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# Influence of gravity, compression and magnetic field on the propagation of rayleigh waves in non-homogeneous orthotropic elastic media 

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#### Abstract

The influence of the gravity and magnetic field on the propagation of Rayleigh wave in a prestressed inhomogeneous, orthotropic elastic solid medium has been discussed. The method of separation of variable has been used to find the frequency equations for the surface waves. The obtained dispersion equations are in agreement with the classical results when gravity, magnetic field, non-homogeneity and initial stress are neglected.


Keywords: Inhomogeneity, Orthotropic elastic solid, Gravity field, Initial stress, Magnetic field.

## NOMENCLATURE

$\overrightarrow{\mathrm{E}}$ is the electric intensity,
$\mu_{0}$ is permeability of vacuum,
$\overrightarrow{\mathrm{H}}_{i}$ is the perturbed magnetic field,
$\sigma^{o}$ is the conductivity of the material,
g is the earth gravity,
$\vec{V}=\frac{\overrightarrow{\partial v}}{\partial t}$ is velocity of conductor,
$\overrightarrow{\mathrm{H}}_{0}$ is the initial magnetic field intensity along z-axis,
$\overrightarrow{\mathrm{H}}$ is magnetic field intensity,
$\vec{F}$ is the Lorentz's force,
$\mathrm{C}_{\mathrm{ij}}$ is elastic constant,
$T_{j}$ are the body forces,
$\overrightarrow{\mathrm{B}}$ is magnetic field induction,
$\mathcal{E}_{0}$ is permittivity of vacuum,
$\vec{J}$ is the current density, $w_{i j}$ is the vector,
$\sigma_{i j}$ is the stress component,
$\vec{u}$ is the component of displacement vector,
$\mu_{e}$ is the magnetic permeability of the medium,
$\rho$ is the density of the material,
P is the initial stress,
$t$ is the time,
$\alpha_{i j}, \rho_{0}, P_{0}$ are dimensionless constants.

## INTRODUCTION

The theory of elasticity is an approximation to the stress-strain behavior of real materials. An ideal elastic material regains its original configuration on the removal of deforming force. Therefore an ideal "elastic wave" is that wave
which propagates through a material in such a way that the particles oscillates about their mean positions without causing any change. The term "initial compression" is meant by compressions developed in a medium before it is being used for study. The earth is an initially compressed medium. These compressions may have significant effect on elastic waves.

The earth has a layered structure, and this exerts a significant influence on the propagation of elastic waves. The simplest cases of influence exerted on the propagation of seismic waves by a single plane boundary which separates two half-spaces with different properties, and by two parallel plane boundaries forming a layer. Earth is being treated as an elastic body in which three types of waves can occur.

1. Dilatational and equivoluminal waves in the interior of the earth.
2. In the neighborhood of its surface known as Rayleigh waves [1].
3. Third type of waves occurs near the surface of contact of two layers of the earth known as love waves [2].

The Rayleigh waves are observed far from the disturbance source near the surface. Since the energy carried by these waves is concentrated over the surface, its dissipation is slower than the Dilatational and equivoluminal waves where the energy is dissipated over the volume of the disturbed region. Therefore, during earth quakes for an observer remote from the source of disturbance, the Rayleigh waves represent the greatest danger. In the case of Love waves, the energy is concentrated near the interface; hence they are dissipated more slowly. In the problem of propagation of Love type seismic wave in inhomogeneous isotropic media of finite depth lying over a infinite half space, it is shown that the distortional wave velocity in the layer is greater than in the semi infinite half space.

The propagation of Love waves in an in homogeneous layer is of considerable importance in earth quakes engineering and seismology on account of occurrence of in homogeneities in the crust of the earth as the earth is supposed to be made up of different layers. This problem has been studied by Sezawa [3], Wilson [4] Das and Gupta [5], Deresiewicz [6], Scholte [7] by considering different models of by considering different models of a layer changing either density or rigidity and established the presence of Love waves in each case. Also Gogna [8], Karsel [9], Chadwick [10], Zhang [11] also studied the propagation of Love waves through non-homogeneous media. Bromwich [12] was the first who taken the case of gravity in wave propagation through elastic solid media. Taking into account, the effect of initial stresses and using Biot's theory of incremental deformations, Jones [13], De and Sengupta [14] studied many problems of elastic waves and vibrations under the influence of gravity field. Abd-Alla and Ahmed [15] studied the Rayleigh waves in an orthotropic magneto-elastic medium under gravity field and initial stress. Recently, Love waves in a non-homogeneous elastic media, Rayleigh waves in a non-homogeneous granular media, Stoneley, Rayleigh and Love waves in viscoelastic media, Love waves in a non-homogeneous orthotropic layer under compression ' $P$ ' overlying semi-infinite non-homogeneous medium, effect of initial stress and gravity on Rayleigh waves propagating in non-homogeneous orthotropic elastic media were studied by Kakar et al. [16, 17, 18, 19, 20].

