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MHD boundary layer flow of a non-newtonian power-law fluid on a moving flat plate

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ABSTRACT

The problem of steady, two-dimensional laminar flow of a power-law fluid passing through a moving flat plate under the influence of transverse magnetic field is studied. The resulting governing partial differential equation is transformed into a non linear ordinary differential equation using appropriate transformation. This non linear ordinary differential equation is linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. The system of algebraic equations is solved by using Gauss-Seidal iterative method. The solution is found to be dependent on various governing parameters including magnetic field parameter M, power-law index n and velocity ratio parameter ε. A systematical study is carried out to illustrate the effects of these major parameters on the velocity profiles. It is found that dual solutions exits when the plate and the fluid move in opposite directions, near the region of separation.

Keywords: Magnetic field parameter, Non-Newtonian fluids, power-law index, Quasi-linearization and finite difference method.

INTRODUCTION

Fluids for which the relationship between the shear stress and rate of strain is non linear through the origin at given temperature and pressure are said to be non-Newtonian. The subject of boundary-layer flow on a continuously moving surface traveling through a quiet ambient fluid is currently one of important in view of its relevance to a number of engineering processes. Flows due to a continuously moving surface is encountered in several processes for thermal and moisture treatment of materials, particularly in processes involving continuous pulling of a sheet through a reaction zone, as in metallurgy , in textile and paper industry, in the manufacture of polymeric sheets, sheet glass and crystalline materials. An example for a continuously moving surface is a polymer sheet or filament extruded continuously from die, or a long thread traveling between a feed roll and wind-up roll. Sakiadis [1] was the first to investigate the flow due to sheet issuing with constant speed from a slit into a fluid at rest, he has considered the problem of forced convection along an isothermal moving plate. Tsou et.al [2] studied flow and heat transfer in the boundary layer on a continuously moving surface whereas Soundalgekar and Murthy [3] studied the heat transfer problem by assuming the plate temperature to be variable.

Klemp and Acrivos [4] demonstrated a method for integrating the boundary layer equations through a region of reverse flow and applied it to the problem of uniform flow past a parallel flat plate of finite length whose surface has a constant velocity directed opposite to that of main stream. Similar problems were considered by Abdelhafez [5], Hussaini et. al [6] and Ishak et. al [7]. All of the above investigators, however restrict their analysis to the flow of Newtonian fluids. Most fluids such as molten plastics, artificial fibres, drilling of petroleum, blood and polymer solutions are considered non-Newtonian fluids. Schowalter [8] has introduced the concept of the boundary layer in the theory of non-Newtonian power-law fluids. Acrivos, Shah and Petersen [9] have investigated the steady laminar flow of non- Newtonian fluids over a plate.

Howell et.al [10] and Rao et.al [11] have studied the momentum and heat transfer on a continuous moving surface in a power-law fluid. Kumari and Nath [12] discussed over a continuously moving surface with a parallel free stream.

Mahmoud and Mahmoud [13] had given the analytical solutions of hydromagnetic boundary-layer flow of a non-Newtonian power-law fluid past a continuously moving surface. Recently, Anuar Ishak [14] have investigated the steady boundary-layer flow of a non-Newtonian power-law fluid over a flat plate in a moving fluid.

The object of the present paper is to study the magnetic effects on a steady, two-dimensional laminar flow of a power-law fluid passing through a moving flat plate. The numerical solutions are carried out by using the implicit finite difference scheme.

Mathematical Formulation:

Consider a steady, two dimensional laminar flow of a power-law fluid passing through a moving flat plate with constant velocity U_w, in the same or opposite direction to the free stream U_∞ . The x – axis extends parallel to the plate, while the y-axis extends upwards, normal to it. Also, a magnetic field of strength B is applied in the positive y-direction, which produces magnetic effect in the x-direction. The boundary layer equations governing the flow in a power-law fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad \text{-----(1)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}-\frac{\sigma B^2}{\rho}u, \qquad \text{-----(2)}$$

where u and v are the velocity components along the x and y axes, respectively, τ_{xy} is the shear stress and ρ is the fluid density. The boundary conditions are

$$u = U_w,\ v = 0 \quad \text{at} \quad y = 0,$$
$$u \to U_\infty \quad \text{as} \quad y \to \infty \qquad \text{-----(3)}$$

