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Mathematical modelling on water pollution and self-purification of river Ganges

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ABSTRACT

The Pollution of India's National River Ganges has become grim task for today. After observing the data it has been found that Ganges in high profile cities like Kanpur, Varanasi and Allahabad are highly polluted. In this study pollution is analysed in these highly polluted areas and a dynamical model is formulated to solve the pollution problem. In the paper it is shown mathematically that if sewage treatment capacity is increased then how pollution in the river goes on decreasing. Self-purification of Ganges is taken into account using de-oxygenation and re-aeration process. After analysing self-purification concept certain results are concluded about the pollution of river. It is advisable that sewage treatment network should be increased on a large scale in highly polluted stretches like KANPUR-VARANASI. The target goal of this paper is to draw attention towards dying condition of Ganges in these regions and through mathematical model an idea is given that how poorest condition of Ganges in these areas can be recovered.

Keywords: River, Biological oxygen demand, ph values, sewage, Industrial waste, self-purification, de-oxygenation, re-aeration, sewage treatment plants

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INTRODUCTION

Ganges is trans-boundary river of Asia which flows in India and Bangladesh.[3] Most sacred river to Hindus The Ganges , declared national river in 2009 ,is moral symbol of development of Indian Culture. The Ganges Basin accounts for at-most 26% of India's graphical part.[2] In the world it has 5th place in the list of most polluted river in 2007.

The project of thousand crore of rupees to clean up the river is proving like a white elephant .The discharge of untreated sewage and other pollutants into Ganges is major cause of increased pollution. Nevertheless the past magnificence, Ganges River is today failing to save her perseverance against continued exploitation of dominant forces of frugality and general torpor. [6]

Leading source of increased pollution in the river is domestic sewage. As far as CPCB data are considered, 2,723 million litre a day (MLD) of sewage is brought about by 50 cities located along the river, which accumulates over 85 % of the river's contaminant load [2]