In the present study, the influence of gravity, magnetic field and initial stress on the propagation of Rayleigh type waves in a non-homogeneous, orthotropic elastic solid medium has been discussed. The dispersion equation so obtained is in well agreement with the corresponding classical results.

## FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

The problem is dealing with magnetoelasticity. Therefore the basic equations will be electromagnetism and elasticity. The Maxwell equations of the electromagnetic field in a vacuum (in the absence of displacement current), are
$\vec{\nabla} \cdot \overrightarrow{\mathrm{E}}=0, \vec{\nabla} \cdot \overrightarrow{\mathrm{~B}}=0, \vec{\nabla} \times \overrightarrow{\mathrm{E}}=-\frac{\overrightarrow{\partial \mathrm{B}}}{\partial t}, \vec{\nabla} \times \overrightarrow{\mathrm{B}}=\mu_{0} \varepsilon_{0} \frac{\overrightarrow{\partial \mathrm{E}}}{\partial t}$.
The current displacement vector and electric field are related as
$\vec{J}=\sigma^{o} \vec{E}$,

If the conductor is moving with velocity $\vec{V}$ in applied magnetic field, then
$\vec{J}=\sigma^{o}(\vec{E}+\vec{V} \times \vec{B})=\sigma^{o}\left(\vec{E}+\frac{\overrightarrow{\partial v}}{\partial t} \times \vec{B}\right)$.
The electromagnetic wave equation through a vacuum is given by
$\left(\nabla^{2}-\mu_{0} \grave{o}_{\mathrm{o}} \frac{\partial^{2}}{\partial t^{2}}\right) \overrightarrow{\mathrm{E}}=0,\left(\nabla^{2}-\mu_{0} \grave{o}_{\mathrm{o}} \frac{\partial^{2}}{\partial t^{2}}\right) \overrightarrow{\mathrm{B}}=0$.
Let us consider an orthotropic, non-homogeneous elastic solid under an initial compression P along x -direction further it is also under the influence of gravity, and magnetic field. Here we consider Oxyz Cartesian coordinates system where O is any point on the plane boundary and Oz is normal to the medium and Rayleigh wave propagation is taken in the positive direction of x-axis. It is also assumed that at a great distance from center of disturbance, the wave propagation is two dimensional and is polarized in ( $\mathrm{x}, \mathrm{z}$ ) plane. So, displacement components along x and z direction i.e. u and w are non-zero while $\mathrm{v}=0$. Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary.

Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary. Let g be the acceleration due to gravity and $\rho$ is the density of the material medium.

