

Mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink through porous media

***¹M. Jayabharath Reddy, ²Jayarami Reddy and ³G. Sivaiah**

¹Dept. of Mathematics, OXFORD Degree College, Srikalahasthi, Chittoor Dist. A.P.(India)

²Dept. of Mathematics, Priyadarsini Institute of Technology, Tirupati, Chittoor Dist, A.P.(India)

³Dept. of Mathematics, Govt. Degree College, Jammalamadugu, Kadapa Dist., A.P.(India)

ABSTRACT

The aim of the present work is to study the influence of mass transfer on unsteady hydromagnetic free convective memory flow of incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink through porous medium. Approximate solutions have been derived for the mean velocity, mean temperature and mean concentration using multi-parameter perturbation technique and these are presented in graphical form. The effects of different physical parameters such as magnetic parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number, Eckert number and heat sink strength parameter are discussed.

Keywords: MHD, Viscous dissipation, Heat sink, Porous media, Suction.

INTRODUCTION

Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing and environmental pollution.

Viscoelastic flows arise in numerous processes in chemical engineering systems. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects. An extensive range of mathematical models has been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. An eloquent exposition of viscoelastic fluid models has been presented by Joseph [1]. Examples of such models are the Oldroyd model [2], Johnson-Seagalman model [3], the upper convected Maxwell model [4], and the Walters-B model [5]. Both steady and unsteady flows have been investigated at length in a diverse range of geometries using a wide spectrum of analytical and computational methods. Siddappa and Khapate [6] studied the second order Rivlin-Ericksen viscoelastic boundary layer flow along a stretching surface. Rochelle and Peddieson [7] used an implicit difference scheme to analyze the steady boundary-layer flow of a nonlinear Maxwell viscoelastic fluid past a parabola and a paraboloid. Ji et al. [8] studied the Von Karman Oldroyd-B viscoelastic flow from a rotating disk using the Galerkin method with B-spline test functions. Rao and Finlayson [9] used an adaptive finite element technique to analyze viscoelastic flow of a Maxwell fluid.

Ramanamurthy et al. [10] have discussed the MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Mustafa et al. [11] obtained the analytical solution of unsteady MHD memory flow with oscillatory suction, variable free stream and heat source. Numerical study of transient free convective mass transfer

in a Walters-B viscoelastic flow with wall suction was analysed by Chang et al. [12]. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [13]. Gireesh Kumar et al. [14] analyzed the effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink.

The present study is to study the mass transfer effects on unsteady hydromagnetic free convective memory flow of incompressible and electrically conducting fluid flow an infinite vertical plate through porous medium. Our main interest is to observe how various parameters affect the flow past an infinite vertical accelerated plate.

2. FORMULATION OF THE PROBLEM

Consider unsteady hydromagnetic free convective flow of incompressible and electrically conducting fluid past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink through porous media. Let the x-axis be taken in the vertically upward direction along the infinite vertical plate and y-axis normal to it. Boussineq's approximation, for the equations of the flow is governed as:

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + v \frac{\partial^2 u}{\partial y^2} - B_1 \left(\frac{\partial^3 u}{\partial t \partial y^3} + v \frac{\partial^3 u}{\partial y^3} \right) - \sigma B_0^2 \frac{u}{\rho} - \frac{vu}{k} \quad (2)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + S(T - T_\infty) + \frac{v}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

From (1) we have,

$$v = -v_0 \quad (5)$$

On disregarding the Joulean heat dissipation, the boundary conditions of the problem are:

$$\left. \begin{aligned} u = 0, \quad v = -v_0, \quad T = T_w + \varepsilon(T_w - T_\infty)e^{i\omega t}, \quad C = C_w + \varepsilon(C_w - C_\infty)e^{i\omega t} : y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty : y \rightarrow \infty \end{aligned} \right\} \quad (6)$$

Introducing the non-dimensional quantities and parameters:

$$\left. \begin{aligned} u^* = \frac{u}{v_0}, \quad t^* = \frac{tv_0^2}{4\nu}, \quad y^* = \frac{yv_0}{\nu}, \quad \omega^* = \frac{4\nu\omega}{v_0^2}, \quad T = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C_\infty}{C_w' - C_\infty}, \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{v_0^3}, \quad Gc = \frac{g\beta^*\nu(C_w' - C_\infty)}{v_0^3}, \quad Pr = \frac{\nu}{k}, \quad K = \frac{K_0}{\rho C_p}, \quad \nu = \frac{\eta_0}{\rho} \\ Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \quad Ec = \frac{u_0^2}{C_p(T_w - T_\infty)}, \quad R_m = \frac{B_1 v_0^2}{\nu^2}, \quad S^* = \frac{4S\nu}{v_0^2}, \quad k = \frac{k^* v_0^2}{\nu^2} \end{aligned} \right\} \quad (7)$$

