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Hydromagnetic two-layered fluid slip-flow between two parallel porous walls

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ABSTRACT

An attempt is made to investigate the hydromagnetic two-layered fluid flow driven by a constant pressure gradient in a horizontal channel under the action of uniform strong magnetic field in the slip-flow regime. Closed form solutions for the velocity and temperature distributions in the two regions are obtained by assuming that the fluids in the two regions are incompressible, immiscible and electrically conducting. Further, the two fluids are assumed to have different viscosities, electrical and thermal conductivities. The transport properties of the two fluids are taken to be constant and the bounding walls are maintained at constant and equal temperatures. The profiles are plotted after obtaining the numerical values for different sets of values of the governing parameters involved and discussed in detail by analyzing these parameters such as, slip parameter, porous parameter, Hartmann number, ratio's of the viscosities, heights, electrical and thermal conductivities.

Keywords: MHD, Immiscible fluids/Two-layered fluid flows, slip-flow, porous boundaries

INTRODUCTION

The problem of hydromagnetic two-layered fluid/two-phase flows in channels has been attracted by several investigators due to their numerous applications in science, engineering and various power generating industries: such as MHD generators, nuclear reactors, geothermal energy extractions and many such applications. The questions pertaining to nuclear-reactor safety have led to insist for an understanding of the detailed phase-distribution mechanisms involved in two-phase flows. Transportation and extraction of the products of oil are other obvious applications using a two-phase system to obtain increased flow rates in an electromagnetic pump from the possibility of reducing the power required to pump oil in a pipe line by suitable addition of water [20]. Consequently, extensive publications on experimental and theoretical aspects of the two-phase flow systems with or without considering the heat transfer problems associated with MHD generators, as the generator channels are significantly influenced by the presence of magnetic field have been appeared in the literature during the past several decades due to the pioneer research works of the various investigators notably, Shercliff [21], Alad'Yev et al [1], Thatcher [24], Postlethwaite and Sluyter [14], Michiyoshi [11-12], Dobran [6]. In [8], Lohrasbi and Sahai have studied the MHD heat transfer aspects in two-phase flow with the fluid in one phase being electrically conducting. Malashetty and Leela [9] carried out a theoretical study on magnetohydrodynamic heat transfer in twofluid flow in case of short circuit type. Subsequently, in [10] these authors have analyzed the problem of magnetohydrodynamic heat transfer in two-phase flow by assuming that the fluids in both regions are electrically conducting for the open circuit case. However, the use of liquid metals as heat transfer agents and as working fluid in a rotating MHD power generator and nuclear reactor technology has created a growing interest in the behaviour of liquid metal flows, and in particular the nature of interaction with magnetic field. Recently, Raju and Murty [15] studied magnetohydrodynamic two-phase flow and heat transfer in a rotating system. Umavathi et.al [26]

investigated oscillatory Hartmann two-fluid flow and heat transfer in a horizontal channel, and Tsuyoshi Inoue and Schu-Ichiro Inutsuka [25] studied two-fluid magnetohydrodynamic simulations of converging Hi flows in the interstellar medium. Raju and Sreedhar [16] have studied usteady two-fluid flow and heat transfer of conducting fluids in channels under transverse magnetic field.

In most of the above mentioned studies, the investigators have considered the no-slip conditions at the boundary walls. But, in the slip flow regime, the fluid flow still behaves the Navier-Stokes equation (Navier, 1827, Street, [22] and Davis, [5]). Keeping in view of this fact, there are some investigations on the slip flow regime, which are readily available in the literature due to the works of Basset [2], Schaaf and Chambre [19], Lance and Rogers [7], Street [22], Sastry and Bhadram [18], Tamada and Murali [23], Bhatt and Sacheti [3], Michael et al. [13], Chand et al. [4] and many more. But research articles on magnetohydrodynamic two-phase/two fluid slip flows are very rarely found in the literature. Recently, Raju et al. [17] studied magnetohydrodynamic two-layered fluid slip-flow between two parallel walls, in view of the fact that, there may be the cases where partial slip does occur on the walls. These situations may include rarefied gas flows, rough or porous walls. In these situations, the usual no-slip condition may be replaced by the partial slip condition at the boundaries. Also, the flow is supposed to undergo slipping at the walls with velocity proportional to the shear stress there. And so far, no attempt has been made on this study as far as the author's knowledge is concerned as it is evident in the available literature. Owing to these studies and many other applications, an investigation on hydromagnetic two-layered fluid flow system of an incompressible, electrically conducting fluid with different properties in a horizontal channel bounded by two insulating walls accounting for wall porosity is carried out in this paper by introducing the slip boundary conditions at both walls. This type of studies is expected to be useful in the development of high altitude flights, space science and in nuclear fusion research etc. Also, these studies are likely to be useful to carry out experiments to produce power on a large scale in stationary plants with large magnetic fields, such as in the design of MHD generators, MHD pumps and flow meters etc.