Here states of initial stresses are given by ( $\sigma$ is a function of z )
$\sigma_{i j}=\sigma ; i=j, \quad \sigma_{i j}=0 ; i \neq j$.
(where $i, j=1,2,3$ ), (where $i, j=1,2,3$ )
Equations of equilibrium of initial compression are
$\frac{\partial \sigma}{\partial x}=0, \frac{\partial \sigma}{\partial y}=0, \frac{\partial \sigma}{\partial z}-\rho g=0$.
The value of magnetic field intensity is
$\overrightarrow{\mathrm{H}}(0,0, \mathrm{H})=\overrightarrow{\mathrm{H}}_{0}+\overrightarrow{\mathrm{H}}_{i}$
Equations governing the propagation of small elastic disturbances in a perfectly conducting elastic solid having electromagnetic force $\vec{F}=(\vec{J} \times \overrightarrow{\mathrm{B}})$ (the Lorentz force, $\vec{J}$ is the current density and $\overrightarrow{\mathrm{B}}$ being magnetic induction vector) as the only body force are (Biot [21])
$\sigma_{11}, \mathrm{x}+\sigma_{12, \mathrm{y}}+\sigma_{13, \mathrm{z}}+\mathrm{P}\left(\mathrm{w}_{\mathrm{z}, \mathrm{y}}-\mathrm{w}_{\mathrm{y}, \mathrm{z}}\right)-\rho \mathrm{g} \mathrm{u}_{3, \mathrm{x}}+\mathrm{F}_{\mathrm{x}}=\rho u_{, t t}$
$\sigma_{12, \mathrm{x}}+\sigma_{22, \mathrm{y}}+\sigma_{23, \mathrm{z}}-\mathrm{Pw}_{\mathrm{z}, \mathrm{x}}+\mathrm{F}_{\mathrm{y}}=\rho \mathrm{v}, \mathrm{tt}$,
$\sigma_{13, \mathrm{x}}+\sigma_{23, \mathrm{y}}+\sigma_{33, \mathrm{Z}}-\mathrm{Pw}_{\mathrm{y}, \mathrm{x}}+\rho \mathrm{g} \mathrm{u}_{1, \mathrm{x}}+\mathrm{F}_{\mathrm{z}}=\rho w_{, t t}$
where $\mathrm{u}, \mathrm{v}, \mathrm{w}$ are displacement components in $\mathrm{x}, \mathrm{y}$ and z direction and $\mathrm{w}_{\mathrm{x}}, \mathrm{w}_{\mathrm{y}}, \mathrm{w}_{\mathrm{z}}$ are al components and are given by
$w_{x}=\frac{1}{2}\left(w_{, y}-v_{, z}\right), w_{y}=\frac{1}{2}\left(u_{, z}-w_{, x}\right), w_{z}=\frac{1}{2}\left(v_{, x}-u_{, y}\right)$.
Further dynamical Eq. (7) in (x, z) directions in terms of Eq. (5) are given by
$\sigma_{11}, \mathrm{x}+\sigma_{12, \mathrm{y}}+\sigma_{13}, \mathrm{z}+\mathrm{P}\left(\mathrm{w}_{\mathrm{z}, \mathrm{y}}-\mathrm{w}_{\mathrm{y}}, \mathrm{z}\right)-\rho \mathrm{g} \mathrm{u}_{3}, \mathrm{x}+\mathrm{F}_{\mathrm{x}}=\rho u_{, t t}$
$\sigma_{13}, \mathrm{x}+\sigma_{23}, \mathrm{y}+\sigma_{33, \mathrm{z}}-\mathrm{Pw} \mathrm{y}_{\mathrm{y}, \mathrm{x}}+\rho \mathrm{g} \mathrm{u}_{1}, \mathrm{x}+\mathrm{F}_{\mathrm{z}}=\rho w_{, t t}$
where stress components are given by
$\sigma_{11}=\left(C_{11}+P\right) u_{1, x}+\left(C_{13}+P\right) u_{3, z}$,
$\sigma_{33}=C_{31} u_{1}, x+C_{33} u_{3, \mathrm{Z}}$,
$\sigma_{13}=\mathrm{C}_{44}\left(\mathrm{u}_{1, \mathrm{Z}}+\mathrm{u}_{3, \mathrm{x}}\right)$,
where $\mathrm{C}_{\mathrm{ij}}$ are elastic constants. Since the problem is treated in two-dimensions ( $\mathrm{x}, \mathrm{z}$ ), therefore $\mathrm{C}_{12}=\mathrm{C}_{22}=\mathrm{C}_{23}=0$
Let us take the assumption that $\mathrm{C}_{44}=\frac{1}{2}\left(\mathrm{C}_{11}-\mathrm{C}_{13}\right)$.
Substituting Eq. (10) in Eq. (19) ; we have
$\left(C_{11}+P\right)\left(2 u_{1}, x x+u_{1}, z z+u_{3}, x z\right)+C_{13}\left(u_{3}, x z-u_{1}, z z\right)+\left(u_{1, z}+u_{3}, x\right)\left(C_{11}-C_{13}\right), z+2 u_{1, x}\left(C_{11}+P\right), x+2 u_{3, z}$
$\left(\mathrm{C}_{13}+\mathrm{P}\right)_{, \mathrm{x}}-2 \rho g \mathrm{u}_{3, \mathrm{x}}+2 \mu_{e} H_{0}^{2}\left(u_{, x x}+w_{, x z}\right)=\rho u_{, t t}$
$\mathrm{C}_{11}\left(\mathrm{u}_{1}, \mathrm{xz}+\mathrm{u}_{3}, \mathrm{xx}\right)+\left(\mathrm{C}_{13}+\mathrm{P}\right)\left(\mathrm{u}_{1}, \mathrm{xz}-\mathrm{u}_{3}, \mathrm{xx}\right)+2 \mathrm{C}_{33} \mathrm{u}_{3}, \mathrm{zz}+2 \rho \mathrm{~g} \mathrm{u}_{1}, \mathrm{x}+2 \mu_{e} H_{0}^{2}\left(u_{, x x}+w_{, x z}\right)+\left(\mathrm{u}_{1, \mathrm{z}}+\mathrm{u}_{3}, \mathrm{x}\right)\left(\mathrm{C}_{11}\right.$
$\left.-\mathrm{C}_{13}\right), \mathrm{x}+\mathrm{u}_{1}, \mathrm{x} \mathrm{C}_{13}, \mathrm{z}+\mathrm{u}_{3, \mathrm{z}} \mathrm{C}_{33}, \mathrm{z}=\rho w_{, t t}$.
Now we assume the non-homogeneity for the elastic half space, density and compression are given by
$C_{i j}=\alpha_{i j} e^{m z}, \rho=\rho_{0} e^{m z}, P=P_{0} e^{m z}$,
where $\alpha_{\mathrm{ij}}, \rho_{0}, \mathrm{P}_{0}$ and m are constants.
Substituting Eq. (13) in Eq. (11) and in Eq. (12), we get
$e^{m z}\left(\alpha_{11}+P_{0}\right)\left(2 u_{1}, x x+u_{1}, z z+u_{3}, x z\right)+\alpha_{13}\left(u_{3}, x z-u_{1}, z z\right) e^{m z}+\left(u_{1, z}+u_{3}, x\right)\left(\alpha_{11}-\alpha_{13}\right) m e^{m z}-2 \rho_{0} g u_{3, x}$ $\mathrm{e}^{\mathrm{mz}}+2 \mu_{e} H_{0}^{2}\left(u_{, x x}+w_{, z z}\right)=\rho u_{, t t}$
$\alpha_{11}\left(\mathrm{u}_{1}, \mathrm{xz}^{+\mathrm{u}_{3}, \mathrm{xx}}\right)+\left(\alpha_{13}+2 \mathrm{P}_{0}\right)\left(\mathrm{u}_{1}, \mathrm{xz}\right)-\left(\alpha_{13}+2 \mathrm{P}_{0}\right) \mathrm{u}_{3}, \mathrm{xx}+2 \alpha_{33} \mathrm{u}_{3, \mathrm{zz}}+2 \rho_{0} \mathrm{~g} \mathrm{u}_{1, \mathrm{x}}+2 \mu_{e} H_{0}^{2}\left(u_{, x x}+w_{, x z}\right)+2 \alpha_{13} \mathrm{~m}$
$\mathrm{u}_{1, \mathrm{x}}+2 \alpha_{33} \mathrm{mu}_{3, \mathrm{z}}=\rho w_{, t t}$.