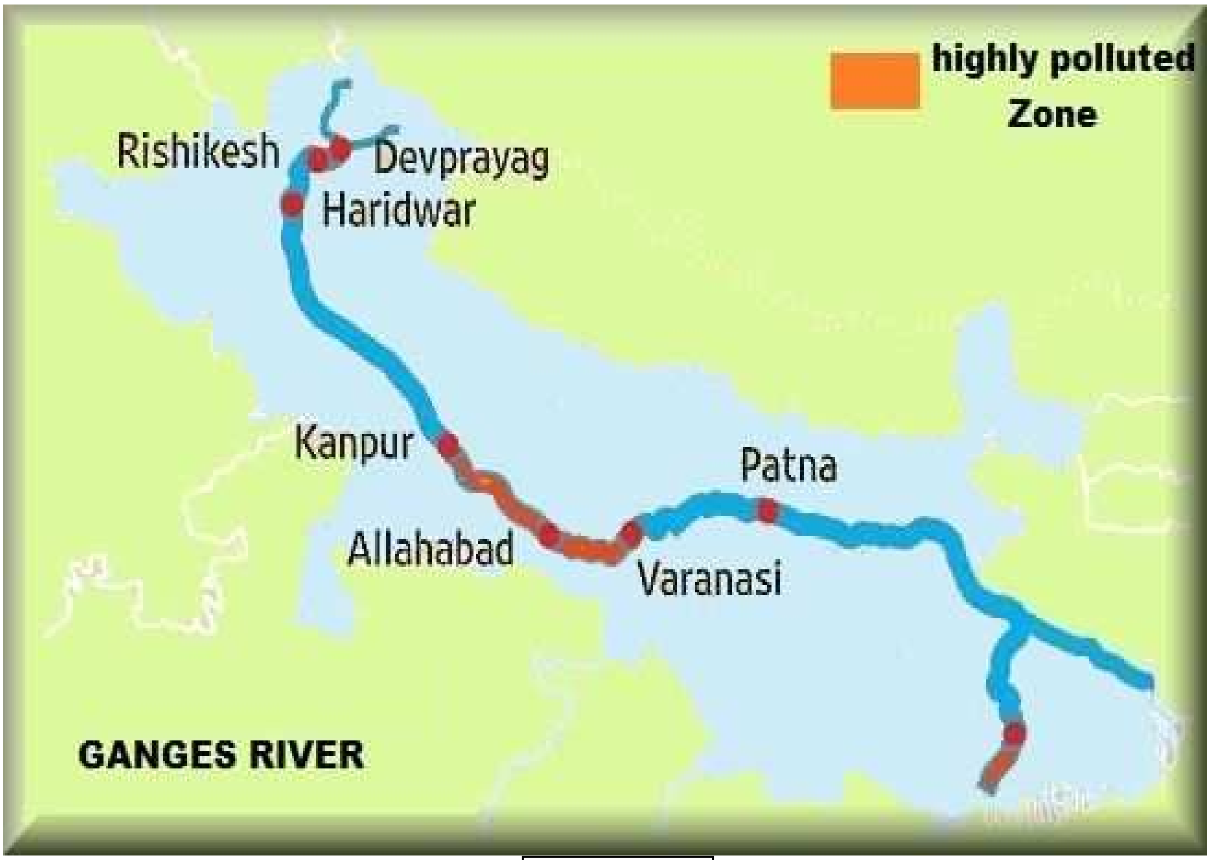

Figure No. 1

According to a study of Central Board for Preservation and Control of Water Pollution the long stretches of the Ganges near Kanpur and Varanasi are unsuitable for any beneficial human use. The water quality is generally good (B type) up to Bithoor, except near of Kannauj where polluted water from the kali river and city sewage combine to exceed the assimilation capacity of the river.

At Kanpur inpouring of Industrial waste/ sewage at Ganges is extremely high due to which water quality is declined to category ‘D’ and ‘E’. When river reaches to Sangam water quality improves to ‘B’ and ‘C’. After that when the river enters into Varanasi it recovers its frailty of water quality up-to ‘B’. But again discharge of heavy contaminants/ pollutants into Ganges diminish quality of water to ‘D’ and ‘E’ which is not even fit for bathing.

We develop a discrete dynamical model as an example of a mathematical model that describes the pollution problem more clearly. Dynamical system is about the evolution of some quantities over time. This evolution can be as continuous over time or in discrete time steps. We can formulate dynamical systems where the state of the system obtains in discrete time steps.

Flowing Water of river has a effective quality of getting purify as it moves forward with distance. This is known as Self-Purification. Actually this is a natural capacity of river to purify itself from pollutants/sewage. [8]. So to develop a simple model of purification of industrial waste we shall consider the self-purification property of river into account using simple first order decay equations.

2. Analyzation of Data:(Data Source: CPCB water quality criteria)
If data of Central Pollution Control Board are considered as evidence then some conclusions come out. Biological oxygen demand on some places is described in figure no. 2.