The stress tensor is defined as

i.e. $$\tau_{xy} = 2K\left|2D_{kl}D_{kl}\right|^{(n-1)/2} D_{ij}, \qquad \text{-----(4)}$$

where $$D_{ij}\ \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \qquad \text{-----(5)}$$

denotes the stretching tensor, K is the consistency coefficient and n is the power-law index. The index n is non-dimensional, and the dimension of K depends on the value of n. The two- parameter rheological eq. (4) is known as the Ostwald-de-waele model or, more commonly, the power-law model. The parameter n is an important index to subdivide fluids into pseudo plastic fluids (n<1) and dilatant fluids (n>1). For n = 1, the fluid is simply the Newtonian fluid. Therefore, the deviation of n from unity indicates the degree of deviation from Newtonian behaviour. With n ≠ 1, the constitutive eq. (4) represents shear thinning (n<1) and shear thickening (n>1) fluids. Using eq.s (4) and (5), the shear appearing in eq. (2) can be written as

$$\tau_{xy} = K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}. \qquad \text{-----(6)}$$

Now the momentum equation (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) - \frac{\sigma B^2}{\rho}u \qquad \text{-----(7)}$$

Method of Solution :
We shall transform equation (2) into a ordinary differential equation amenable to a numerical solution. For this purpose we introduce a similarity transformation given as

$$\eta = \left(\frac{\text{Re}}{x/L}\right)^{\frac{1}{n+1}}\frac{y}{L}, \qquad \psi = LU_\infty\left(\frac{x/L}{\text{Re}}\right)^{\frac{1}{n+1}} f(\eta) \qquad \text{-----(8)}$$

where η is the similarity variable, *f*(η) is the dimensional stream function, L is the characteristic length and Re is the generalized Reynolds number defined as

$$\text{Re} = \frac{\rho U_\infty^{2-n}L^n}{K} \qquad \text{-----(9)}$$

The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial\psi}{\partial y} \qquad and \qquad v = -\frac{\partial\psi}{\partial x} \qquad \text{-----(10)}$$

Using the similarity transformation, equation (2) is transformed into the ordinary differential equation of the form

$$\left(\left|f''\right|^{n-1}f''\right)' + \frac{1}{n+1}ff'' - Mf' = 0 \qquad \text{-----(11)}$$

where $M = \dfrac{\sigma B^2}{\rho U_\infty}x$ is the magnetic parameter

The transformed boundary conditions are

$$f(0) = 0, \qquad f'(0) = \varepsilon \qquad and \qquad f'(\infty) = 1 \qquad \text{-----(12)}$$

where $\varepsilon = U_w / U_\infty$ is the velocity ratio parameter. We note that when ε = 0 (stationary plate) an n = 1 (Newtonian fluid), the present problem reduces to the classical Blasius problem. When ε < 0, the fluid and the plate move in the opposite directions, while they move in the same direction when ε >0. The case 0 < ε < 1 is when the speed of the plate is less than those of the fluid and the opposite is true when ε >1. Moreover, ε = 1 corresponds to the case when the plate and the fluid move with the same velocity. For brevity, in this study we consider only the case ε ≤ 1.

To solve the transformed governing equation (11) with the boundary conditions (12), first equation (11) is linearized using the Quasi linearization technique [15].
Then equation (11) is transformed to

$$n\left[F'''(f'')^{n-1} + f'''(F'')^{n-1} - F'''(F'')^{n-1}\right] + \frac{1}{n+1}\left[Ff'' + fF'' - FF''\right] - Mf' = 0 \qquad (13)$$

where F is assumed to be known and the above equation (13) can be expressed in the simplified form as

$$A_0 f''' + A_2 f'' + A_3 f' + A_4 f = A_5 - A_1 [f'']^{n-1} \qquad \text{-----(14)}$$

where $A_0[i] = n(F'')^{n-1}$, $A_1[i] = nF'''$, $A_2[i] = \frac{1}{n+1}F$,

$A_3[i] = -M$, $A_4[i] = \frac{1}{n+1}F''$

$A_5[i] = nF'''(F'')^{n-1} + \frac{1}{n+1}FF''$

Using implicit finite difference formulae, the equations (14) is transformed to

$$B_0[i]f[i+2] + B_1[i]f[i+1] + B_2[i]f[i] + B_3[i]f[i-1] = B_4[i] \qquad (15)$$

where $B_0[i] = 2A_0[i]$ $B_1[i] = -6A_0[i] + 2hA_1[i] + h^2A_3[i]$
$B_2[i] = 6A_0[i] - 4hA_2[i] + 2h^3A_4[i]$ $B_3[i] = -2A_0[i] + 2hA_2[i] - h^2A_3[i]$
$B_5[i] = 2h^3\{ A_5[i] - A_1 [F''[i]]^{n-1} \}$

here 'h' represents the mesh size in η direction. Equation (15) are solved under the boundary conditions (12) by Gauss-Seidel iteration method and computations were carried out by using C programming. The numerical solutions of f are considered as $(n+1)^{th}$ order iterative solutions and F are the n^{th} order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$.

