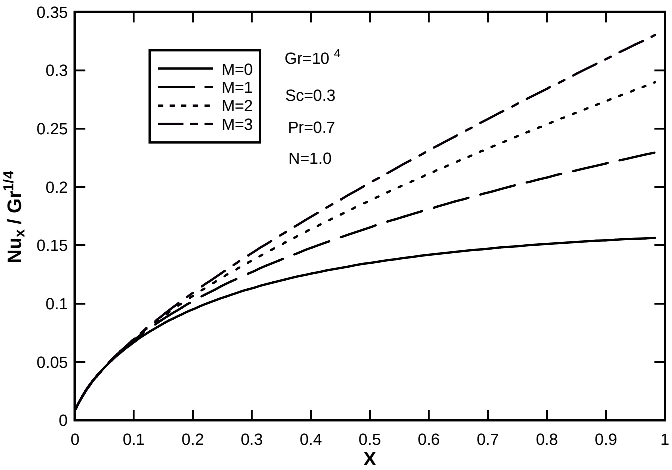

Figure 17. Effect of magnetic parameter on Local Nusselt number

Fig. 18 show the axial evolution of the local Sherwood number $Sh_x / Gr^{\frac{1}{4}}$ in a steady-state situation. From this figure it is observed that an increase in the magnetic parameter M leads to a decrease in Sh_x .

Fig. 19 show the axial evolution of the local Sherwood number $Sh_x / Gr^{\frac{1}{4}}$ in a steady-state situation. From this figure it is observed that an increase in the Schmidth number (Sc) leads to a increase in Sh_x .

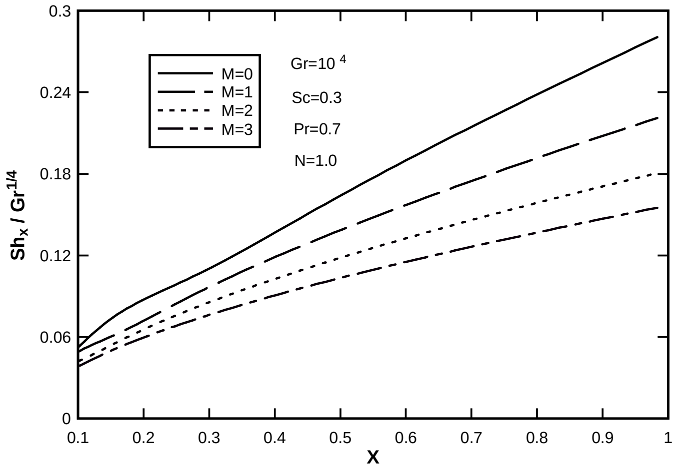

Figure 18. Effect of magnetic parameter on local Sherwood number

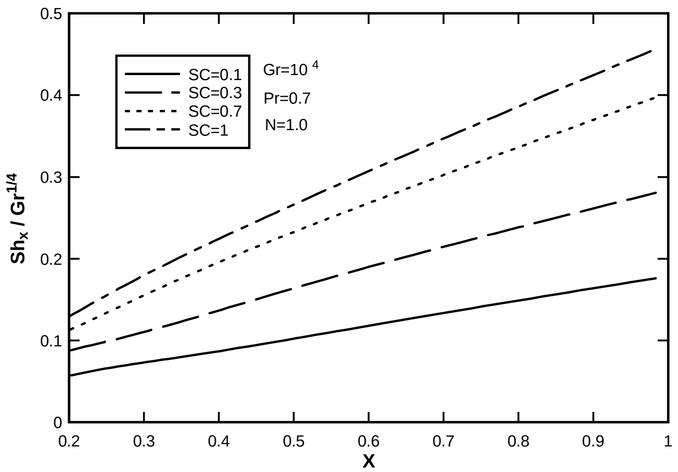

Figure 19. Effect of Schmidth number (Sc) on local Sherwood number

REFERENCES

[1] E. Pohlhausen, *ZAMM* 1 (**1921**) 115–125.
[2] S.D. Harris, L. Elliot, D.B. Ingham, I. Pop, *Int. J. Heat Mass Tran*. 41 (**1998**) 357–372.
[3] G. Polidori, C. Popa, T. Hoang-Mai, *Mech. Res. Commun*. 30 (**2003**) 515–621.
[4] D.A.S. Rees, *Int. J. Heat Mass Tran*. 42 (**1999**) 2455–2464.
[5] J. Li, D.B. Ingham, I. Pop, I. J. *Heat Mass Tran*. 44 (**2001**) 2311–2322.
[6] M. Kassem, *J. Comput. Appl. Math*. 187 (**2006**).
[7] B. Gebhart, *J. Fluid Mech*. 14 (**1962**) 225–232.
[8] H.S. Takhar, V.M. Soundalgekar, *Appl. Sci. Res.* 46 (**1989**).
[9] A. Pantokratoras, *Appl. Math. Model.* 29 (**2005**).
[10] V.M. Soundalgekar, B.S. Jaiswal, A.G. Uplekar, H.S. Takhar, *Appl. Mech. Eng*. 4 (**1999**).
[11] V.M. Soundalgekar, P. Ganesan, *Int. J. Eng. Sci*. 19 (**1981**) 757–770.
[12] M.Y. Gokhale, F.M. Al Samman, *Int. J. Heat Mass Tran*. 46 (**2003**) 999–1011.
[13] B. Shanker, N. Kishan, *J. Eng. Heat Mass Tran*. 19 (**1997**) 273–278.
[14] P. Gounder Ganesan, P. Hari Rani, *Int. J. Therm. Sci*. 39 (**2000**) 265–272.
[15] M.A. Hossain, S.K. Das, I. Pop, *Int. J. Non-linear Mech*. 33 (**1998**) 541–553.
[16] E.M. Aboeldahad, E.M.E. Elbarbary, *Int. J. Eng. Sci*. 39 (**2001**) 1641–1652.
[17] Sreehari Reddy, P., Nagarajan, A.S. and Sivaiah, M.(**2008**): *Journal of Naval Architecture and Marine Engng*, 2, 47 – 56,(**2008)**.

[18] Suneetha, S., Bhaskar Reddy, N., Ramachandra Prasad,V., (**2008**): *Journal of Naval Architecture and Marine* engineering, 2, 57 – 70.
[19] Suneetha, S., Bhaskar Reddy, N., Ramachandra Prasad, V., (**2009**): *Thermal Science,* 13, 2, 71 – 181.
[20] Hitesh Kumar: (**2009**): *Thermal Science* 13, 2, 163 – 169.
[21] B. Carnahan, H.A. Luther, and J.O. Wilkes., (**1969**). Applied Numerical Methods, Wiley, New York.