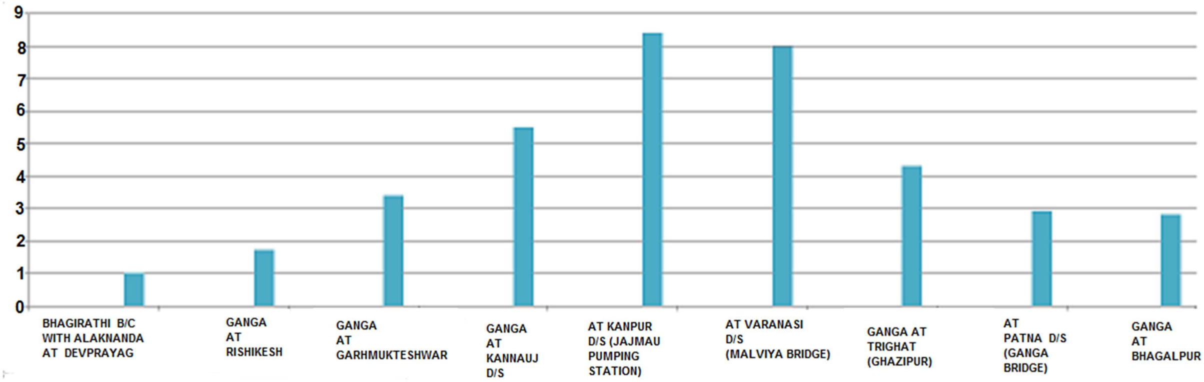

Figure No. 2

From graph it can be easily seen that at Kanpur and Varanasi BOD values are paranormally high.Accepted level of dissolved oxygen is more than 6 mg/l and biological oxygen demand is less than 2 mg/l. dissolved oxygen for Ganges at various cities is mentioned in figure 3.

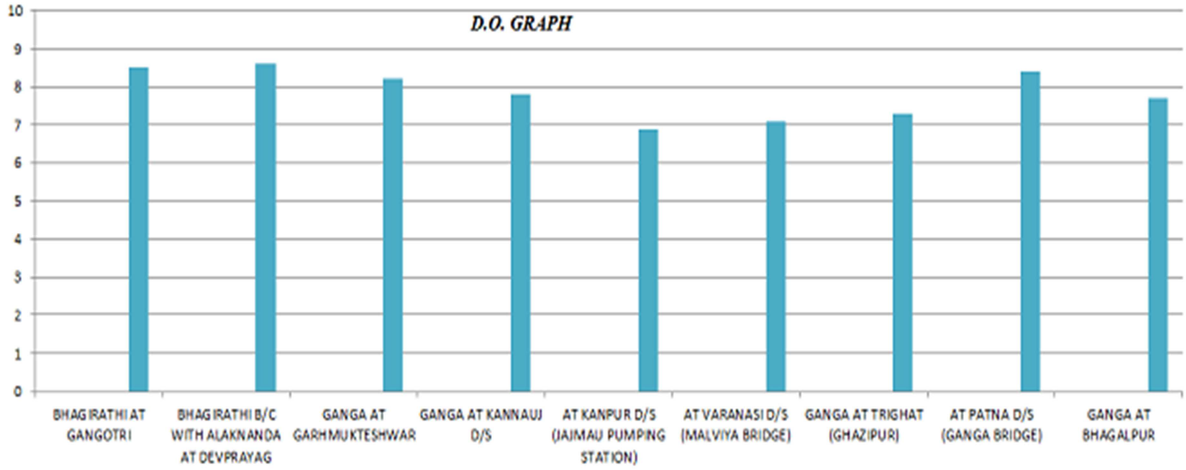

Figure No 3

As far as Fecal coliform is concern in this region, river has hundreds times more fecal coliform than the prescribed limit. Fecal coliform graph is shown in figure 4.