RESULTS AND DISCUSSION

For negative values of ε there is a critical value ε_c with two solution branches for $\varepsilon_c<\varepsilon<0$, unique solution for $\varepsilon \geq \varepsilon_c$, a saddle-node bifurcation at $\varepsilon = \varepsilon_c$ and no solution for $\varepsilon < \varepsilon_c$. The boundary layer approximation breakdown at $\varepsilon = \varepsilon_c$ and thus no solution is obtained for for $\varepsilon < \varepsilon_c$. These values of ε_c are given in table 1 which are in good agreement with Klemp and Acrivos [4], Hussaini et .al [6] and Anuar Ishak and Norfifah Bachok [14] for Newtonian fluid. It is seen from the table 1 that the increase of power-law index n decreases the critical value of velocity ratio parameter ε_c.

The fluid velocity profiles $f'(\eta)$ are shown in figures 1-4 for various flow parameters magnetic field parameter M, power-law index n and velocity ratio parameter ε. Figure 1 shows that with the increase in the values of power-law index n, velocity profiles $f'(\eta)$ increases for a positive value of velocity ratio parameter ε = 0.5 in the absence of magnetic field. The velocity profiles $f'(\eta)$ for seleted values of n presented in the absence of magnetic field show that the far field boundary condition is approached asymptotically (i.e. the velocity gradient at large distance from the plate is zero). It is evident from the figure 2 that for each value of power-law index n considered, there exist two different profiles for velocity ratio parameter ε = -0.25, which support the dual nature of the solution. Figure 2(a) drawn for lower branch and figure 2(b) drawn for upper branch with velocity ratio parameter ε = -0.25 shows that the velocity profiles $f'(\eta)$ increases with the increase of power-law index n. It is also observed that the effect of power-law index n is more in upper branch for pseudo-plastic fluids (n < 1). The effect of magnetic field M is shown in figure 3 with velocity ratio parameter ε = 0.5 for different fluids such as (a) pseudo-plastic fluid (n = 0.6), (b) Newtonian fluid (n = 1.0) and (c) dilatant fluid (n = 1.4). The effect of magnetic field parameter M is to decelerate the fluid flow velocity $f'(\eta)$ in all the cases (a), (b) and (c). Figure 4 is shown for velocity profiles $f'(\eta)$ (upper branch) for different values of magnetic field parameter M with velocity ratio parameter ε - -0.25 for different fluids such as (a) pseudo-plastic fluid (n = 0.6), (b) Newtonian fluid (n = 1.0) and (c) dilatant fluid (n = 1.4). The effect of magnetic field parameter M is to reduces the velocity profiles $f'(\eta)$ far away from the plate and reverse phenomenon is observed near the plate.