## SOLUTION OF THE PROBLEM

To investigate the surface wave propagation along Ox, we introduce displacement potentials in terms of displacements components are given by
$\mathrm{u}=\phi_{, \mathrm{x}}-\psi_{, \mathrm{z}} ; \mathrm{w}=\phi_{, \mathrm{z}}+\psi_{, \mathrm{x}}$
Introducing Eq. (16) in Eqs (14) and (15) we get
$2\left(\alpha_{11}+\mathrm{P}_{0}+\mu_{e} H_{0}^{2}\right) \nabla^{2} \phi-2 \rho_{0} \mathrm{~g} \psi_{, \mathrm{x}}+\mathrm{m}\left(\alpha_{11}-\alpha_{13}\right)\left(2 \phi_{, \mathrm{Z}}+\psi_{, \mathrm{X}}\right)=2 \rho_{0} \phi_{. t t}$,
$\left(\alpha_{11}+\mathrm{P}_{0}-\alpha_{13}\right) \nabla^{2} \psi+2 \rho_{0} \mathrm{~g} \phi, \mathrm{x}-\mathrm{m}\left(\alpha_{11}-\alpha_{13}\right) \psi_{, \mathrm{Z}}=2 \rho_{0} \psi_{, t t}$,
and
$\alpha_{11} \phi_{, \mathrm{xx}}+\alpha_{33} \phi_{, \mathrm{zz}}-\rho_{0} \mathrm{~g} \psi_{, \mathrm{x}^{-}}-2 \alpha_{13} \mathrm{~m} \psi_{, \mathrm{x}}+2 \alpha_{33} \mathrm{~m} \phi_{, \mathrm{Z}}=2 \rho_{0} \psi_{, t t}$
$\left(\alpha_{11}-\alpha_{13}-2 \mathrm{P}_{0}+\mu_{e} H_{0}^{2}\right) \psi_{, \mathrm{xx}}+\left(2 \alpha_{33}-\alpha_{13}-\alpha_{11}-2 \mathrm{P}_{0}\right) \psi_{, \mathrm{zz}}+\left(2 \rho_{0} \mathrm{~g}+2 \alpha_{13} \mathrm{~m}\right) \phi_{, \mathrm{x}}+2 \alpha_{33} \mathrm{~m} \psi_{, \mathrm{z}}=2 \rho_{0} \phi_{. t t}$
where, $\nabla^{2}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial z^{2}}$.
Since the velocity of waves are different in $x$ and $z$ direction. Now Eq. (17) and Eq. (18) represent the compressive wave along x and z -direction while Eq. (19) and Eq. (20) represents the shear waves along these directions. Since we consider the propagation of Rayleigh waves in x-direction, therefore we consider only Eq. (17) and Eq. (20).

