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Mass transfer effects on mhd flows exponentially accelerated isothermal vertical plate in the presence of chemical reaction through porous media

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ABSTRACT

A finite difference solution of mass transfer effects on MHD flow of incompressible viscous dissipative fluid past an exponentially accelerated isothermal vertical plate, on taking into account of viscous dissipative heat, under the influence of chemical reaction through porous medium is evaluated. The velocity, temperature and concentration are studied for different parameters such as the magnetic field parameter, Grashof number, mass Grashof number, chemical reaction parameter, Schmidt number, Prandtl number, permeability parameter. The numerical values of Skin-friction are in tabulated.

Key words: MHD, chemical reaction, porous medium, viscous dissipation etc.

INTRODUCTION

Simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes, solidification of binary alloy and chemical engineering.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous process. This depends on whether they occur at an interface or as a single phase volume reaction. One of the simplest chemical reactions is the first order reaction in which the rate of reaction is directly proportional to the species concentration. Muthucumaraswamy [4] studied the effect of a chemical reaction on a moving isothermal vertical surface with suction. Mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction were studied by Muthucumaraswamy and Janakiraman [5].

The effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction were studied by Das et al. [2]. Anjali Devi and Kandasamy [1] studied the steady laminar flow along a semi-infinite horizontal plate in the presence of species concentration and chemical reaction. Mass transfer effect on exponentially accelerated isothermal vertical plate was discussed by Muthucumaraswamy et al. [6]. Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature was studied by Rajesh and Vijaya Kumar Varma [7]. Radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation was studied by Sudheer Babu et al. [8]

Magneto convection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting. Ramachandra Prasad and Bhaskar Reddy [9] studied the Radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation. MHD Unsteady free convective heat and mass transfer flow past a vertical porous plate with viscous dissipation was studied by Rushikumar [10]. Mansour et al. [3] analyzed the effect of chemical reaction and viscous dissipation on MHD natural convection flow saturated in porous media with suction or injection. Recently the effects of heat transfer on MHD unsteady free convection flow pat an infinite/semi infinite vertical plate was analyzed by [12-16].

The aim of present paper is to study an unsteady free convection flow of a viscous, incompressible, Newtonian, electrically conducting and chemically reacting fluid past an exponentially accelerated infinite isothermal vertical plate in the presence of variable mass diffusion and taking into account of viscous dissipation heat, under the influence of transverse magnetic field. The dimensionless governing equations of momentum, energy and diffusion, which govern the flow field, are solved by using Crank-Nicolson method. The behavior of the velocity, temperature, concentration and skin-friction has been discussed for various parameters of governing equations.

FORMULATION OF THE PROBLEM:
The two-dimensional unsteady, chemically reacting and electrically conducting flow of a viscous incompressible fluid past an exponentially accelerated isothermal infinite vertical plate with variable mass diffusion has been considered. The $x^{'}$ - axis is taken along the plate in the vertically upward direction and the $y^{'}$ - axis is taken normal to the plate. At time $t^{'} \leq 0$, the plate and fluid are at the same temperature $T_{\infty}^{'}$ with concentration level $C_{\infty}^{'}$. At time $t^{'} > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp\left(a^{'}t^{'}\right)$ in its own plane and the temperature from the plate is raised to $T_w^{'}$ and the mass is diffused from the plate to the fluid at a uniform rate. A magnetic field of uniform strength is applied perpendicular to the plate and the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

Momentum equation

$$\frac{\partial u^{'}}{\partial t^{'}} = g\beta\left(T^{'} - T_{\infty}^{'}\right) + g\beta^{*}\left(C^{'} - C_{\infty}^{'}\right) - v\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma B_0^2}{\rho}u^{'} - \frac{v}{k^{'}}u^{'} \tag{1}$$

Energy equation

$$\frac{\partial T^{'}}{\partial t^{'}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \frac{\upsilon}{C_p}\left(\frac{\partial u^{'}}{\partial y^{'}}\right)^2 \tag{2}$$