Where ρ , κ , C_p , Pr , Gr , Gc , S , Sc , Ec , M and R_m are acceleration due to density, thermal conductivity specific heat at constant pressure, Prandtl number, Grashof number, Solutal Grashof number, sink strength, Schmidt number, Eckert number, Hartmann number and Reynolds number respectively.

Using (7), (6) and (5), equations (2), (3) and (4) become

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GrT - GcC - \frac{\partial^2 u}{\partial y^2} - R_m \left(\frac{1}{4} \frac{\partial^3 u}{\partial t \partial y^3} - \frac{\partial^3 u}{\partial y^3} \right) - \left(M + \frac{1}{k} \right) u \quad (8)$$

$$\frac{\text{Pr}}{4} \frac{\partial T}{\partial t} - \text{Pr} \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \text{Pr} \frac{ST}{4} + \text{Pr} Ec \left(\frac{\partial u}{\partial y} \right)^2 \quad (9)$$

$$\frac{Sc}{4} \frac{\partial C}{\partial t} - Sc \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \quad (10)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} u = 0, \quad T = 1 + \varepsilon e^{i\omega t}, \quad C = 1 + \varepsilon e^{i\omega t} : y = 0 \\ u \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0, \quad y \rightarrow \infty \end{aligned} \right\} \quad (11)$$

To solve equations (8), (9) and (10), we assume ω to be very small and the velocity, temperature and concentration in the neighbourhood of the plate as

$$\left. \begin{aligned} u(y, t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) \\ T(y, t) &= T_0(y) + \varepsilon e^{i\omega t} T_1(y) \\ C(y, t) &= C_0(y) + \varepsilon e^{i\omega t} C_1(y) \end{aligned} \right\} \quad (12)$$

Where u_0 , T_0 and C_0 are mean velocity, mean temperature and mean concentration respectively.

Using (12) in equations (8), (9) and (10), equating harmonic and non-harmonic terms for mean velocity, mean temperature and mean concentration, after neglecting coefficient of ε^2 , we get

$$R_m u_0^{111} + u_0^{11} + u_0^1 - \left(M + \frac{1}{k} \right) u_0 = -[Gr T_0 + Gc C_0] \quad (13)$$

$$T_0^{11} + \text{Pr} T_0^1 + \frac{\text{Pr} S T_0}{4} = -\text{Pr} Ec (u_0^1)^2 \quad (14)$$

$$C_0^{11} + Sc C_0^1 = 0 \quad (15)$$

The equation (13) is third order differential equation due to presence of elasticity. Therefore u_0 is expanded using (Beard and Walters rule, 1964).

$$u_0 = u_{00} + R_m u_{01} \quad (16)$$

Zero-Order of R_m

$$u_{00}^{11} + u_{00}^1 - \left(M + \frac{1}{k} \right) u_{00} = -[Gr T_0 + Gc C_0] \quad (17)$$

First-Order of R_m

$$u_{01}^{11} + u_{01}^1 - \left(M + \frac{1}{k} \right) u_{01} = -u_{00}^{111} \quad (18)$$

