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Mass Transfer Effects on MHD Viscous flow past an impulsively started infinite vertical plate with constant Mass flux

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ABSTRACT

In this paper, we study the heat and mass transfer on the unsteady visco-elastic second order Rivlin-Erickson fluid past an impulsively started infinite vertical plate in the presence of a foreign mass and constant mass flux on taking into account of viscous dissipative heat at the plate under the influence of a uniform transverse magnetic field. The flow is governed by a coupled non-linear system of partial differential equations. The velocity, the temperature, the concentration, the skin-friction and the rate of heat transfer are obtained by using finite difference method. Numerical results are graphically discussed for various values of physical parameters of interest.

Keywords: Mass Transfer effects on MHD Viscous Flow; Rivlin-Erickson Fluid; Mass flux, foreign mass, Non-linear Systems;

INTRODUCTION

Engineering processes in which a fluid supports an exothermal chemical or nuclear reaction are very common today and the correct process design requires accurate correlation for the heat transfer coefficients at the boundary surfaces. Despite its increasing importance in technological and physical problems, the unsteady MHD free convection flows of dissipative fluids past an infinite plate have received much attention because of non-linearity of the governing equations. Without taking into account viscous dissipative heat and MHD, this problem was solved by Siegal [12] by integral method. The experimental confirmations of these results were presented by Goldstein and Eckert [8]. Other papers in this field are by Gebhart [4], Schetz and Eichhorn [11], Monold and Yang [10], Sparrow and Gregg [17], Chung and Anderson [1], Goldstein and

Briggs [7], etc. In all these papers, the effect of viscous dissipative heat and MHD was assumed to be neglected. Recently, Sreekanth *et al.* [18] have studied transient MHD free convection flow of an incompressible viscous dissipative fluid and Ganesan *et al.* [2] have studied Radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving vertical cylinder. Flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate was first studied by Stokes [20]. Stewartson [19] presented analytic solution to the viscous flow past an impulsively started semi-infinite horizontal plate whereas Hall [9] solved the same problem by finite difference method. Soundalgekar [13] first presented an exact solution to the flow of a viscous incompressible fluid past an impulsively started infinite vertical plate by the Laplace transform technique. The effect of the presence of impurities is studied in scientific literature by considering it as a foreign mass. It is usually a very complicated phenomenon; however, by introducing suitable assumptions, the governing equations can be simplified. Gebhart [5] derived these simplified equations by assuming the concentration level to be low. This enabled us to neglect Soret-Dufer effects. Thus Soundalgekar [14] first studied mass-transfer effects on flow past an impulsively started infinite vertical isothermal plate. Soundalgekar *et al.* [15] have studied mass-transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux. This flow was governed by coupled linear differential equations and the solution is obtained by using Laplace transformation technique. Free convection flow with mass transfer past a semi-infinite vertical plate was presented by Gebhart and Pera [6]. Recently, Ganesan *et al.* [3] have studied Finite Difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass transfer.

It is now proposed to solve the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux, on taking into account of viscous dissipative heat under the influence of a uniform transverse magnetic field. The problem is governed by a coupled non-linear system of partial differential equations whose exact solution is not possible. So, we employ finite difference method for its solution.

Formulation and solution of the problem

Consider the flow of a viscous incompressible visco-elastic second order Rivlin-Erickson fluid past an impulsively started infinite vertical plate. The x'– axis is taken along the plate in the vertically upward direction and the y'- axis is chosen normal to the plate. Initially the temperature of the plate and the fluid T'_∞, and the species concentration at the plate C'_w and in the fluid throughout C'_∞ are assumed to be the same. At time $t' > 0$, the plate temperature is changed to T'_w causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate and the plate starts moving upward due to impulsive motion, gaining a velocity of U_0. A uniform magnetic field of intensity H_0 is applied in the y-direction. Therefore the velocity and the magnetic field are given by $\overline{q} = (u, 0, 0)$ and $\overline{H} = (0, H_0, 0)$. The flow being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow and Cess [16]) in the absence of any input electric field, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} + K_0^*\frac{\partial^3 u'}{\partial y'^2 \partial t'} - \frac{\sigma\mu_e^2 H_0^2}{\rho}u' \qquad (2.1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \tag{2.3}$$

the initial and boundary conditions are

$$\begin{aligned} & u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \quad \text{for all } y', t' \le 0 \\ & u' = U_0,\ T' = T'_w,\ \frac{dC'}{dy'} = -\frac{j''}{D} \quad \text{at } y' = 0 \\ & u' = 0,\ T' \to T'_\infty,\ C' \to C'_\infty \quad \text{as } y' \to \infty \end{aligned} \tag{2.4}$$