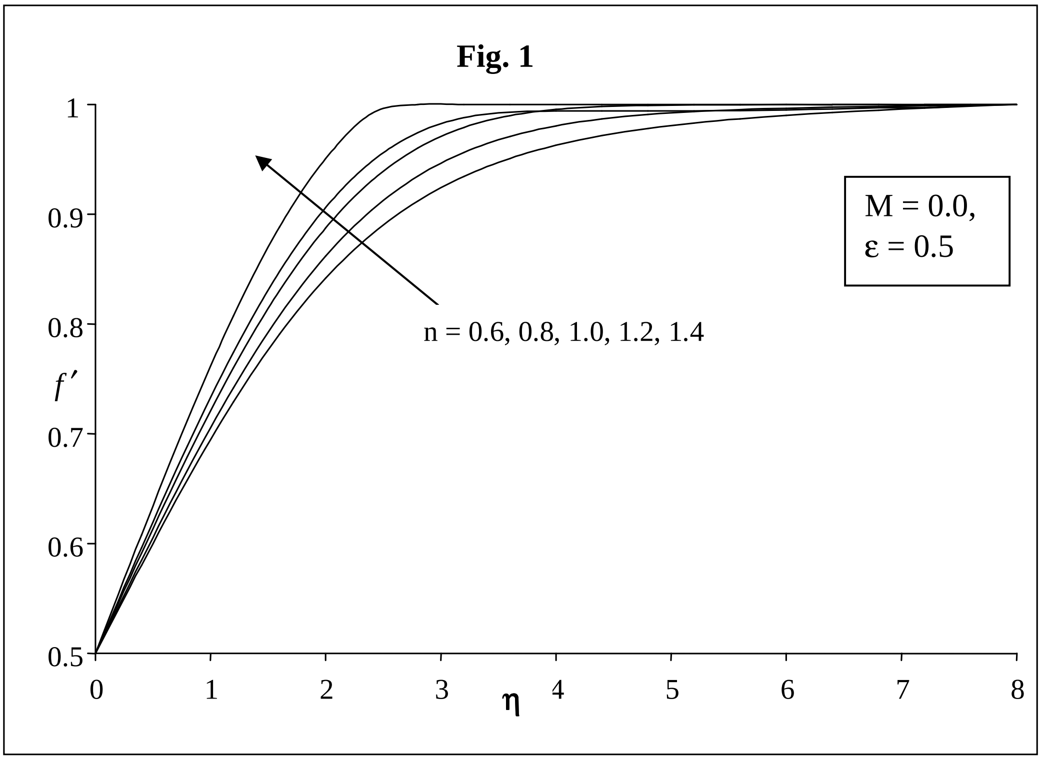
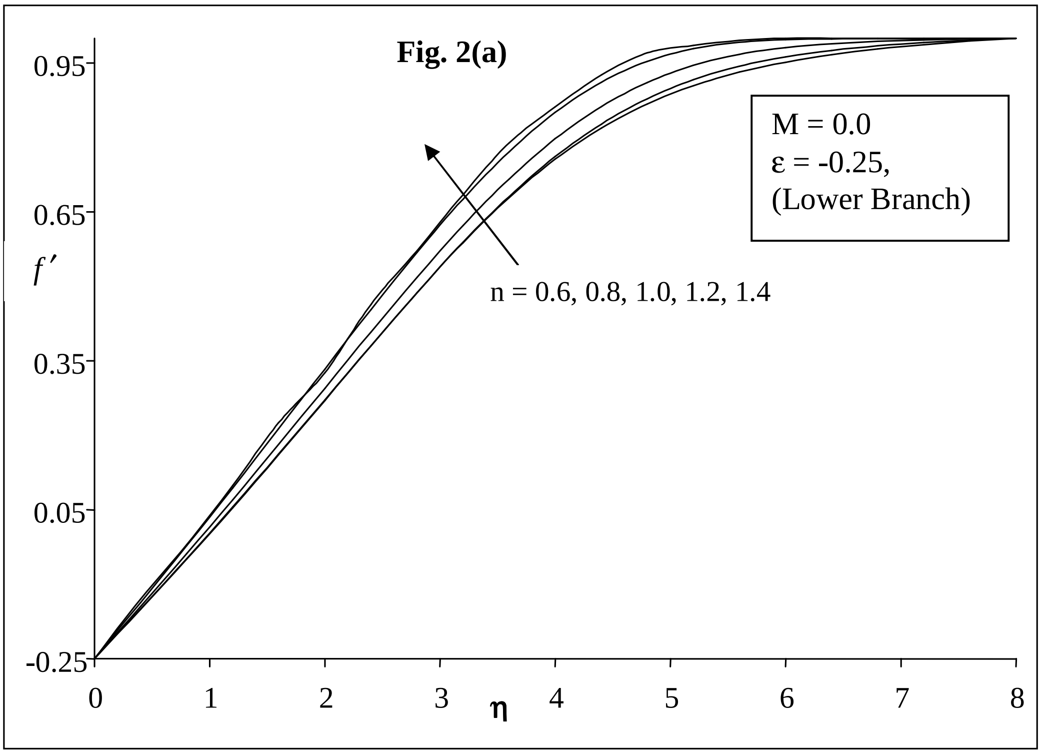


Fig. 1. Velocity Profiles f' for different values of power-law index n with ε = 0.5 and M = 0.0.