To solve Eq. (18) and Eq. (21) we introduce
$\phi(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{f}(\mathrm{z}) e^{i \alpha(x-c t)}$,
$\psi(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{h}(\mathrm{z}) e^{i \alpha(x-c t)}$.
putting Eq. (22) in Eq. (18) and Eq. (21) we get
$\mathrm{f}_{, \mathrm{zz}}+\mathrm{A}_{\mathrm{f}, \mathrm{z}}+\mathrm{Bf}+\mathrm{Ch}=0$,
$h_{, z z}+A^{\prime} h_{, Z}+B^{\prime} h+C^{\prime} f=0$,
where

$$
\begin{align*}
& \mathrm{A}=\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}}, \mathrm{~B}=\frac{\alpha^{2}\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right)}{\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}}, \mathrm{C}=\frac{\left[-2 \rho_{0} g+m\left(\alpha_{11}-\alpha_{13}\right)\right] i \alpha}{2\left(\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}\right)}, \\
& \mathrm{A}^{\prime}=\frac{2 m \alpha_{33}+\mu_{e} H_{0}^{2}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}, \mathrm{~B}^{\prime}=\frac{\alpha^{2}\left(2 c^{2} \rho_{0}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}, \mathrm{C}^{\prime}=\frac{\left(2 \rho_{0} g+2 m \alpha_{13}\right) i \alpha}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}} . \tag{25}
\end{align*}
$$

Now Eq. (23) and Eq. (24) have exponential solution in order that $\mathrm{f}(\mathrm{z})$ and $\mathrm{h}(\mathrm{z})$ describe surface waves and also they varnish as $\mathrm{z} \rightarrow \infty$ hence Eq. (18) takes the form,

$$
\begin{align*}
& \phi(\mathrm{x}, \mathrm{z}, \mathrm{t})=\left[C_{1} e^{-\lambda_{1} z}+C_{2} e^{-\lambda_{2} z}\right] e^{i \alpha(x-c t)} \\
& \psi(\mathrm{x}, \mathrm{z}, \mathrm{t})=\left[C_{3} e^{-\lambda_{1} z}+C_{4} e^{-\lambda_{2} z}\right] e^{i \alpha(x-c t)}, \tag{26}
\end{align*}
$$

where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \mathrm{C}_{4}$ are arbitrary constants and $\lambda_{1}, \lambda_{2}$ are the roots of the equation
$\lambda^{4}+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}}+\frac{2 m \alpha_{33}+\mu_{e} H_{0}^{2}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}\right] \lambda^{3}$
$+\alpha^{2}\left[\frac{2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}+\frac{\rho_{0} c^{2}-\alpha_{11}-P_{0}}{\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}}\right] \lambda^{2}$
$+m \alpha^{2}\left[\frac{\left(\alpha_{11}-\alpha_{33}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)+\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right) 2 \alpha_{33}}{\left(\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right] \lambda$
$+\left[\frac{\alpha^{4}\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)}{\left(\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right.$
$\left.+\alpha^{2} \frac{\left\{m\left(\alpha_{11}-\alpha_{13}\right)-2 \rho_{0} g\right\}\left(2 \rho_{0} g+2 m \alpha_{13}\right)}{2\left(\alpha_{11}+P_{0}+\mu_{e} H_{0}^{2}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right]=0$.
Here we consider only real roots of Eq. (27). Now the constants $\mathrm{C}_{1}, \mathrm{C}_{2}$ and $\mathrm{C}_{3}, \mathrm{C}_{4}$ are related by the Eq (23) and Eq. (24).