Diffusion equation

$$\frac{\partial C^{'}}{\partial t^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} - K_l\left(C^{'} - C_{\infty}^{'}\right) \tag{3}$$

Where $x^{'}$, $y^{'}$ and $t^{'}$ are the dimensional distances along and perpendicular to the plate and dimensional time respectively, $u^{'}$ and $v^{'}$ are the components of dimensional velocities along $x^{'}$ and $y^{'}$ directions respectively. $T^{'}$ and $C^{'}$ are the dimensional temperature and concentration, ρ is the fluid density, v is the kinematic viscosity, C_p is the specific heat at constant pressure, σ is the electrical conductivity of fluid, g is the acceleration due to gravity, β and β^{*} are the thermal and concentration expansion coefficients, $k^{'}$ is the permeability of the porous medium, B_0 is the magnetic induction, D is the chemical molecular diffusivity, K_l is the chemical reaction parameter and κ is the fluid thermal conductivity. The first and second terms on the right hand side of the momentum equation (2) denote the thermal and concentration buoyancy effects respectively.

With the following initial and boundary conditions:

$$\left.\begin{array}{llll} u' = 0, & T' = T'_\infty, \quad C' = C'_\infty & forall & y', t' \le 0 \\ t' > 0: \; u' = u_0 \exp(a't'), & T' = T'_w, \quad C' = C'_\infty + (C'_w - C'_\infty)At' & at & y' = 0 \\ u' \to 0, & T' \to T'_\infty, \quad C' \to C'_\infty & as & y' \to 0 \end{array}\right\} \tag{4}$$

where $A = \dfrac{u_0^2}{\nu}$

On introducing the following non –dimensional quantities:

$$\left.\begin{array}{l} U = \dfrac{u}{u_0}, \; t = \dfrac{t' u_0^2}{\nu}, \; Y = \dfrac{y u_0}{\nu}, \; \theta = \dfrac{T - T_\infty}{T_w - T_\infty}, \; C = \dfrac{C' - C'_\infty}{C'_w - C'_\infty}, \; k = \dfrac{u_0^2 k'}{\nu^2} \\ Gr = \dfrac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \; Gc = \dfrac{g\beta^* \nu(C'_w - C'_\infty)}{u_0^3}, \; \Pr = \dfrac{\mu C_p}{k} \\ a = \dfrac{a'\nu}{u_0^3}, \; Sc = \dfrac{\nu}{D}, \; M = \dfrac{\sigma B_0^2 \nu}{\rho u_0^3}, \; Ec = \dfrac{u_0^2}{C_p (T_w - T_\infty)}, Kr = \dfrac{\nu K_l}{u_0^2} \end{array}\right\} \tag{5}$$

In view of (5) equations (1) to (3) reduce to the non-dimensional forms

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Y^2} + Gr\,\theta + Gc\,C - \left(M + \frac{1}{k}\right)U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} + Ec\left(\frac{\partial U}{\partial Y}\right)^2 \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - Kr\,C \tag{8}$$

with the following initial and boundary conditions in non-dimensional quantities are:

$$\left.\begin{array}{lllll} U(Y,t) = 0, & \theta(Y,t) = 0, & C(Y,t) = 0 & for\ all & Y, t \le 0 \\ t > 0: U(Y,t) = \exp(at), & \theta(Y,t) = 1, & C(Y,t) = t & at & Y = 0 \\ U(Y,t) \to 0, & \theta(Y,t) \to 0, & C(Y,t) \to 0 & as & Y \to \infty \end{array}\right\} \tag{9}$$