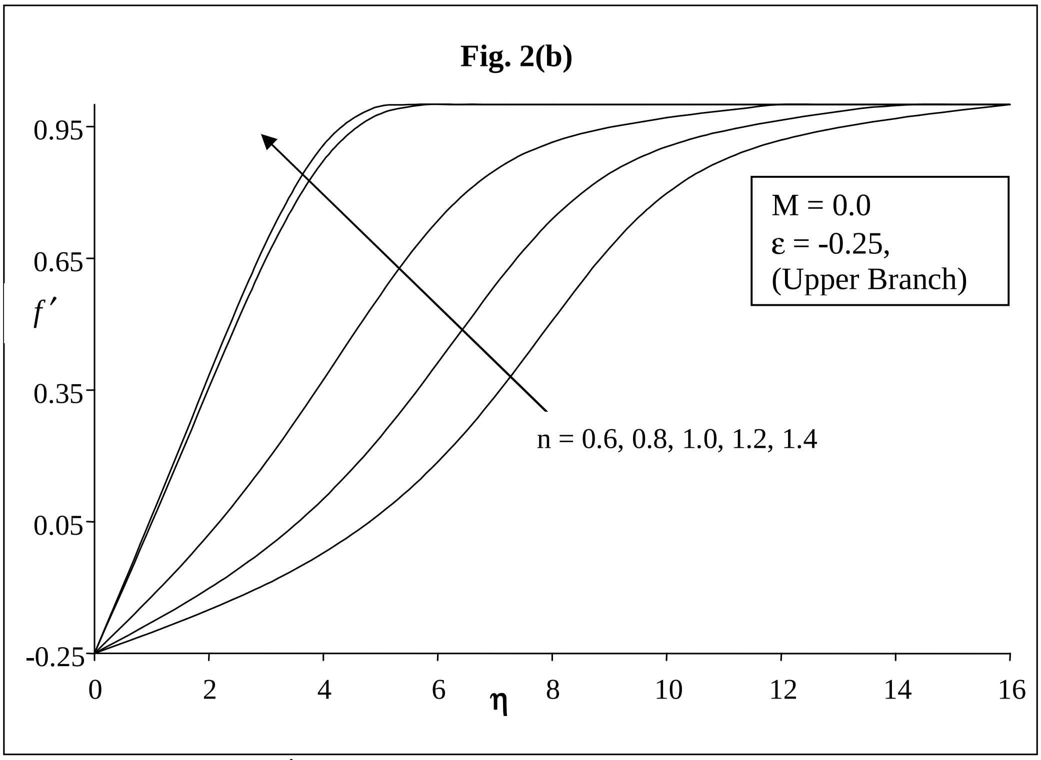
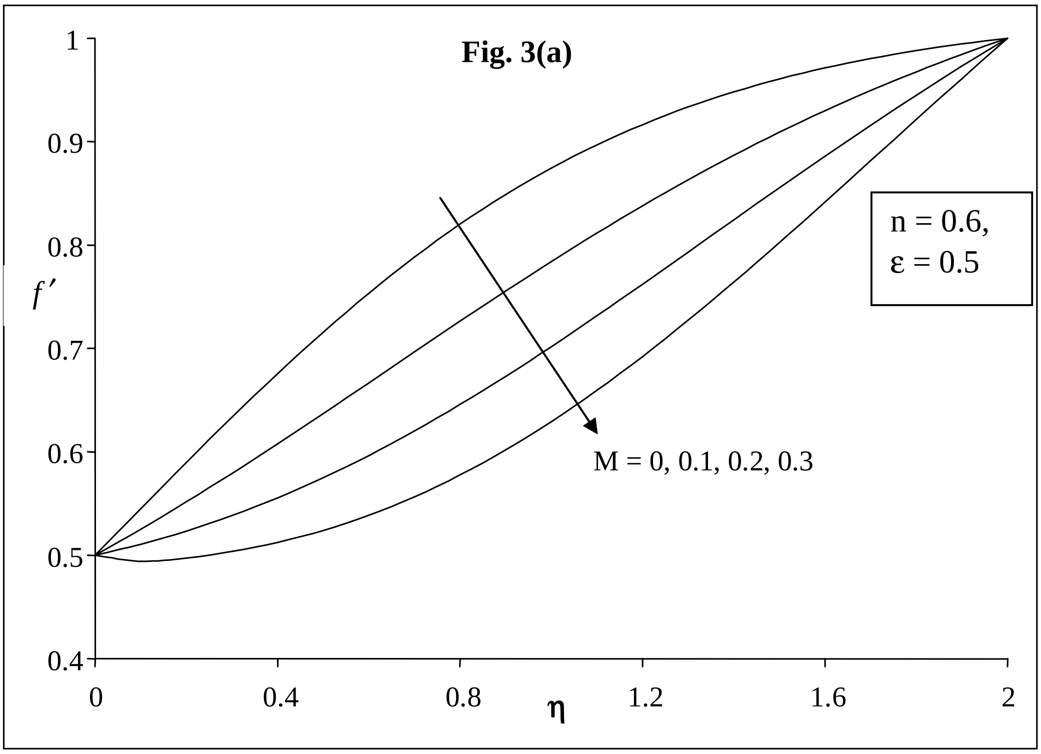
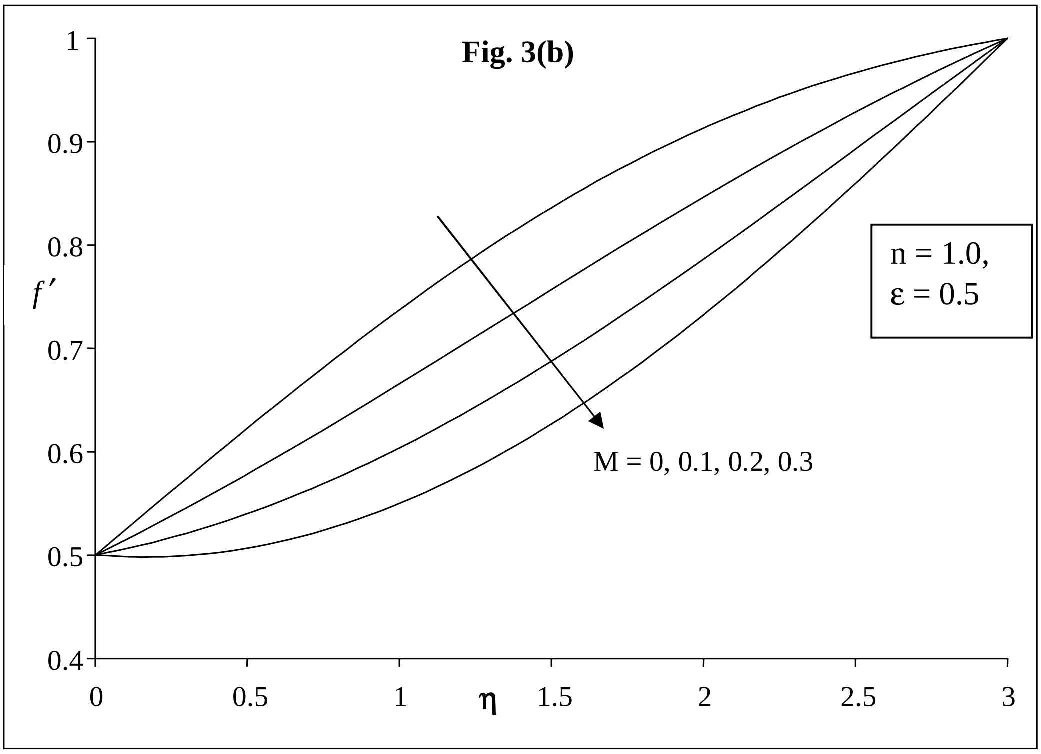