By equating the co-efficients of $e^{-\lambda_{1} z}$ and $e^{-\lambda_{2} z}$ to zero, Eq. (23) gives,
$C_{3}=\gamma_{1} C_{1}, C_{4}=\gamma_{2} C_{2}$,
where
$\gamma_{j}=\frac{2 i\left[\left(\alpha_{11}+\mathrm{P}_{0}+\mu_{e} H_{0}^{2}\right) \lambda_{j}^{2}-\mathrm{m}\left(\alpha_{11}-\alpha_{13}\right) \lambda_{j}+\left(\rho_{0} c^{2}-\alpha_{11}-\mathrm{P}_{0}\right)\right]}{\alpha\left[m\left(\alpha_{11}-\alpha_{13}\right)-2 \rho_{0} g\right]}$
( $j=1,2$.

## BOUNDARY CONDITIONS

The plane $\mathrm{z}=0$ is free from stresses i.e. $\sigma_{13}=\sigma_{33}=0$ at $\mathrm{z}=0$,
$\sigma_{13}=\frac{1}{2}\left(\alpha_{11}-\alpha_{13}\right)\left[2 \phi_{, \mathrm{xz}}-\psi_{, \mathrm{zz}}+\psi_{, \mathrm{xx}}\right] \mathrm{e}^{\mathrm{mz}}$,
$\sigma_{33}=\alpha_{31}\left[\phi, \mathrm{xx}-\psi_{, \mathrm{xz}}\right] \mathrm{e}^{\mathrm{mz}}+\alpha_{33}\left[\phi_{, \mathrm{zz}}+\psi_{, \mathrm{zx}}\right] \mathrm{e}^{\mathrm{mz}}$.

Introducing Eq. (31) and Eq. (32) in Eq. (30) we have
$\mathrm{C}_{1}\left(2 \lambda_{1} \mathrm{i} \alpha+\gamma_{1} \lambda_{1}^{2}+\alpha^{2} \gamma_{1}\right)+\mathrm{C}_{2}\left[2 \lambda_{2} \mathrm{i} \alpha+\gamma_{2} \lambda_{2}^{2}+\alpha^{2} \gamma_{2}\right]=0$,
$\mathrm{C}_{1}\left[-\alpha^{2} \lambda_{13}+\lambda_{1}^{2} \alpha_{33}-\alpha_{1} \gamma_{1} \mathrm{i} \alpha\left(\alpha_{33}-\alpha_{13}\right)\right]+\mathrm{C}_{2}\left[-\alpha^{2} \lambda_{13}+\lambda_{2}^{2} \alpha_{33}-\lambda_{1} \gamma_{1} \mathrm{i} \alpha\left(\alpha_{33}-\alpha_{13}\right)\right]=0$.

Eliminating $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ from Eq. (33) and Eq. (34) ; we have
$\left[2 \lambda_{1} \mathrm{i} \alpha+\gamma_{1} \lambda_{1}{ }^{2}+\alpha^{2} \gamma_{1}\right]\left[-\alpha^{2} \lambda_{13}+\lambda_{2}^{2} \alpha_{33}-\lambda_{1} \gamma_{1} \mathrm{i} \alpha\left(\alpha_{33}-\alpha_{13}\right)\right]-\left[2 \lambda_{2} \mathrm{i} \alpha+\gamma_{2} \lambda_{2}^{2}+\alpha^{2} \gamma_{2}\right]\left[-\alpha^{2} \lambda_{13}+\lambda_{1}^{2} \alpha_{33}-\right.$ $\left.\alpha_{1} \gamma_{1} \mathrm{i} \alpha\left(\alpha_{33}-\alpha_{13}\right)\right]=0$,
where $\gamma_{j}(\mathrm{j}=1,2)$ are given by Eq. (29) and $\lambda_{\mathrm{j}}(\mathrm{j}=1,2)$ are roots of Eq. (27).
Now Eq. (35) gives the wave velocity equation for Rayleigh waves in a non-homogeneous elastic half space of orthotropic material under the initial compression, magnetic field and influence of gravity. From Eq. (35), it follows that Rayleigh waves depends on gravity, initial compression, magnetic field and non-homogeneous character of the medium and nature of the material, we conclude that if $\alpha$ is large i.e. length of wave i.e. $\frac{2 \pi}{\alpha}$ is small then gravity, magnetic field and compression have small effects on Rayleigh waves in non-homogeneous orthotropic half space and if $\alpha$ is small i.e. $\frac{2 \pi}{\alpha}$ is large then gravity, magnetic field and compression plays a vital role for finding out the wave velocity c .