Where u' is the velocity of the fluid along the plate in the x'- direction, t' is the time, g is the acceleration due to gravity, β is the coefficient of volume expansion, β^* is the coefficient of thermal expansion with concentration, T' is the temperature of the fluid near the plate, T'_∞ is the temperature of the fluid far away from the plate, T'_w is the temperature of the fluid, C' is the species concentration in the fluid near the plate, C'_∞ is the species concentration in the fluid far away from the plate, j'' is the mass flux per unit area at the plate, ν is the kinematic viscosity, K_0^* is the coefficient of kinematic visco-elastic parameter, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, H_0 is the strength of applied magnetic field, ρ is the density of the fluid, C_p is the specific heat at constant pressure, K is the thermal conductivity of the fluid, μ is the viscosity of the fluid, D is the molecular diffusivity, U_0 is the velocity of the plate.

Equations (2.1)-(2.3) can be made dimensionless by introducing the following dimensionless variables and parameters:

$$u = \frac{u'}{U_0},\ t = \frac{t' U_0^2}{\nu},\ y = \frac{y' U_0}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{\left(j'' \nu / DU_0 \right)}$$

$$G = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3},\ (\text{ the Grashof number })$$

$$Gc = \frac{\nu g \beta^* \left(j'' \nu / DU_0 \right)}{U_0^3},\ (\text{ the modified Grashof number })$$

$$\lambda = \frac{K_0^* U_0^2}{\nu^2},\ (\text{ the visco - elastic parameter })$$

$$M = \frac{\sigma \mu_e^2 H_0^2 \nu}{\rho U_0^2},\ (\text{ the magnetic parameter })$$

$$\Pr = \frac{\nu \rho C_p}{K},\ (\text{ the Prandtl number })$$

$$E = \frac{\mu U_0}{\nu \rho C_p (T'_w - T'_\infty)},\ (\text{ the Eckert number })$$

$$Sc = \frac{\nu}{D},\ (\text{ the Schmidt number }) \qquad (2.5)$$

In terms of the above dimensionless quantities, Eqs. (2.1)-(2.3) reduces to

$$\frac{\partial u}{\partial t} = G \cdot \theta + Gc \cdot C + \frac{\partial^2 u}{\partial y^2} + \lambda \left(\frac{\partial^3 u}{\partial y^2 \partial t} \right) - M \cdot u \qquad (2.6)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \Pr \cdot E \left(\frac{\partial u}{\partial y} \right)^2 \qquad (2.7)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \qquad (2.8)$$

with the following initial and boundary conditions:

$$u = 0, \theta = 0, C = 0 \text{ for all } y, t \le 0$$

$$\left.\begin{array}{l} u = 1, \theta = 1, \dfrac{dC}{dy} = -1 \quad \text{at } y = 0 \\ u = 0, \theta = 0, C = 0 \quad \text{as } y \to \infty \end{array}\right\} (t > 0) \qquad (2.9)$$

Method of solution

Eqs. (2.6) - (2.8) are coupled non-linear partial differential equations, and are to be solved by using the initial and boundary conditions (2.9). However, exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference scheme of equations for (2.6)-(2.8) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} = G.\theta_{i,j} + Gc.C_{i,j} + \frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2} + \lambda\left[\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}-u_{i-1,j}+2u_{i,j}-u_{i+1,j}}{\Delta t\cdot(\Delta y)^2}\right] - Mu_{i,j} \tag{3.1}$$

$$Pr\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right) = \left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right) + Pr\cdot E\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)^2 \tag{3.2}$$

$$Sc\left(\frac{C_{i,j+1}-C_{i,j}}{(\Delta t)}\right) = \left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right) \tag{3.3}$$

Here, index i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$.