Fig. 2. Velocity Profiles f' for different values of power-law index n with ε = -0.25 and M = 0.0.
(a) Lower Branch (b) Upper Branch





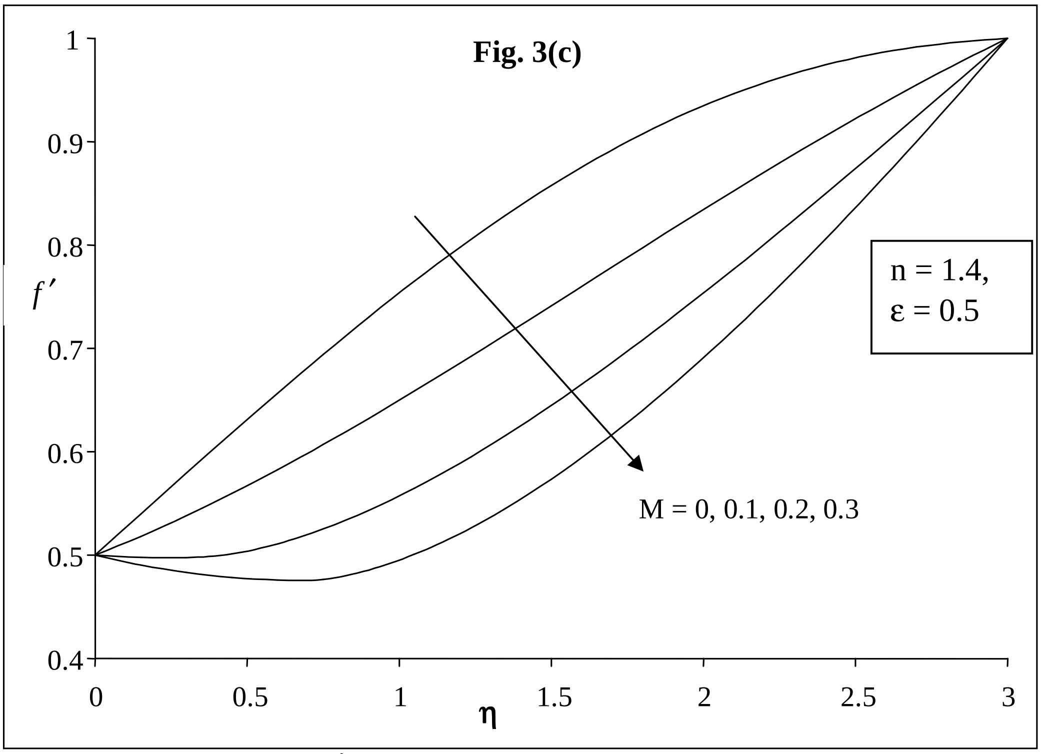


Fig. 3. Velocity Profiles f' for different values of Magnetic parameter M with ε = 0.5.
(a) n = 0.6 (Pseudo plastic fluid) (b) n = 1.0 (Newtonian fluid) (c) n = 1.4 (dilatant fluid)