Case 1.When the medium is isotropic, Eq. (35) becomes
$\left[2 \lambda_{1} \mathrm{i} \alpha+\gamma_{1} \lambda_{1}{ }^{2}+\alpha^{2} \gamma_{1}\right]\left[\mathrm{K}_{1}^{2}\left(\lambda_{2}^{2}-\alpha^{2}\right)+2 \mathrm{~K}_{2}{ }^{2}\left(1-\mathrm{i} \alpha \gamma_{2} \lambda_{2}\right)\right]-\left[2 \lambda_{2} \mathrm{i} \alpha+\gamma_{2} \lambda_{2}{ }^{2}+\alpha^{2} \gamma_{2}\right]\left[\mathrm{K}_{1}^{2}\left(\lambda_{1}^{2}-\alpha^{2}\right)\right.$
$\left.+2 \mathrm{~K}_{2}{ }^{2}\left(1-\mathrm{i} \alpha \gamma_{1} \lambda_{1}\right)\right]=0$,
where $\mathrm{K}_{1}^{2}=\frac{\lambda+2 \mu+P}{\rho}, \mathrm{~K}_{2}^{2}=\frac{\mu-P / 2}{\rho},(\lambda, \mu$ are Lame's constants $)$.
Eq. (37) determines the Rayleigh waves in a non-homogeneous isotropic elastic solid under the influence of gravity, magnetic field and compression.

Case2. When initial compression and magnetic field are absent i.e. $P_{0}=0, H_{0}=0$, then equation (36) reduces to,
$\left[2 \lambda_{1} \mathrm{i} \alpha+\gamma_{1} \lambda_{1}^{2}+\alpha^{2} \gamma_{1}\right]\left[\mathrm{K}_{1}^{2}\left(\lambda_{2}^{2}-\alpha^{2}\right)+2 \mathrm{~K}_{2}^{2}\left(1-\mathrm{i} \alpha \gamma_{2} \lambda_{2}\right)\right]-\left[2 \lambda_{2} \mathrm{i} \alpha+\gamma_{2} \lambda_{2}^{2}+\alpha^{2} \gamma_{2}\right]\left[\mathrm{K}_{1}^{2}\left(\lambda_{1}^{2}-\alpha^{2}\right)\right.$
$\left.+2 \mathrm{~K}_{2}^{2}\left(1-\mathrm{i} \alpha \gamma_{1} \lambda_{1}\right)\right]=0$,
where $\mathrm{K}_{1}^{2}=\frac{\lambda+2 \mu}{\rho}, \mathrm{~K}_{2}^{2}=\frac{\mu}{\rho}$.
Eq. (38) determines the Rayleigh surface waves in a non-homogeneous isotropic elastic solid under the influence of gravity which is similar to corresponding classical result given by Das et al.

Case2. When non-homogeneity, $\mathrm{H}_{0}=0$ of the material is absent, we get same dispersion Equation as Eq. (35) with $\gamma_{\mathrm{j}}=\frac{-i\left[\left(\alpha_{11}+\mathrm{P}_{0}\right) \lambda_{j}^{2}+\left(\rho_{0} \mathrm{c}^{2}-\alpha_{11}-\mathrm{P}_{0}\right)\right]}{\alpha \rho_{0} g} ; \mathrm{j}=1,2$,
where $\lambda_{1}, \lambda_{2}$ are the roots of the equation

$$
\begin{align*}
& \lambda^{4}+\alpha^{2}\left[\frac{2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}+\frac{\rho_{0} c^{2}-\alpha_{11}-P_{0}}{\alpha_{11}+P_{0}}\right] \lambda^{2} \\
& +\left[\frac{\alpha^{4}\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)-2 \alpha^{2} \rho_{0}^{2} g^{2}}{\left(P_{0}+\alpha_{11}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right]=0 . \tag{39}
\end{align*}
$$