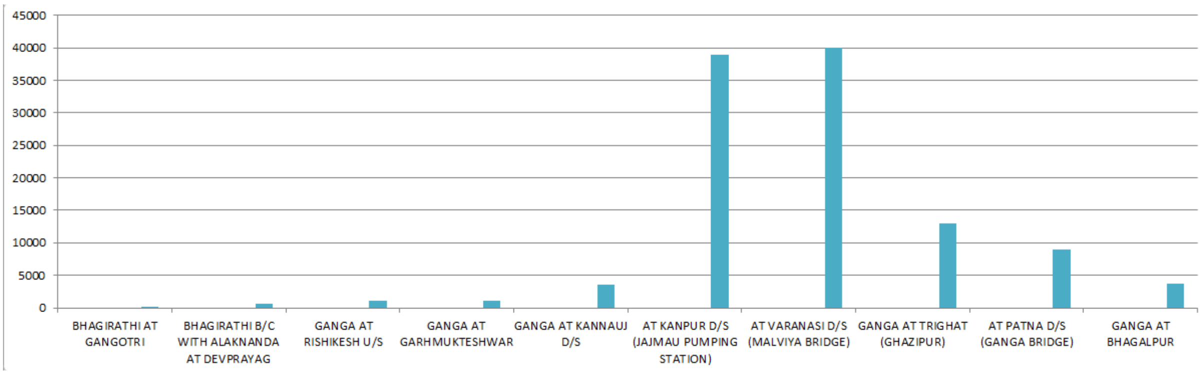

Figure No. 4

Fecal coliform bacteria are very harmful for our health. Report says that water of Varanasi is not even fit for bathing what to say about drinking. When its quality is increased waterborne diseases may occur at high risks.

3. Formulation of Mathematical Model on water pollution in Highly Polluted Kanpur-Varanasi Stretch:
This is a dynamical model reveals the growing pollution in highly polluted Kanpur-Varanasi Stretch. Here in the model we assume that Concentration of pollutant in the entire highly polluted stretch remains same. Also Mixing of sewage/industrial waste in the polluted stretch is instantaneous, uniform and proper. Daily flow at the initial and the end point of the stretch is same.

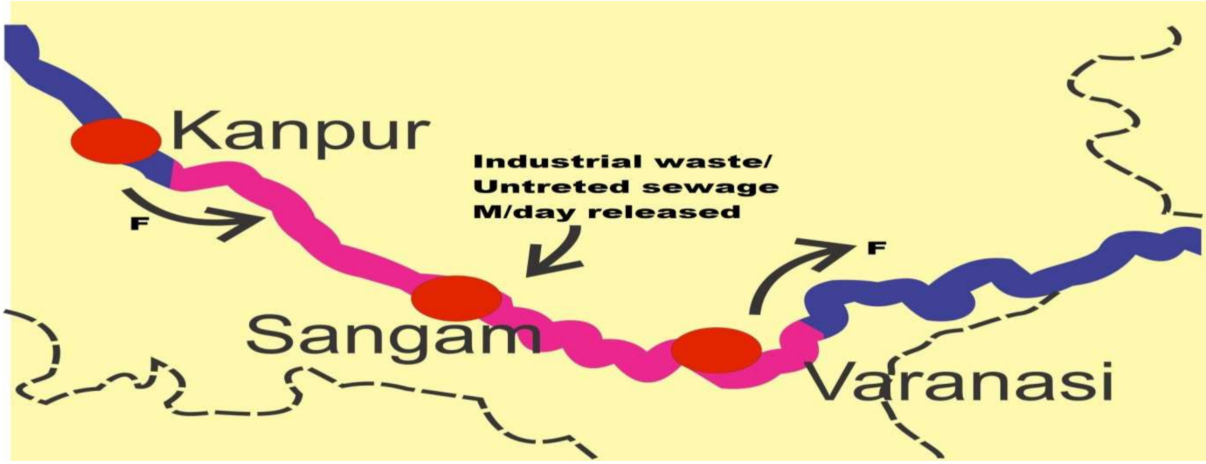

Figure No 5

Let at any initial time quantity of sewage and industrial waste in polluted stretch is S_0 after t day's quantity of industrial waste/ sewage increases to S_t. So total increment in sewage / industrial waste in t days= $S_t - S_0 = \Delta S_t$

Assume that M be the amount of untreated sewage/ industrial waste releasing in the most polluted stretch per day. F be the daily flow of river. C be the concentration of pollutants in the water entering the most polluted stretch. Then total sewage / industrial waste/ pollutants entering the polluted stretch= CF

If total volume of water in stretch is V then initial concentration of pollutant water in the stretch= S_0/V

After t days concentration changes to = S_t/V

so during period of t days we can assume that concentration of river in polluted stretch remains as $(S_0 + S_t)/2V$.
From mass conservation equation-

$$S_t - S_0 = Mt + FCt - F\frac{(S_0 + S_t)t}{2V}$$

$$\Delta S_t = \{M + FC - F\frac{(S_0+S_t)}{2V}\}t \qquad ....(1)$$

If flow rate at initial point of stretch is v_1 and at end point of the stretch is v_2 then total daily flow=average velocity of flow/day×cross sectional area of the channel