So in this paper, the hydromagnetic two-layered fluid flow driven by a constant pressure gradient in a horizontal channel bounded by two parallel porous walls under the action of a uniform strong magnetic field in the slip flow regime is studied. Closed form solutions for the velocity and temperature distributions in the two fluid regions are obtained by assuming that the fluids in the two regions are incompressible, immiscible and electrically conducting. Further, the two fluids are assumed to have different viscosities, thermal and electrical conductivities. The transport properties of the two fluids are taken to be constant and the bounding walls are maintained at constant and equal temperatures. The profiles are plotted after obtaining the numerical values for different sets of values of the governing parameters involved such as, porous parameter, slip parameter, magnetic parameter, the ratio of viscosities, ratio of electrical conductivities, ratio of thermal conductivities and the height ratio, also it is discussed the behavior of the flow and temperature distributions by analyzing these parameters. The results of Malashetty and Leela [10] have been recovered, when

 $\lambda = 0$ (that is, no-slip at the boundaries).

The paper is organized as follows: The introduction is given in section 1. The formulation of the flow problem accounting for wall porosity, mathematical analysis for equations of motion, energy and the boundary conditions are presented in section 2. Section 3 deals with the solutions of the problem for velocity and temperature distributions. In section 4, the discussion of the results is presented in detail from the graphs which are shown in figures 2 to 15.

2. Mathematical analysis for equations of motion, energy and the boundary conditions

A steady magnetohydrodynamic two-phase flow driven by a common pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$ in a horizontal channel with parallel porous walls at $y = -h_1$ and $y = -h_2$, which are infinite in extent along x- and z-directions subject to the uniform suction v_0 applied normal to both walls is considered. Fig.1 depicts the flow model and coordinate system. It is assumed that, the regions $0 \le y \le h_1$ and $-h_2 \le y \le 0$ are occupied by two different immiscible, incompressible fluids with different viscosities, electrical and thermal conductivities. The transport properties of the two fluids are considered as constant and the bounding walls are maintained at constant and equal temperatures T_w . It is also assumed that the induced magnetic field is small when compared with the applied field, so that it is negligible. With these assumptions and conditions, the resulting governing equations of motion, energy

and the corresponding boundary and interface conditions in non-dimensional form in the two fluid regions are given as:

Region-I

$$\frac{d^2 u_1}{d y^2} + P - S^2 (R_e + u_1) = -\lambda \frac{d u_1}{d y}$$
(1)

$$\frac{d^{2}\theta_{1}}{dy^{2}} + \lambda \frac{d\theta_{1}}{dy} + \left(\frac{du_{1}}{dy}\right)^{2} + S^{2} (R_{e} + u_{1})^{2} = 0$$
⁽²⁾

Region-II

$$\frac{d^{2}u_{2}}{dy^{2}} + Ph^{2}\alpha - \alpha\sigma h^{2}S^{2}(R_{e} + u_{2}) = -\lambda \frac{du_{2}}{dy}$$
(3)

$$\frac{d^2\theta_2}{dy^2} + \lambda \frac{d\theta_2}{dy} + \frac{\beta}{\alpha} \left(\frac{du_2}{dy}\right)^2 + \beta \sigma h^2 S^2 (R_e + u_2)^2 = 0$$
(4)

The slip boundary conditions on velocity are

$$u_1(1) = -\Gamma \frac{du_1}{dy} \quad \text{at } y = 1 \tag{5}$$

$$u_{2}(-1) = \Gamma \frac{du_{2}}{dy}$$
 at y=-1 (6)

And the interface conditions on velocity are $u_1(0) = u_2(0)$ at y = 0

$$\frac{du_1}{dy} = \frac{1}{\alpha h} \frac{du_2}{dy}, \quad \text{at } y = 0$$
(8)

The slip boundary conditions on temperature are:

$$\theta_1(1) = -\Gamma \frac{d\theta_1}{dy} \quad \text{at } y=1$$
(9)

$$\theta_2(-1) = \Gamma \frac{d \theta_2}{d y} \quad \text{at } y = -1 \tag{10}$$

and interface conditions on temperature are:

$$\theta_1(0) = \theta_2(0)$$
 at y =0 (11)

$$\frac{d\theta_1}{dy} = \frac{1}{\beta h} \frac{d\theta_2}{dy} \qquad \text{at } y = 0 \tag{12}$$