Case3. When gravity field and magnetic field are absent, we get same velocity equation for Rayleigh waves in nonhomogeneous elastic solid under initial compression as Eq. (35) with
$\gamma_{\mathrm{j}}=\frac{2 i\left[\left(\alpha_{11}+\mathrm{P}_{0}\right) \lambda_{j}^{2}-\mathrm{m}\left(\alpha_{11}-\alpha_{13}\right) \lambda_{j}+\left(\rho_{0} \mathrm{c}^{2}-\alpha_{11}-\mathrm{P}_{0}\right)\right]}{\left[\alpha m\left(\alpha_{11}-\alpha_{13}\right)\right]} ; \mathrm{j}=1,2$,
where $\lambda_{1}, \lambda_{2}$ are roots of the equation
$\lambda^{4}+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_{0}}+\frac{2 m \alpha_{33}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}\right] \lambda^{3}$
$+\alpha^{2}\left[\frac{2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}}+\frac{\rho_{0} c^{2}-\alpha_{11}-P_{0}}{\alpha_{11}+P_{0}}\right] \lambda^{2}$
$+m \alpha^{2}\left[\frac{\left(\alpha_{11}-\alpha_{33}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)+\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right) 2 \alpha_{33}}{\left(\alpha_{11}+P_{0}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right] \lambda$
$+\left[\frac{\alpha^{4}\left(\rho_{0} c^{2}-\alpha_{11}-P_{0}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}+2 P_{0}\right)}{\left(\alpha_{11}+P_{0}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right.$
$\left.+\alpha^{2} \frac{\left\{m\left(\alpha_{11}-\alpha_{13}\right)\right\}\left(m \alpha_{13}\right)}{\left(\alpha_{11}+P_{0}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}-2 P_{0}\right)}\right]=0$.
Case4. When medium is initially unstressed i.e. $\mathrm{P}_{0}=0$
We get, velocity equation for Rayleigh waves is similar to Eq. (35) with
$\gamma_{\mathrm{j}}=\frac{2 i\left[\left(\alpha_{11} \lambda_{j}^{2}-\mathrm{m}\left(\alpha_{11}-\alpha_{13}\right) \lambda_{j}+\left(\rho_{0} \mathrm{c}^{2}-\alpha_{11}\right)\right]\right.}{\alpha\left[m\left(\alpha_{11}-\alpha_{13}\right)-2 \rho_{0} g\right]} ; \mathrm{j}=1,2$,
where $\lambda_{1}, \lambda_{2}$ are roots of the equation
$\lambda^{4}+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}}+\frac{2 m \alpha_{33}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}}\right] \lambda^{3}$
$+\alpha^{2}\left[\frac{2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}}{2 \alpha_{33}-\alpha_{11}-\alpha_{13}}+\frac{\rho_{0} c^{2}-\alpha_{11}}{\alpha_{11}}\right] \lambda^{2}$
$+m \alpha^{2}\left[\frac{\left(\alpha_{11}-\alpha_{33}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}\right)+\left(\rho_{0} c^{2}-\alpha_{11}\right) 2 \alpha_{33}}{\left(\alpha_{11}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right] \lambda$
$+\left[\frac{\alpha^{4}\left(\rho_{0} c^{2}-\alpha_{11}\right)\left(2 \rho_{0} c^{2}-\alpha_{11}+\alpha_{13}\right)}{\left(\alpha_{11}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}\right)}+\alpha^{2} \frac{\left\{m\left(\alpha_{11}-\alpha_{13}\right)-2 \rho_{0} g\right\}\left(2 \rho_{0} g+2 m \alpha_{13}\right)}{2\left(\alpha_{11}\right)\left(2 \alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right]=0$.

Case5. When the non-homogeneity of the material, $\mathrm{H}_{0}=0$ and gravity field are absent further medium is initially unstressed and isotropic, Eq. (35) reduces to,
$4 \sqrt{\left(1-\frac{c^{2}}{K_{1}{ }^{2}}\right)\left(1-\frac{c^{2}}{K_{2}{ }^{2}}\right)}=\left(2-\frac{c^{2}}{K_{2}{ }^{2}}\right)^{2}$,
where $\mathrm{K}_{1}^{2}=\frac{\lambda+2 \mu}{\rho}, \mathrm{~K}_{2}^{2}=\frac{\mu}{\rho}$.
Eq. (42) is similar to the equation given by Rayleigh.

## CONCLUSION

1. Equation (35) represents the wave velocity equation for the Rayleigh waves in a non-homogeneous, orthotropic elastic solid medium under the influence of gravity, magnetic field and initial compression.
2. It also depends upon the wave number and confirming that waves are dispersive. Moreover, the dispersion equation contains terms involving gravity, initial compression, magnetic field and non-homogeneity, so the phase velocity ' $c$ ' not only depends upon the gravity field, magnetic field, $r$ and initial compression but also on the nonhomogeneity of the material medium.
3. The explicit solutions of this wave velocity equation cannot be determined by analytical methods. However, these equations can be solved with the help of numerical method, by a suitable choice of physical parameters involved in medium.

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