$= Av$

So from equation (1) we have

$$S_t - S_0 = Mt + v_1A_1Ct - \frac{(S_0 + S_t)}{2V}v_2A_2t$$

$$\frac{S_t - S_0}{t} = M + v_1A_1C - \frac{S_0}{2V}v_2A_2 - S_t\frac{v_2A_2}{2V}$$

$$S_t\left\{\frac{1}{t} + \frac{V_2A_2}{2V}\right\} = M + v_1A_1C - \frac{S_0}{2V}v_2A_2$$

$$S_t = \frac{M+v_1A_1C-\frac{S_0}{2V}v_2A_2}{\frac{1}{t}+\frac{v_2A_2}{2V}} \qquad .... (2)$$

If velocity flow rate at the initial and the end point of the stretch is same i.e.

$v_1 = v_2 = v$

Then from equation 2 we have

$$S_t = \frac{M + v(A_1 C - \frac{S_0}{2V} A_2)}{\frac{1}{t} + \frac{v A_2}{2V}} \quad \text{….. (3)}$$

4. Ending of Releasing Untreated sewage into Ganges:

At Kanpur more than 200 million litres of waste is discharged into the river through 16 major drains. At Allahabad, the famous centre of Hindus pilgrimage, 13 drains discharge 112 mid of sewage into Ganges and its tributary Yamuna which contain 32,164 kg of polluted material.[7]

Table 1: Drains that discharge into Ganges in the Kanpur-Varanasi stretch

Stretch	Discharge(MLD)	BOD load(kg/day)
Kanpur	600	634,915
Unnao	78	12,068
Fatehpur-Raibareilly	1,491	36,148
Allahabad	294	35,943
Mirzapur	149	9,471
Varanasi	411	9,607
Total	**3,023**	**738,152**

Table 2: All Drains in Kanpur-Varanasi stretch-

Drain	Flow(MLD)	BOD Load(KG/DAY)	Stretch
Sisamau nala	197.00	544,980	Kanpur
Bhagwatdas nala	11.00	1,144	Kanpur
Golaghat nala	0.80	114	Kanpur
Satti Chaura	1.10	97	Kanpur
Loni drain	41.90	4,860	Unnao
City Jail drain	35.90	7,208	Unnao
Permiya nala	186.00	11,485	Kanpur
Dabka nala-2	25.00	3,475	Kanpur
Dabka nala-3 (Pakka nala)	0.30	10	Kanpur
Dabka nala-1 (Kachha nala)	94.00	15,792	Kanpur
Wazidpur nala	54.00	45,522	Kanpur
Pandu river	1,396.00	34,900	Fatehpur to Raibareilly
Arihari drain	34.30	127	Fatehpur to Raibareilly
NTPC drain	60.30	1,121	Fatehpur to Raibareilly
Rasulabad-1 (Pakka nala)	29.80	20,264	Allahabad
Rasulabad-2 (Pakka nala)	20.20	5,656	Allahabad
Rasulabad-3 (Kachha nala)	14.20	1,320	Allahabad
Rasulabad-4 (Kachha nala)	48.50	2,376	Allahabad
Kodar drain	20.00	1,040	Allahabad
Ponghat drain	8.00	161	Allahabad
Solari drain	34.80	1,087	Allahabad
Maviya drain	65.00	3,380	Allahabad
Mualaha drain	46.00	598	Allahabad
Ghore Shaheed drain	86.40	4,121	Mirzapur
Khandwa drain	62.20	5,350	Mirzapur
Nagwa drain	66.50	4,060	Varanasi
Ramnagar drain	23.70	963	Varanasi
Varuna drain	304.50	3,776	Varanasi