From the initial condition in (2.9), we have the following equivalent:

$$u(i,0)=0,\ \theta(i,0)=0,\ C(i,0)=0 \ \text{ for all } i \tag{3.4}$$

The boundary conditions from (2.9) are expressed in finite-difference form as follows

$$u(0,j)=1,\ \theta(0,j)=1,\ C_{i-1,j}-C_{i+1,j}=-2\cdot\Delta y \quad \text{for all } j$$

$$u(i_{max},j)=0,\ \theta(i_{max},j)=0,\ C(i_{max},j)=0 \quad \text{for all } j \tag{3.5}$$

(Here i_{max} was taken as 50)

First the velocity at the end of time step viz, u(i,j+1)(i=1,50) is computed from (3.1) in terms of velocity, temperature and concentration at points on the earlier time-step. Then $\theta(i, j+1)$ is computed from (3.2) and $C(i, j+1)$ is computed from (3.3). The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was chosen as 0.001.

To judge the accuracy of the convergence and stability of finite difference scheme, the same program was run with different values of Δt i.e., $\Delta t = 0.0009, 0.0001$ and no significant change was observed. Hence, we conclude that the finite-difference scheme is stable and convergent.

Skin-friction:

We now calculate Skin-friction from the velocity field. It is given in non-dimensional form as: $\tau = -\left(\frac{du}{dy}\right)_{y=0}$, where $\tau = \frac{\tau'}{\rho U_0^2}$ (3.6)

Numerical values of τ are calculated by applying the Newton's forward interpolation formula for five points.

Rate of heat transfer:

The dimensionless rate of heat transfer is given by $Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$ (3.7)

We have computed Nu using the same procedure as in Skin-friction.

Graphs:

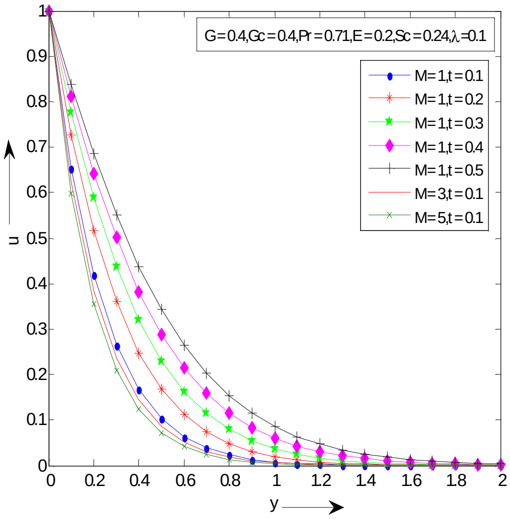

Fig. 1. u against y for different M and time t

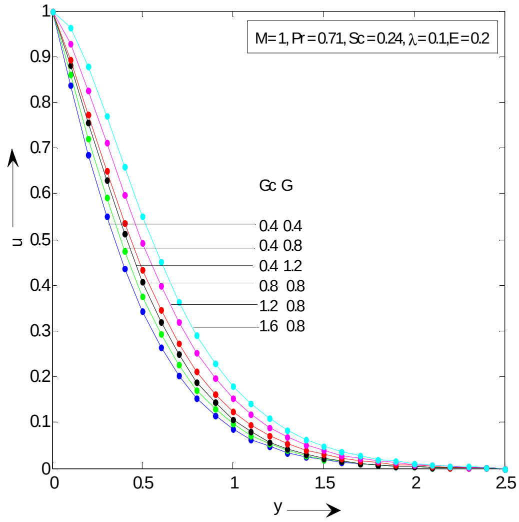

Fig. 2. u against y for different Gc and G

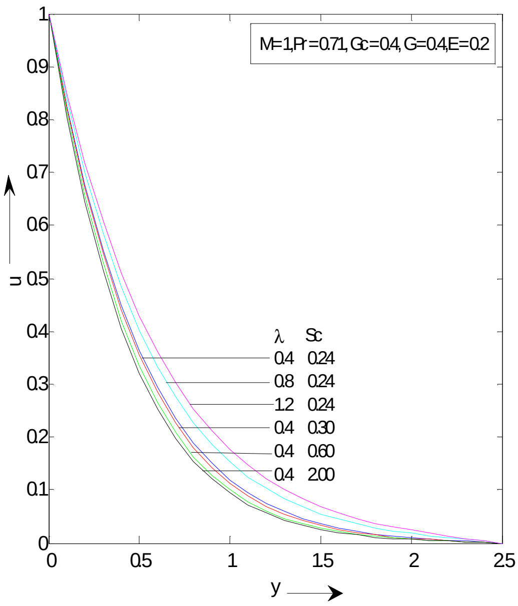

Fig. 3. u against y for different λ and Sc

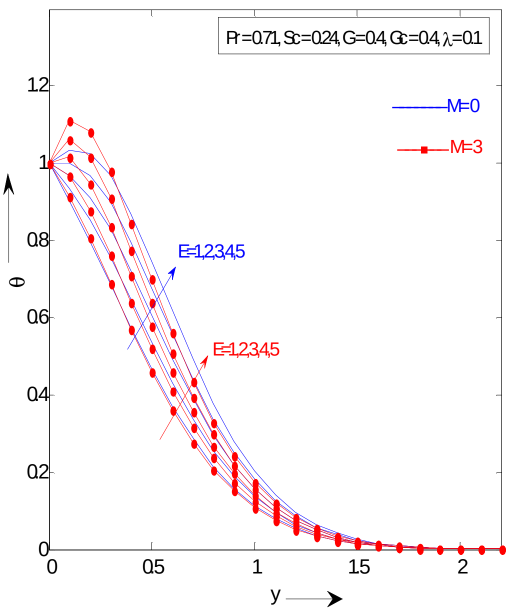

Fig. 4. T against y for different E

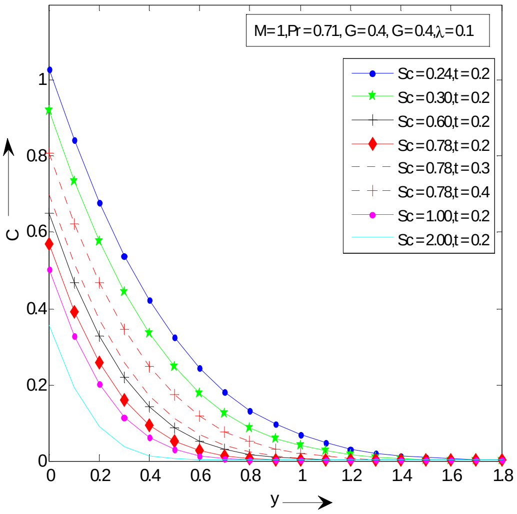

Fig. 5. C against y for different Sc and time t

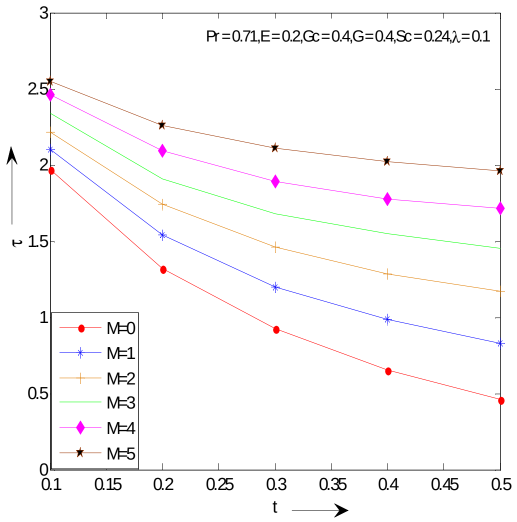

Fig. 6. Skin-friction τ against t for Different M

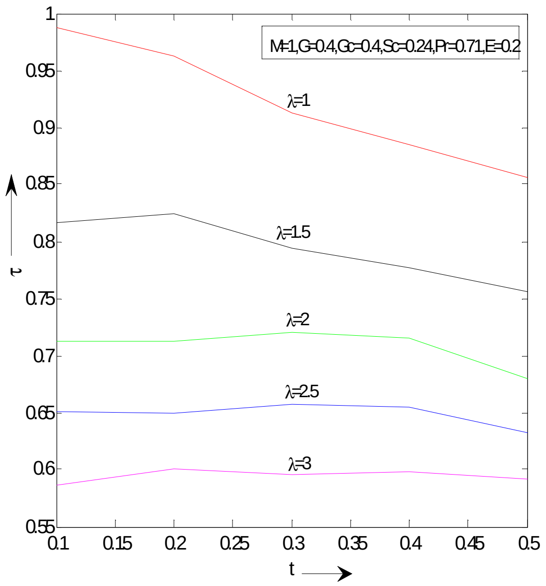

Fig.7.Skin-friction τ against t for different λ

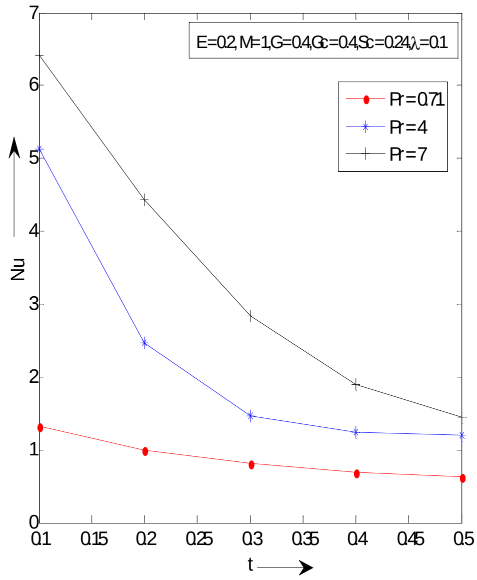

Fig. 8.Rate of heat transfer Nu against t for different Pr

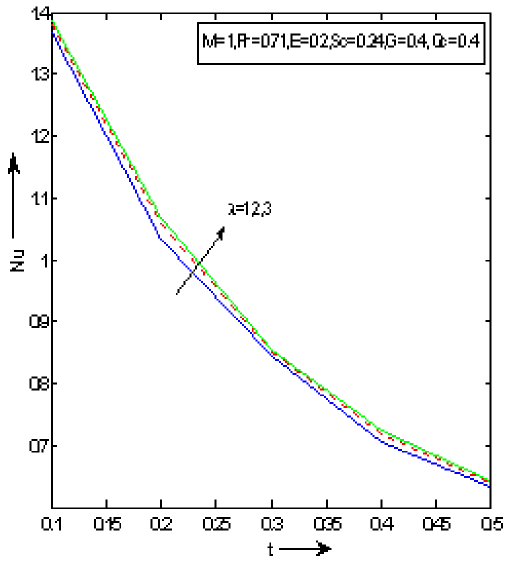