Using multi parameter perturbation technique and assuming $Ec \ll 1$, we write

$$u_{00} = u_{000} + Ec u_{001} \quad (19)$$

$$u_{01} = u_{011} + Ec u_{012} \quad (20)$$

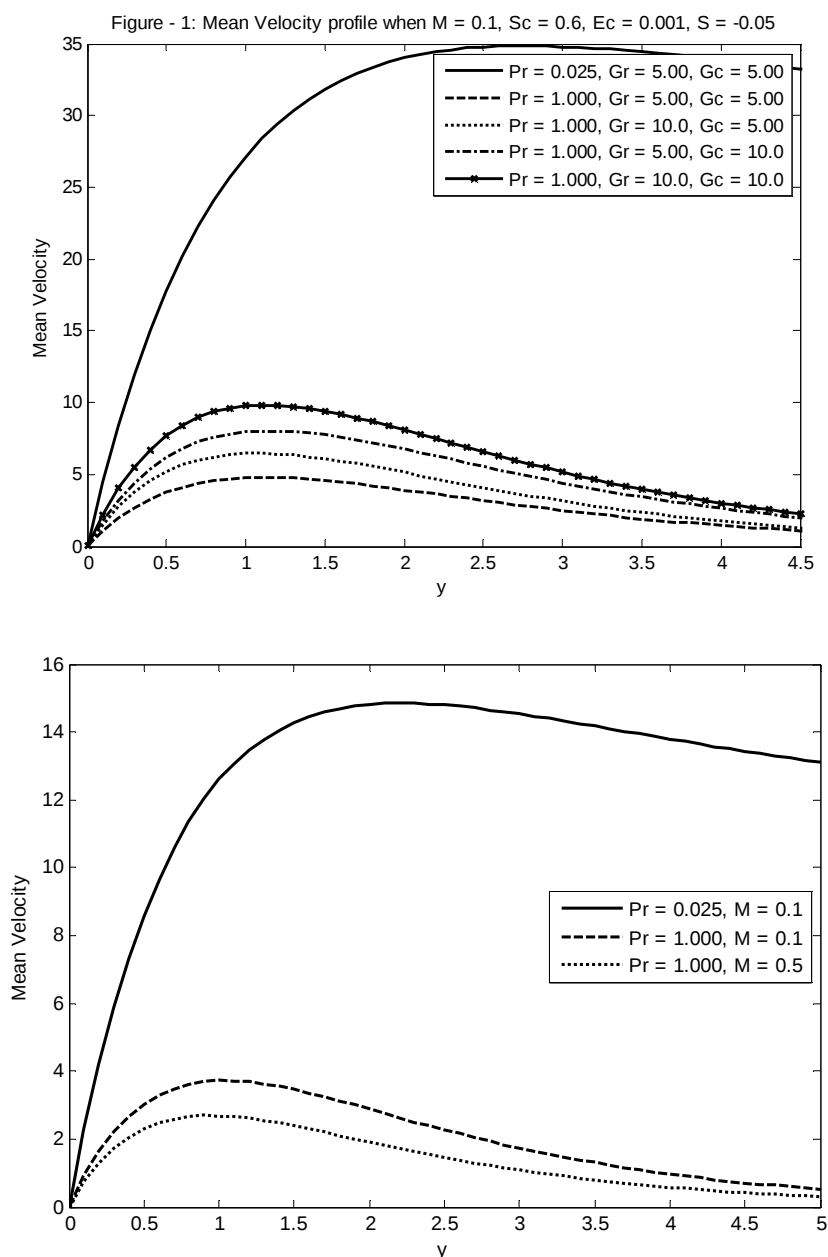
$$T_0 = T_{00} + Ec T_{01} \quad (21)$$

$$C_0 = C_{00} + Ec C_{01} \quad (22)$$

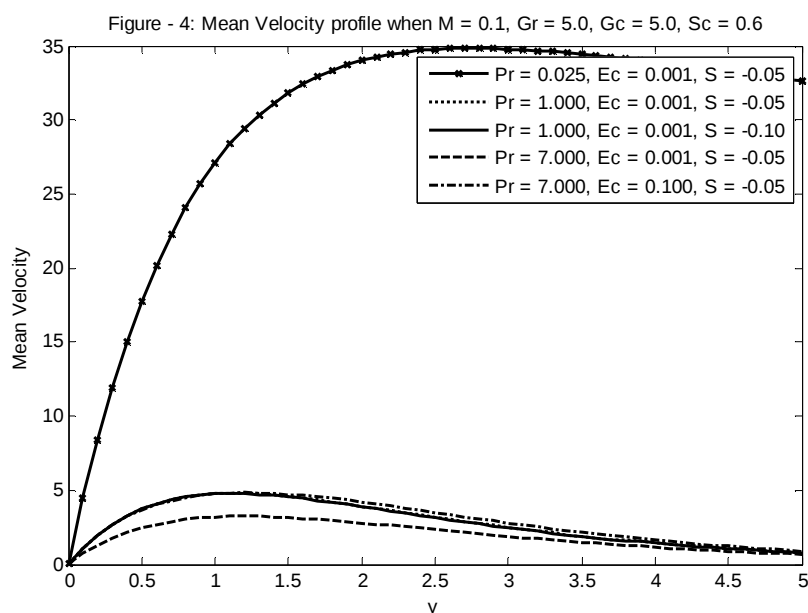
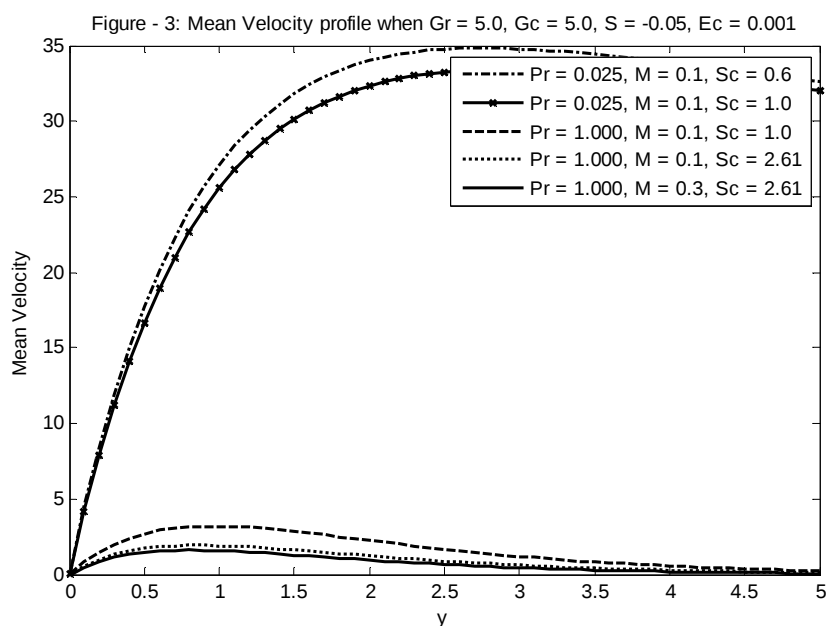
Using equations (19) – (22) in equations (14), (15), (17) and (18) and equating the coefficient of Ec^0 and Ec^1 , we get the differential equations

RESULTS AND DISCUSSION

The problem of mass transfer on unsteady hydromagnetic free convective memory flow of incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink has been formulated, analysed and solved by using multi-parameter perturbation technique. Approximate solutions have been derived for the mean velocity, mean temperature and mean concentration. The effects of the flow parameters such as magnetic parameter (M), suction parameter (S), Grashof number for heat and mass transfer (Gr , Gc), Schmidt number (Sc), Prandtl number (Pr) and Eckert number (Ec) on the mean velocity, mean temperature and mean concentration profiles of the flow field are presented with help of mean velocity profiles (Figures 1-4), mean temperature profiles (Figures 5-7).