In making these equations dimensionless, we use \overline{u}_1 , $h_i(i = 1, 2)$, $\mu_1 \overline{u}_1 / h_1^2$ and $\mu \overline{u}_1^2 / K_1$ the

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(7)

scales for velocities, distance, pressure and temperature respectively. S (the magnetic parameter) = B_0 $h_1(\sigma_1/\mu_1)^{1/2}$, α (ratio of viscosities) $=\frac{\mu_1}{\mu_2}$, σ (ratio of electrical conductivities) $=\frac{\sigma_2}{\sigma_1}$, β (ratio of thermal conductivities) $=\frac{K_1}{K_2}$, R_e (electric load parameter) $=E_z/B_0\overline{\mu_1}$, h(ratio of the heights) $=\frac{h_2}{h_1}$. Moreover, to

obtain the above dimensionless quantities, we use the non-dimensional parameter, λ (suction number) = hv_0/ν , in addition to the quantities, as already defined above. Further, the equs. (5) and (6) represent the slip conditions on velocities at upper and lower walls respectively, the conditions (7 - 8) represent the continuity of velocity and shear stress at the interface y = 0. The Conditions (9 - 10) represent the slip conditions on temperature at both the walls y = 1 and y = -1. Conditions (11 - 12) represent the continuity of temperature and heat flux at the interface y = 0. The equations (1) to (4) are in linear form, which are to be solved subject to the boundary and interface conditions as stated at equations (5) to (12).

3. SOLUTIONS OF THE PROBLEM

Exact solutions of the governing linear differential equations (1) and (3) using the boundary and interface conditions (5) to (8) for the velocity distributions such as u_1 and u_2 respectively in the two fluid regions are obtained as:

$$u_{1}(y) = c_{1}e^{\frac{(a-\lambda)}{2}y} + c_{2}e^{-\frac{(a+\lambda)}{2}y} - \frac{M}{s^{2}}$$
 (13)

$$u_{2}(y) = c_{3}e^{(\frac{b-\lambda}{2})y} + c_{4}e^{-(\frac{b+\lambda}{2})y} - \frac{N}{S^{2}h^{2}\sigma\alpha}$$
(14)

Further, we solve the non-dimensional energy equations (2) and (4) subject the boundary and interface conditions (9) to (12) for temperature distributions in the two fluid regions, by making use of the solutions (13) and (14), as already obtained for u_1 and u_2 . Hence, the solutions for temperature distributions θ_1 and θ_2 in the two fluid regions are obtained as:

$$\begin{aligned} \theta_{1}(y) &= c_{5} + c_{6}e^{-\lambda y} + \frac{t_{1y}}{\lambda} + \frac{t_{2}}{a(a-\lambda)}e^{(a-\lambda)y} + \frac{t_{3}}{a(a+\lambda)}e^{-(a+\lambda)y} - \frac{yt_{4}}{\lambda}e^{-\lambda y} + 4\frac{t_{5}}{a^{2}-\lambda^{2}}e^{\frac{(a-\lambda)}{2}y} + 4\frac{t_{6}}{a^{2}-\lambda^{2}}e^{\frac{-(a+\lambda)y}{2}y} \\ \theta_{2}(y) &= c_{7} + c_{8}e^{-\lambda y} + t_{44}e^{(b-\lambda)y} + t_{45}e^{-(b+\lambda)y} + t_{46}e^{-\lambda y} + t_{47}e^{\frac{(b-\lambda)y}{2}y} + t_{48}e^{\frac{-(b+\lambda)y}{2}y} + t_{49}y, \ (16) \end{aligned}$$

The numerical computations of the velocity and temperature distributions in both the regions are performed for different sets of values of the governing parameters involved in the study and these results are presented graphically. It is to be noted that, while computing the results, the value of P is fixed at 2. The constants involved in the above mentioned solutions are given as in Appendix.