METHOD OF SOLUTION

Numerical Technique: Equations (6) – (8) are coupled non-linear partial differential equations and are to be solved under the initial and boundary conditions of equations (9). However exact or approximate solutions are not possible for this set of equations and hence we use finite difference technique of implicit type namely Crank-Nicolson implicit finite difference method which is always convergent and stable. The finite difference equations corresponding to equations (9) are as follows:

$$\begin{aligned} \left(\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right) &= \frac{1}{2}\left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2(\Delta Y)^2}\right) + \\ & Gr\left(\frac{\theta_{i,j+1} + \theta_{i,j}}{2}\right) + Gc\left(\frac{C_{i,j+1} + C_{i,j}}{2}\right) - \left(M + \frac{1}{k}\right)\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \end{aligned} \tag{10}$$

$$\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right)=\frac{1}{2\Pr}\left(\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}+\theta_{i+1,j+1}-2\theta_{i,j+1}+\theta_{i-1,j+1}}{2(\Delta Y)^2}\right)+Ec\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta Y}\right)^2 \quad (11)$$

$$\left(\frac{C_{i,j+1}-C_{i,j}}{\Delta t}\right)=\frac{1}{2Sc}\left(\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}+C_{i+1,j+1}-2C_{i,j+1}+C_{i-1,j+1}}{2(\Delta Y)^2}\right)-Kr\left(\frac{C_{i,j+1}+C_{i,j}}{2}\right) \quad (12)$$

$$\left.\begin{array}{llll} U_{i,0}=0, & \theta_{i,0}=0, & C_{i,0}=0 & for\ all\ i \\ U_{0,j}=\exp(aj\Delta t), & \theta_{0,j}=1, & C_{0,j}=j\Delta t & \\ U_{L,j}\to 0, & \theta_{L,j}\to 0, & C_{L,j}\to 0 & \end{array}\right\} \quad (13)$$

Here, the suffix i corresponds to Y, j corresponds to t and $L\to\infty$. Also $\Delta t=t_{j+1}-t_j$ and $\Delta Y=Y_{i+1}-Y_i$ knowing the values of C, θ and u at a time t, we can calculate the values at a time $t+\Delta t$ as follows. We substitute i = 1, 2, 3 …41 in equation (12) which results in a tridiagonal system of equations in unknown values of C. Using initial and boundary conditions, the system can be solved by Thomas algorithm as discussed in Carnahan et al. [4]. Thus C is known at all values of Y at time $t+\Delta t$. Then knowing the values of C and applying the same procedure and using boundary conditions we calculate θ from equation (11). Then knowing the values of θ and applying the same procedure and using boundary conditions, we calculate u from equation (10). This procedure is continued to obtain the solution till desired time t. In order to check the accuracy these results are computed with usual explicit finite difference technique and the results computed from both explicit and implicit method are found to agree well.

Skin – Friction:
We now calculate the skin-friction from the velocity field, which is given in non dimensional form as

$$\tau=-\left(\frac{du}{dy}\right)_{y=0}=\frac{\tau'}{\rho u_0^2} \quad \text{where } \tau'=-\mu\left(\frac{\partial u'}{\partial y'}\right)_{y'=0}$$

Numerical values of t are calculated by applying the Newton's Interpolation formula for four points.

Nusselt number:
From the temperature field, we now study the rate of heat transfer, which is given in non – dimensional form as

$$q=-\left(\frac{d\theta}{dy}\right)_{y=0}$$

and the numerical values of q are listed in table 2.

Sherwood number:
From the concentration field, we now study the rate of mass transfer which is given as

$$Sh=-\frac{1}{C(0)}\left(\frac{dC}{dy}\right)_{y=0}=+\frac{1}{C(0)}$$

and the numerical values of Sh are listed in table 3.