Fig. 9. Rate of heat transfer Nu against t for different λ

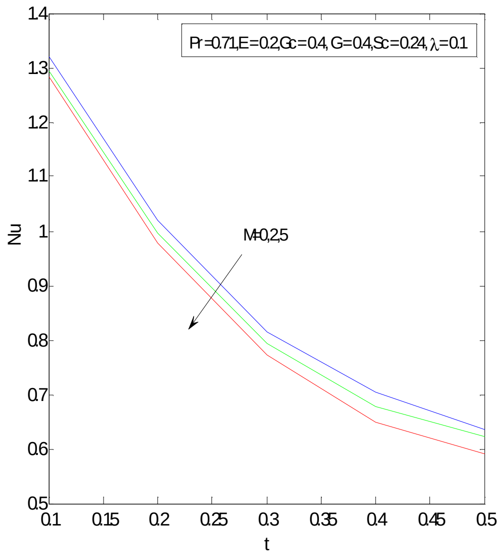

Fig. 10. Rate of heat transfer Nu against t for different M

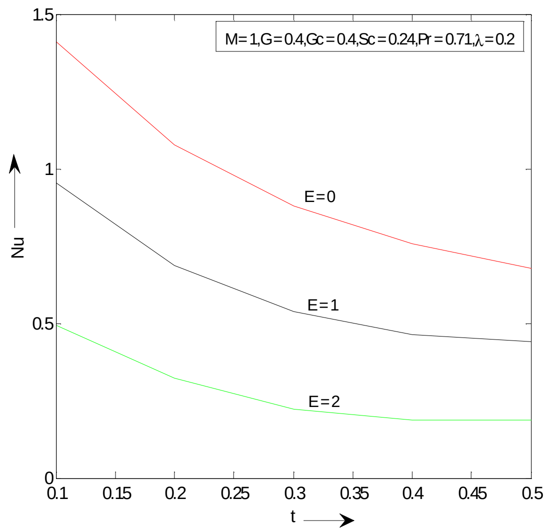

Fig. 11.Rate of heat transfer Nu against t for different E

CONCLUSION

From Fig. 1, we observe that the velocity distribution u is drawn against y for different values of magnetic parameter M and time t for Prandtl number Pr = 0.71(air). We notice that the velocity distribution u increases with the increase in t whereas u decreases with the increase in M. From Fig. 2, u is drawn against y for different values of modified Grashof number Gc and Grashof number G. We notice that the u increases as Gc or G increases.