RESULTS AND DISCUSSION

Hydromagnetic two-layered fluid flow and heat transfer in a horizontal channel, driven by a constant pressure gradient accounting for wall porosity applied normal to the parallel walls is investigated. The flow is supposed to undergo slipping at both the porous walls with a velocity proportional to the shear stress there. The two fluids in the two regions are considered to be incompressible and electrically conducting with different viscosities, thermal and electrical conductivities. The governing linear differential equations are solved analytically to obtain the exact solutions for velocity distributions, such as, u_1 and u_2 respectively in the two fluid regions. Closed form solutions for temperature distributions, namely, θ_1 and θ_2 in the two fluid regions are determined by making use of the already derived solutions of velocity distributions. The profiles are plotted after obtaining the numerical values for different sets of values of the governing parameters involved such as, porous parameter λ , slip parameter Γ . magnetic parameter S, the ratio of viscosities α , ratio of electrical conductivities σ , ratio of thermal

conductivities β and the height ratio h. Also it is discussed the behavior of the flow and heat transfer characteristics by analyzing the governing parameters involved. The graphs for the velocity and temperature distributions are shown in Figures 2 to 15. It is found that there is a good agreement in results between the present study and existing available one in the literature for the reduced cases. Here we note that when $\Gamma = 0$ and $\beta = 0$, that is in the case of no-slip at both walls and for non porous walls, these results are in agreement with those of Malashetty and Leela [10].

From Fig.2, the velocity distributions in both the regions, that is at upper and lower fluid regions are found to increase with an increase in magnetic parameter, S, when all the remaining parameter held fixed. Also it is observed that the velocity distribution in the upper fluid region is higher than that at the lower region. From Fig.3, it is noticed that the velocity distributions in both the regions, that is at upper and lower fluid regions are found to increase with an increase in porous parameter λ . It is also found that the velocity distribution in the upper fluid region is higher than that at the lower region. It is seen from Fig.4 that, the velocity distributions in both the regions are increasing with an increase in σ . Also it is observed that the velocity distribution in the upper fluid region is higher than that at the lower region. From Fig.5, the velocity distributions in both the regions are found to increase with an increase in h. From Fig.6, the velocity distributions in both the regions have a tendency to increase with an increase in α . From Fig. 7, it is noticed that, an increase in β enhances the velocity distributions in both the regions. Also it is observed that the velocity distribution in the upper fluid region is higher than that at the lower region. From Fig.8, the velocity distributions in both the regions are found to increase with an increase in $\hat{\Gamma}$. From Fig. 9, it is observed that as $\hat{\Gamma}$ increases the temperature increases in the two regions when all the remaining parameters are held fixed. From Fig. 10, it is noticed that, as α increases, the temperature increases in the two regions. From Fig. 11, as h increases, there is a raise in temperature at the two regions when all the remaining parameters are fixed. From Fig. 12, the temperature distribution is found to in increase with an increase in β , when all the remaining parameters are kept fixed. From Fig. 13, it is observed that, the temperature distributions in both the regions, that is at upper and lower fluid regions are found to increase with an increase in magnetic parameter S. Also, from figures 14 and 15, it is noticed that the temperature distribution is found to increase with an increase in electrical conductivity ratio σ and the porous parameter λ .

$$\begin{array}{l} \begin{array}{l} \textbf{APPENDIX} \\ a = \sqrt{\lambda^2 + 4S^2}, \quad b = \sqrt{\lambda^2 + 4S^2h^2\sigma\alpha}, \quad c = S^2k_1 - p, \quad d = S^2h^2\sigma\alpha k_1 - ph^2\alpha, \\ d_1 = \frac{a - \lambda}{2}, \quad d_2 = \frac{-(a + \lambda)}{2}, \quad d_3 = e^{d_1}(1 + \Gamma d_1), \quad d_4 = e^{d_2}(1 + \Gamma d_2), \\ d_5 = \frac{c}{S^2}, \quad d_6 = -(\frac{b - \lambda}{2}), \quad d_7 = (\frac{b + \lambda}{2}), \quad d_8 = \frac{d}{S^2h^2\sigma\alpha}, \quad d_9 = e^{d_6}(1 + \Gamma d_6), \\ d_{10} = e^{d_7}(1 + \Gamma d_7), \quad d_{11} = d_5 - d_8, \quad d_{12} = \alpha h(a - \lambda), \quad d_{13} = -\alpha h(a + \lambda), \quad d_{14} = b - \lambda, \\ d_{15} = -(b + \lambda) \quad d_{16} = d_{14} - d_{12}, \quad d_{17} = d_{14} - d_{13}, \quad d_{18} = d_{14} - d_{15}, \quad d_{19} = d_{11}d_{14}, \\ d_{20} = d_9 - d_{10} \quad d_{21} = d_{11}d_9 + d_8, \quad d_{22} = d_{16}d_{20} - d_9d_{18}, \quad d_{23} = d_{17}d_{20} - d_9d_{18}, \end{array}$$