(Data source from Reference No 2)

Total Sewage vs. Treatment Capacity:

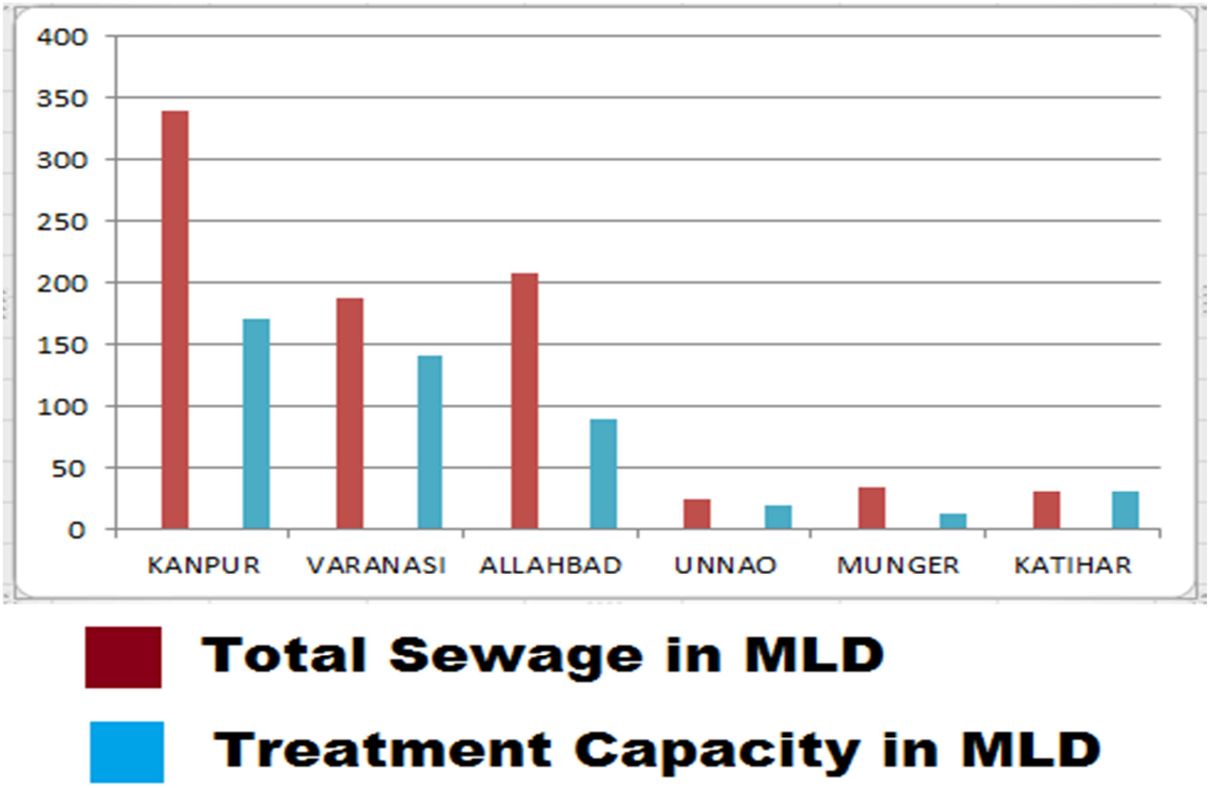

Figure No 6

If daily release of untreated sewage/ industrial waste into river is banned i.e. sewage treatment capacity is increased up-to total sewage generated i.e.
M→ 0 then from equation 1

$$\Delta S_t = \{M + FC - F\frac{(S_0 + S_t)}{2V}\}t$$

$$S_t - S_0 = Mt + FCt - F\frac{(S_0 + S_t)t}{2V}$$

$$(S_t - S_0)\left\{1 + \frac{Ft}{2V}\right\} = FCt - \frac{FS_0}{V}t \qquad \text{as } M\rightarrow 0$$