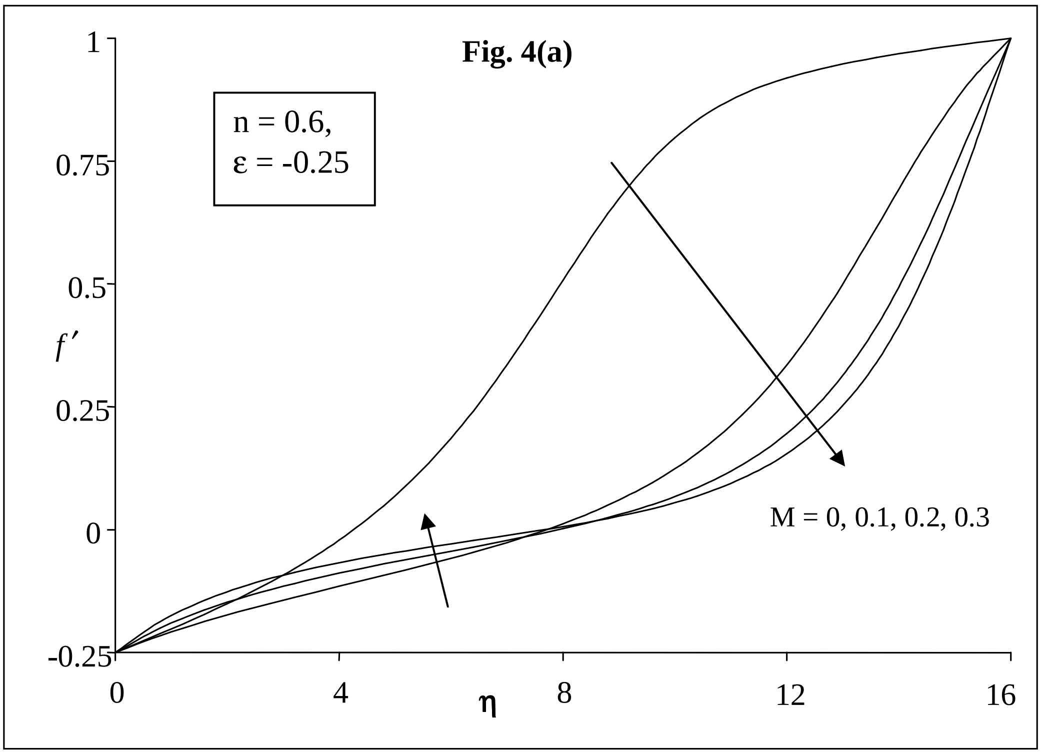


Fig. 4(a)
n = 0.6,
ε = -0.25
M = 0, 0.1, 0.2, 0.3
f′
η
1
0.75
0.5
0.25
0
-0.25
0
4
8
12
16