$$\begin{split} & d_{24} = d_{19}d_{20} - d_{18}d_{21}, \ d_{25} = \frac{d_{24}d_{3} - d_{5}d_{22}}{d_{23}d_{3} - d_{4}d_{22}}, \ d_{26} = \frac{d_{5} - d_{4}d_{25}}{d_{3}}, \\ & d_{27} = \frac{d_{9}d_{26} + d_{9}d_{25} - d_{21}}{d_{20}}, \ d_{28} = \frac{d_{8} - d_{27}d_{10}}{d_{9}}, \ c_{1} = d_{26}, \ c_{2} = d_{25}, \ c_{3} = d_{28}, \\ & c_{4} = d_{27}, t_{1} = \frac{-(S^{2}k_{1} - c)^{2}}{S^{2}}, \ t_{2} = -\frac{c_{1}^{2}}{4}(4S^{2} + (a - \lambda)^{2}), \ t_{3} = -\frac{c_{2}^{2}}{4}(4S^{2} + (a + \lambda)^{2}), \\ & t_{4} = \frac{-c_{1}^{2}}{2}(4S^{2} - a^{2} + \lambda^{2}), \ t_{5} = -2c_{1}(S^{2}k_{1} - c), \ t_{6} = -2c_{2}(S^{2}k_{1} - c), \ t_{7} = \frac{t_{2}}{a(a - \lambda)}, \ t_{8} = \frac{t_{3}}{a(a + \lambda)}, \\ & t_{9} = \frac{4t_{5}}{a^{2} - \lambda^{2}}, \ t_{10} = \frac{4t_{6}}{a^{2} - \lambda^{2}}, \ t_{11} = \frac{t_{1} - t_{4}}{\lambda}, \ t_{12} = \frac{t_{2} - t_{3}}{a}, \ t_{43} = -\frac{2t_{5}}{a + \lambda}, \\ & t_{44} = \frac{-2t_{6}}{a^{2} - \lambda^{2}}, \ t_{5} = t_{7} + t_{8} + t_{9} + t_{10}, \ t_{16} = t_{11} + t_{12} + t_{13} + t_{14}, \ t_{17} = t_{7}e^{a - \lambda}, \ t_{18} = t_{8}e^{-} (a + \lambda) \\ & t_{19} = t_{9}e^{\frac{a - \lambda}{2}}, \ t_{15} = t_{7} + t_{8} + t_{9} + t_{10}, \ t_{16} = t_{11} + t_{12} + t_{13} + t_{14}, \ t_{17} = t_{7}e^{a - \lambda}, \ t_{18} = t_{8}e^{-} (a + \lambda) \\ & t_{19} = t_{9}e^{\frac{a - \lambda}{2}}, \ t_{20} = t_{10}e^{-\frac{(a + \lambda)}{2}}, \ t_{21} = \frac{1}{\lambda}(t_{1} - t_{4}e^{-\lambda}), \ t_{22} = t_{17} + t_{18} + t_{19} + t_{20} + t_{21}, \\ & t_{23} = \frac{t_{23}}{a} + t_{24} + t_{25} + t_{26} + t_{27}, \ t_{29} = c_{3}(\frac{b - \lambda}{2}), \ t_{30} = -c_{4}(\frac{b + \lambda}{2}), \ t_{31} = Re - \frac{d}{s^{2}h^{2}\sigma\alpha}, \\ & t_{22} = -t_{31}c_{3}S^{2}h^{2}\sigma\beta, \ t_{33} = -c_{3}S^{2}h^{2}\sigma\beta, \ t_{38} = -\frac{\beta}{\alpha}t_{29}^{2}^{2}, \ t_{39} = -\frac{\beta}{\alpha}t_{30}^{2}^{2}, \\ & t_{40} = -2\frac{\beta}{\alpha}t_{29}t_{30}, \ t_{41} = t_{38} + t_{33}, t_{42} = t_{39} + t_{34}, \ t_{43} = t_{40} + t_{37}, \ t_{44} = \frac{t_{41}}{b(b - \lambda)}, \\ & t_{45} = \frac{t_{42}}{b(b + \lambda)}, \ t_{46} = -\frac{t_{43}}{\lambda}, \ t_{47} = 4\frac{t_{35}}{b^{2} - \lambda^{2}}, \ t_{48} = 4\frac{t_{36}}{b^{2} - \lambda^{2}}, \ t_{49} = \frac{t_{32}}{\lambda}, \ t_{50} = \frac{t_{41}}{h}, \\ & t_{51} = -\frac{t_{42}}}{b}, \ t_{52} = -\frac{t_{43}$$

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Fig. 1. Physical configuration and flow model



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