Table 1. Values of the Schmidt number Sc

Pr	Species	Sc
0.71	Hydrogen (H)	0.24
	Helium (He)	0.30
	Water (H_20)	0.60
	Ammonia (NH_3)	0.78
	Carbon dioxide (CO_2)	1.00
	Ethyl benzene ($C_6H_5CH_2CH_3$)	2.00

From Fig. 3, u is drawn against y for different values of visco-elastic parameter λ and Schmidt number Sc. Here u increases with the increase of λ whereas the trend reverses with the increase of Sc. Fig. 4. is drawn for Temperature θ against y for different values of Eckert number E. Here we discussed with and without magnetic field. In both cases as E increases T also increases. From Fig. 5. we observe that an increase in Sc leads to decrease in the Concentration profiles C, but C increases with time t. Fig. 6. and Fig. 7. is drawn for Skin-friction τ is drawn against t for different values of M and λ. From these we observe that τ increases with the increase of M but decreases with the increase of λ for Pr=0.71. In Fig. 8, 9,10. and 11. the rate of heat transfer Nu is drawn against t for different Pr, λ, M and E. From Fig 8. and Fig 9. we observe that Nu increases with the increase of Pr and λ whereas from Fig. 10. and Fig. 11. Nu decreases with the increase of M and E for Pr = 0.71.

REFERENCES

[1] Chung, P.M. and Anderson, A.D.: *Trans. ASME J. Heat Transfer* 83c (**1961**) 473-78.
[2] Ganesan, P. and Loganathan, P.: *Int. J. Heat and Mass Transfer* 45 (**2002**) 4281-4288
[3] Ganesan, P. and Palani, G.: *Int. J. Heat and Mass Transfer*, 47 (**2004**), *4449-4457*
[4] Gebhart, B.: *Trans. ASME J. Heat Transfer* 83c (**1961**) 61-70.
[5] Gebhart, B.: *Heat Transfer, New York: McGraw-Hill* **1971**.
[6] Gebhart, B. and Pera, L.: *Int. J. Heat Mass Transfer* 14 (**1971**) 2025.
[7] Goldstein, R.J. and Briggs, D.G.: *Trans. ASME J. Heat Transfer* 86c (**1964**) 490-500.
[8] Goldstein, R.J. and Eckert, E.C.G.: *Int. J. Heat Mass Transfer,* 1 (**1960**) 208-18.
[9] Hall, M.G.: *Proc.Roy. Soc., London* 310A (**1969**) 401.
[10] Menold, E.R. and Yang, K.T.: *Trans. ASME J. Appl. Mech.* 29E (**1962**) 124-26.
[11] Schetz, J.A. and Eichhorn, R.: *Trans. ASME J. Heat Transfer* 84 (**1962**) 334-38.
[12] Siegel, R.:*Trans. Amer. Soc. Mech. Engng* 80 (**1958**) 347-59.
[13] Soundalgekar, V.M.: *Trans. ASME J. Heat Transfer* 99c (**1977**) 499.
[14] Soundalgekar, V.M.: *Trans. ASME J. Appl. Mech.* 46 (**1979**) 757.
[15] Soundalgekar, V.M. Das, U.N. and Ray, S.N.: *Int. J. Heat Mass Transfer* 31 (**1996**) 163-67.
[16] Sparrow, E.M. and Cess, R.D.: *Trans. ASME J. Appl. Mech.* 291 (**1962**) 181-86.
[17] Sparrow, E.M. and Gregg, J.L.: *Trans. ASME J. Heat Transfer* 82c (**1960**) 258-60.
[18] Sreekanth, S. Nagarajan, A.S. and Venkataramana, S: *Indian J.pure appl. Math.,* 32(7), (**2001**) 1051-58.
[19] Stewartson, K.: *Quart. J. Appl. Math. Mech 4* (**1951**).
[20] Stokes, G.G.: Camb. Phil. Trans. IX 8 (**1851**).