But $S_0/_V = C_0$ =initial concentration of sewage/industrial waste of polluted stretch.

$$S_t - S_0 = \frac{F(C - C_0)t}{(1 + ^{Ft}/_{2V})} \qquad \ldots\ldots (4)$$

Now since Kanpur – Varanasi stretch is most polluted part of river therefore

$C < C_0$ OR $C - C_0 < 0$
Therefore $S_t - S_0 < 0$ OR $S_t < S_0$

Hence river will recover from pollution. If equilibrium state is obtained then

$S_t - S_0 = 0$ i.e. $C = C_0$ (After a sufficient time)

5. Self-Cleansing Property of the river:
When water of river is polluted then river has an ability to purify itself using some chemical and biological actions. This is known as self-purification. Self-purification can be proved as a good indicator for the status of a river. Whenever pollutants is there in a river; 2 process takes place simultaneously –

1^{st} is de-oxygenation and 2^{nd} is re-aeration [8].

Due to these processes river has its self-cleansing property.

We will consider now the case of first-order decay equation of industrial wastes. . Equation for the two processes is as follows-

$$\dot{X} = -DC^{\tau}Y + kT_C^{\tau}(S - X) \quad \text{..... (5)}$$

$$\dot{Y} = -DC^{\tau}Y \quad \text{...... (6)}$$

D= decay rate/day
k=proportional coefficient
C= correction coefficient
τ =T-20
X=D.O.
Y=B.O.D.
T_C=Temparature correction coefficient
S=Saturation concentration

RESULTS AND DISCUSSION

Taking-
D= 0.3 Day^{-1}
k= 0.1 Day^{-1}
C= 1.048Day^{-1}
τ=T-20=$(24-20)^0$C
S=8.3374 mg/L

$$\dot{X} = 0.9384 - 0.3619Y - 0.1126X \quad \text{..... (7)}$$

$$\dot{Y} = -0.3619Y \quad \text{..... (8)}$$

At Varanasi D/SMalviya Bridge
B.O.D. = 8.0 mg/l→ $Y_0 = 8.0\ mg/l$taken as initial time $t = 0$
D.O. =7.1 mg/l i.e. $X_0 = 7.1\ mg/l$taken as initial time $t = 0$

Hence solution of equation 7 and 8 is given by-

$$X = 8.3339 + 11.6133e^{-0.3619t} - 12.8472e^{-0.1126t}$$

$$Y = 8.0e^{-0.3619t}$$

At time $t = 0.1$
$X_1 = 6.8310\ mg/l$
$Y_1 = 7.7156\ mg/l$

At time $t = 0.2$
$X_2 = 6.5752\ mg/l$
$Y_2 = 7.4414\ mg/l$

Hence we have$Y_2 < Y_1 < Y_0$

i.e. there is a decrease in the concentration of B.O.D. This simply reveals self-cleansing property of the river. (Although dissolved oxygen concentration is going on decreasing instead of increasing because initially BOD load is so much high i.e. water is so much polluted that Dissolved oxygen quantity decreases. After a certain period when BOD load will come in control, dissolved oxygen will begin to increasing up to saturated value.)

$at\ t \to \infty$
$X \to 8.3339 mg/lt$

Hence concentration of pollutants and industrial waste are decreasing due to self-purification process. Hence Self-purification helps cleansing of the river.

CONCLUSION

River has its self-cleansing ability which allows assimilation and treatment of industrial waste in the river but if releasing of waste in Ganges remains unstopped then self-cleansing process will not remain effective. Where withdrawal from the river is much higher than the discharge of waste, pollution is inevitable. Actually Discharge of untreated sewage is beyond the self-purification capacity of Ganges that's why pollution is continuously going on increasing. All sewage lines from cities and surroundings areas should be connected to sewage treatment plants so that pouring sewage in Ganges can be stopped. Pollution from sewage in maximum (roughly 85%) so immediate actions towards this field may be effective for vision of cleansing Ganges.

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