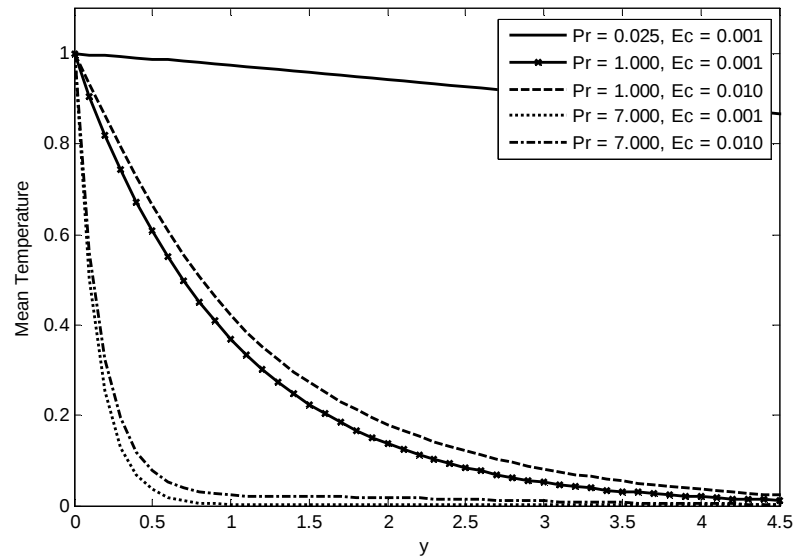
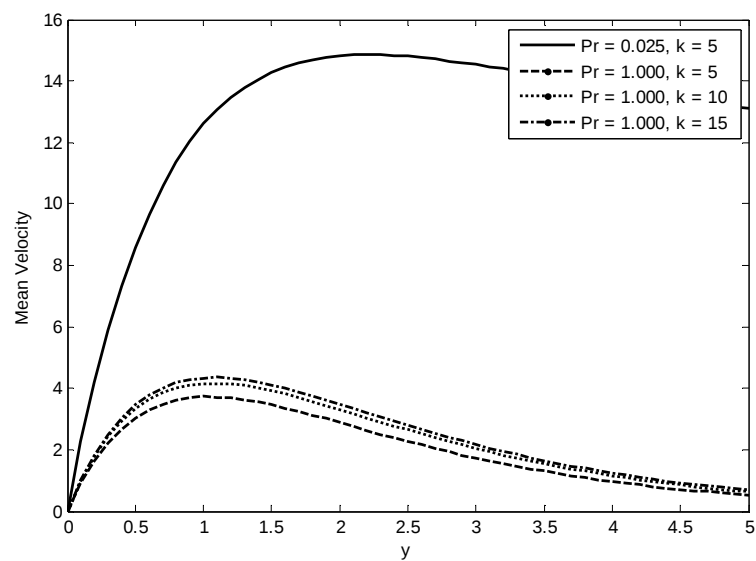
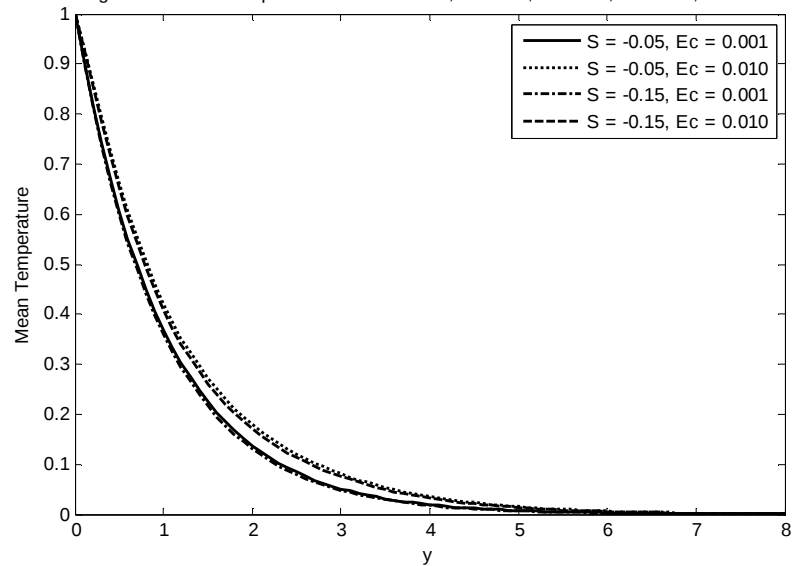
Figures 1 and 2 represent the mean velocity profiles due variations in Gr (Thermal Grashof number), Gc (Solutal Grashof number), M (Magnetic parameter) and S (Sink strength parameter). It is observed that the mean velocity increases with increase of thermal Grashof number and solutal Grashof number. It also observed that mean velocity decrease with increase in magnetic parameter and sink strength parameter.



Figures 3 and 4 reveals the mean velocity profiles due to variations in Sc (Schmidt number), Pr (Prandtl number) and Ec (Eckert number). It is noticed that whenever Schmidt number increases the mean velocity decrease. It is also observed that the increases in Prandtl number and Eckert number causes the decrease in mean velocity.

Figures 5 and 6 reveals the mean temperature profiles due to variations in Pr (Prandtl number) and Ec (Eckert number) and S (Sink strength parameter). It is noticed that whenever Prandtl number and Eckert number increases the mean temperature decrease. It is also observed that the increases in Sink strength parameter and Eckert number causes the decrease in mean temperature.

Figure 7 depicts the mean velocity profile due to the variations in k (porous parameter). It is noticed that whenever porous parameter increases the velocity also increases.

Figure - 5: Mean Temperature profile when $M = 0.1$, $Gr = 5.0$, $Gc = 5.0$, $S = -0.05$ Figure - 6: Mean Temperature when $M = 0.1$, $Pr = 1.0$, $Gr = 5.0$, $Gc = 5.0$, $Sc = 0.6$ 

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