Table – 1: Values of τ

t/M	1	5	10	15
0.1	4.40807	4.82864	5.31386	5.76150
0.2	4.43375	4.90064	5.43215	5.91790
0.3	4.54327	5.04476	5.61381	6.13253
0.4	4.687413	5.222678	5.828848	6.38050

Table – 2: Values of q

t/Pr	0.71	1	7
0.1	1.510258	1.790929	4.680644
0.2	1.125072	1.280887	3.369132
0.3	1.031135	1.104625	2.743608
0.4	1.00776	1.039073	2.371092

Table – 3: Values of Sh

t/Sc	0.6	0.78	1	2.62
0.1	0.02783	0.031719	0.035895	0.057891
0.2	0.04005	0.045321	0.051165	0.082576
0.3	0.05079	0.056684	0.063478	0.101862
0.4	0.06125	0.067432	0.074740	0.118437

RESULTS AND DISCUSSION

In the preceding sections, the problem of an unsteady free convective flow of a viscous, incompressible, chemically reacting fluid past an exponentially accelerated isothermal infinite vertical plate was formulated and solved by finite difference scheme. The effects of different variables like thermal Grashof number (Gr = 5), the solutal Grashof number (Gc = 5), Prandtl number (Pr = 0.71 (air) and Pr = 7 (water)), Schmidt number (Sc = 0.6), Eckert number (E = 0.01), the chemical reaction parameter (Kr = 0.2), the magnetic parameter (M = 5) and the permeability parameter (k = 1).

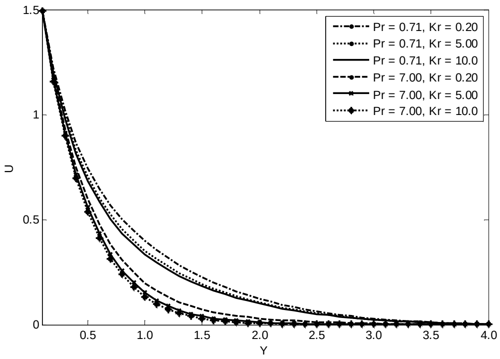

Figure – 1(a)

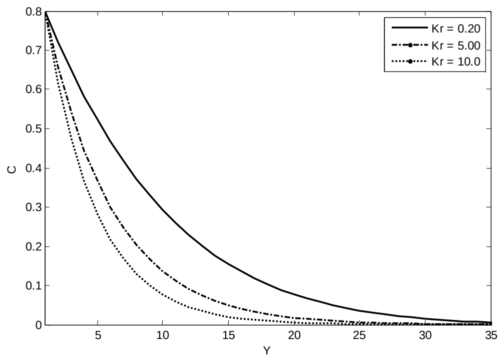

Figure – 1(b)

The velocity and concentration profiles for different values of the chemical reaction parameter (Kr = 0.2, 5.0, 10.0) Gr = 5, Gc = 5, Pr = 7, Sc = 0.6, M = 5, k = 1, E = 0.01 are revealed in the Figures 1(a) and 1(b). It is obvious that an increase in Kr leads to decrease in both the values of velocity and concentration.

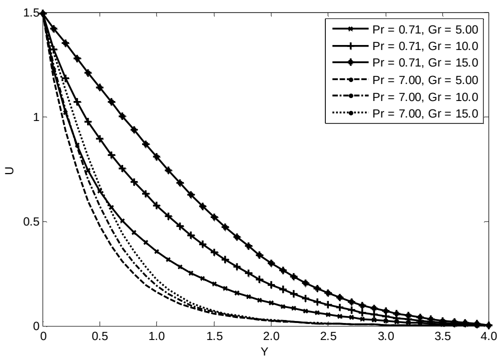

Figure – 2(a)

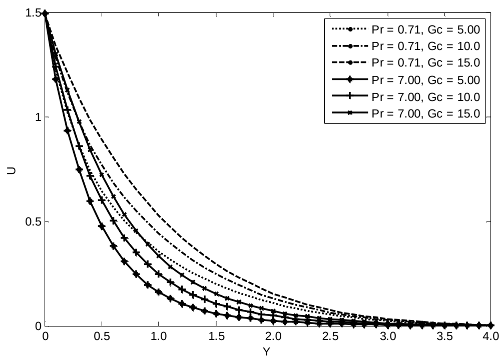

Figure – 2(b)

The velocity profiles for different values of the thermal Grashof number (Gr = 5, 10, 15) Kr = 0.2, Gc = 5, Pr = 0.71 and Pr = 7, Sc = 0.6, M = 5, k = 1, E = 0.01 are described in Figure 2 (a). It is observed that an increase in Gr leads to arise in the values of velocity. For the case of different values of the solutal Grashof number (Gc = 5, 10, 15) Kr = 0.2, Gr = 5, Pr = 0.71 and Pr = 7, Sc = 0.6, M = 5, k = 1, E = 0.01 the velocity profiles are displayed in the Figure 2(b). It is found that an increase in Gr leads to a rise in the values of velocity.