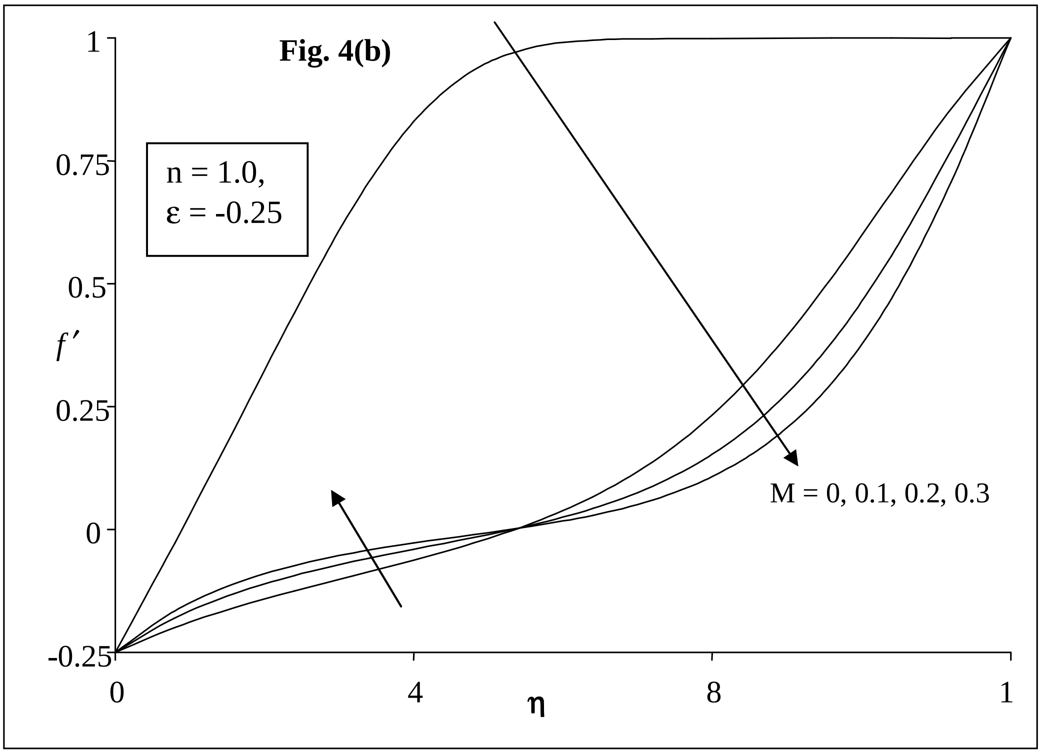


Fig. 4(b)
n = 1.0,
ε = -0.25
M = 0, 0.1, 0.2, 0.3
f′
η
1
0.75
0.5
0.25
0
-0.25
0
4
8
1

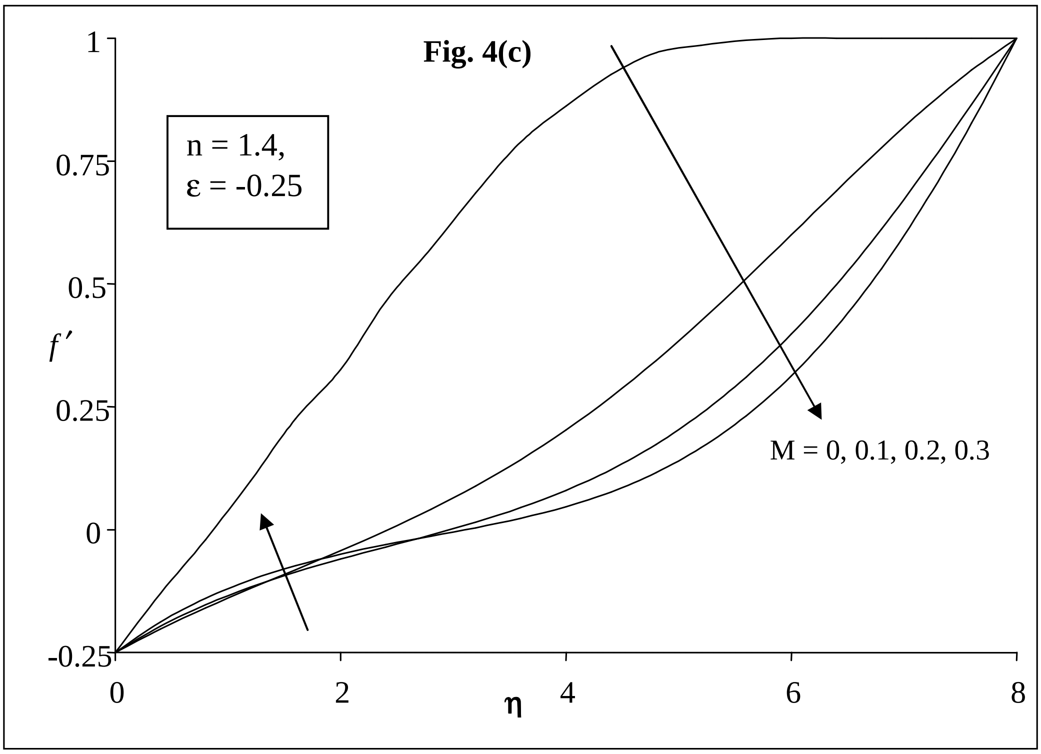


Fig. 4. Velocity Profiles f' for different values of Magnetic parameter M with ε = -0.25.
(a) n = 0.6 (Pseudo plastic fluid) (b) n = 1.0 (Newtonian fluid) (c) n = 1.4 (dilatant fluid)

Table 1 : Variation of ε_c power-law index n

n	ε_c
0.6	-0.3532
0.8	-0.3641
1.0	-0.3816
1.2	-0.3975
1.4	-0.4996

Nomenclature :

B – Magnetic field intensity
D_{ij} – stretching tensor
f - Dimensionless stream function
K – consistency coefficient
L – Characteristic length of the plate
n – power-law index
Re – generalized Reynolds number
U_w – Free stream velocity
u, v – Velocity components along and normal to the plate
x, y - Coordinates along and perpendicular to the plate
η – Dimensionless similarity variable
ψ - stream function
ρ – Density
σ – Electrical conductivity
ε – velocity ratio parameter
τ_{xy} - Shear stress

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