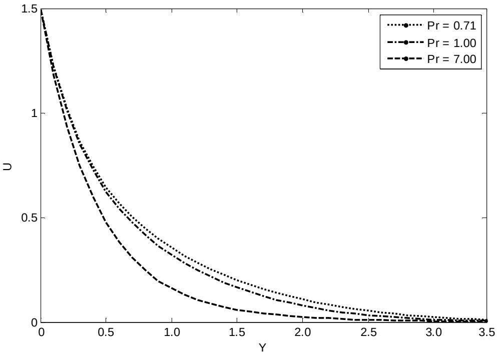

Figure – 3(a)

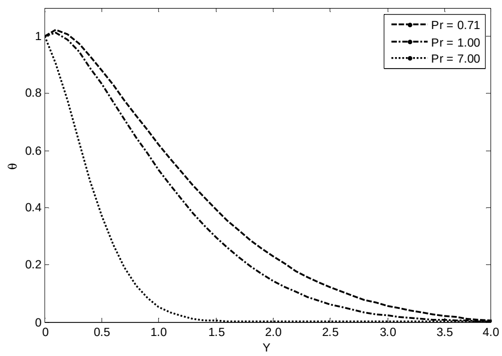

Figure – 3(b)

Figures 3(a) and 3(b) illustrate the behavior of the velocity and temperatures for different values of the Prandtl number (Pr = 0.71, 1.00, 7.00) Kr = 0.2, Gc = 5, Gr = 5, Sc = 0.6, M = 5, k = 1, E = 0.01. The numerical results show that the effect of increasing values of Prandtl number results in a decrease in velocity. From Fig. 3(b), it is observed that an increase in the Prandtl number results in a decrease in temperature.

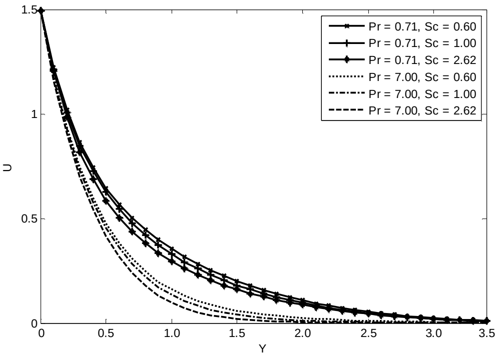

Figure – 4(a)

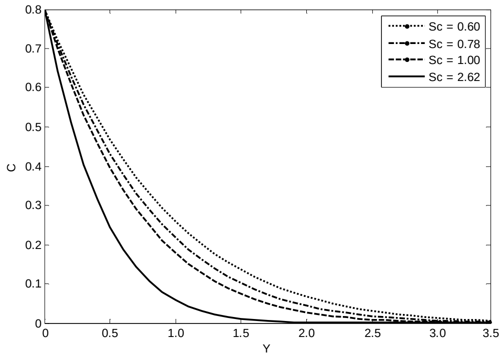

Figure – 4(b)

The effects of Schmidt number (Sc = 0.6, 0.78, 1.00, 2.62) Kr = 0.2, Gc = 5, Pr = 0.71 and Pr = 7, Gr = 5, M = 5, k = 1, E = 0.01 on the velocity and concentration are depicted in Figures 4(a) and 4(b). As the Schmidt number increases, both the velocity and concentration decreases.

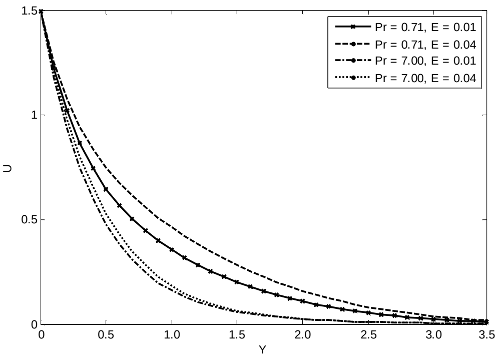

Figure – 5(a)

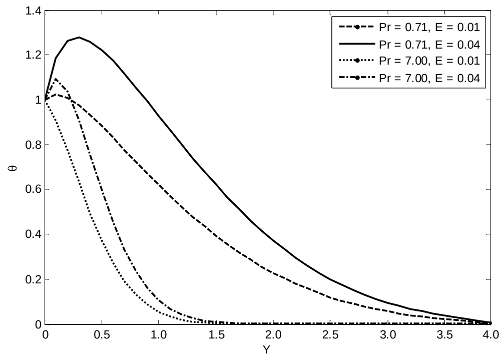

Figure – 5(b)

The velocity and temperature profiles are shown in Figures 5(a) and 5(b) for different values of Eckert number (E = 0.01, 0.02, 0.03, 0.04) Kr = 0.2, Gc = 5, Gr = 5, Pr = 0.71 and Pr = 7, Sc = 0.6, M = 5, k = 1. An increase in Eckert number E leads to increase in both velocity and temperature.

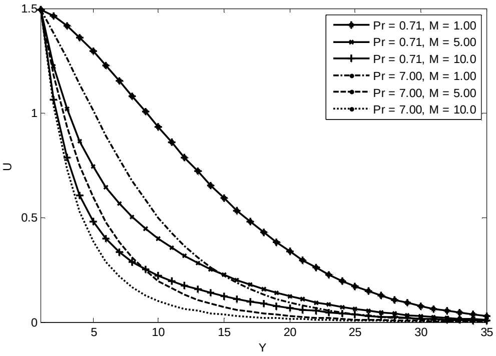

Figure – 6

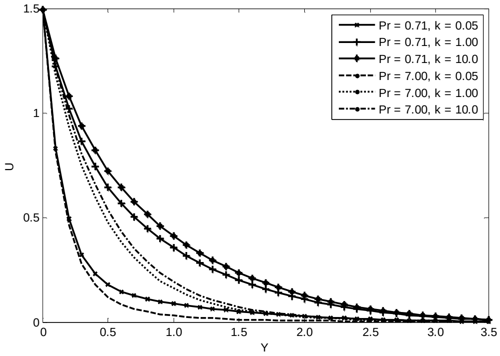

Figure - 7

The effect of different values of Hartmann number (magnetic parameter M) (M = 1, 5, 10, 15) Kr = 0.2, Gc = 5, Pr = 0.71 and Pr = 7, Sc = 0.6, Gr = 5, k = 1, E = 0.01 on velocity profile is shown in Figure-6. It is observed that an increase in M leads to decrease in velocity. Figure 7 concerns the velocity profile for different values of permeable parameter (k = 0.05, 1.00, 10.0) Kr = 0.2, Gc = 5, Pr = 7, Sc = 0.6, M = 5, E = 0.01. From this figure it is clear that the velocity increases with increase of k.

Table No.1 reveals the skin-friction against time t for various values of parameters M. It is noticed that the skin-friction increases with an increase in magnetic parameter.

Table No.2 depicts the Nusselt number against time t. It is found that the rate of heat transfer increases with increasing Pr. Nusselt number for Pr = 1 is higher than that of Pr = 0.71 and for Pr = 7 is higher than that of Pr = 7. The reason is that smaller values of Pr are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Prandtl number. Hence, the rate of heat transfer is reduced.

Table No.3 shows the variation of Sherwood number Sh against time t. It is observed that the Sherwood number increases with an increase in Schmidt number

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