# Heat transfer in MHD flow of a dusty fluid over a stretching sheet with viscous dissipation

## B.J.Gireesha*, G.K.Ramesh and C.S.Bagewadi

*Department of Studies and Research in Mathematics, Kuvempu University,
Shankaraghatta, Shimoga, Karnataka, INDIA.*

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**ABSTRACT**

*An analysis has been carried out to study the magnetohydrodynamic boundary layer flow and heat transfer characteristics of a dusty fluid over a flat stretching sheet in the presence of viscous dissipation. The basic equations governing the flow and heat transfer are in the form of partial differential equations, the same have been reduced to a set of non-linear ordinary differential equations by applying suitable similarity transformation. The transformed equations are solved numerically by applying Runge Kutta Fehlberg fourth-fifth order method (RKF45 Method). The effects of fluid-particle interaction parameter, Chandrasekhar number, Prandtl number, Eckert number on heat transfer characteristics for two general cases namely, the prescribed surface temperature (PST) case and the prescribed wall heat flux (PHF) case are presented graphically and discussed. The skin friction and heat transfer coefficients are tabulated for a range of values of the parameters. Comparison of the obtained numerical results is made with existing literature.*

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## INTRODUCTION

The flow and heat transfer of a viscous and incompressible fluid induced by a continuously moving or stretching surface in a ambient fluid is relevant to the field of chemical engineering processes. Many chemical engineering processes like metallurgical process, polymer extrusion process involves cooling of a molten liquid being stretched into a cooling system. In such processes the fluid mechanical properties of the penultimate product would mainly depend on two things, one is the cooling liquid used and other is the rate of stretching. Some of the polymer fluids such as Polyethylene oxide, polyisobutylene solution in cetane, having better electromagnetic properties, are recommended as their flow can be regulated by external magnetic fields. An extreme care has to be given to control the rate at which in place of cooling liquids the extradite is stretched, rapid stretching results in sudden solidification thereby destroying the properties expected for the outcome. The problem addressed here is a fundamental one that arises in many practical situations such as polymer extrusion process. To name some of them, drawing, annealing and tinning of copper wires, continuous stretching, rolling and manufacturing of plastic films and artificial fibres, materials manufactured by extrusion process and heat treated materials traveling between a feed roll and windup rolls or on conveyer belts, glass blowing, crystal growing, paper production.

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The behavior of boundary layer flow due to a moving flat surface immersed in an otherwise quiescent fluid was first studied by Sakiadis [1], who investigated it theoretically by both exact and approximate methods. Crane [2] presented a closed form exponential solution for the planar viscous flow of linear stretching case. Later this problem has been extended to various aspects by considering non-Newtonian fluids, more general stretching velocity, magnetohydrodynamic (MHD) effects, porous sheets, porous media and heat or mass transfer. Andreson et.al [3] extended the work of Crane [2] to non-Newtonian power law fluid over a linear stretching sheet. Chakrabarti and Gupta [4] have discussed the hydromagnetic flow and heat transfer over a stretching sheet.

Gebhart [5] was the first author who studied the problem taking into account the viscous dissipation. The MHD and viscous dissipation effects of the heat transfer analysis were studied many authors such as Mahmoud [6], Vajravelu and Hadjinicalaou [7], Samad et.al [8] and Anjali Devi [9]. Further, Grubka and Bobba [10] analyzed heat transfer studies by considering the power-law variation of surface temperature. Cortell [11] studied the magnetohydrodynamics flow of a power-law fluid over a stretching sheet. Chen [12] analyzed mixed convection of a power law fluid past a stretching surface in the presence of thermal radiation and magnetic field. Power law model has some limitations as, it does not exhibit any elastic properties such as normal stress differences in shear flow. In certain polymer processing applications, flow of a viscoelastic fluid over a stretching sheet is important. On the basis of this reason Cortell [13] studied the effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Abel et.al [14] extended the work of [13] and studied the viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipation. Tsai et.al [15] studied an unsteady flow over a stretching surface with non-uniform heat source. Ishak et.al [16] obtained the solution to unsteady laminar boundary layer over a continuously stretching permeable surface.

To study the two-phase flows, in which solid spherical particles are distributed in a fluid are of interest in a wide range of technical problems, such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles, and blood rheology etc., The study of the boundary layer flow of fluid-particle suspension flow is important in determining the particle accumulation and impingement of the particle on the surface. In view of these applications, Chakrabarti [17] analyzed the boundary layer flow of a dusty gas. Datta and Mishra [18] have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Further, researches in these fields have been studied by many mathematicians such as Evgeny and Sergei [19], XIE Ming-liang et.al [20], Palani et.al [21], Agranat [22] and Vajravelu et.al [23]. Abdul Aziz [24] obtained the numerical solution for laminar thermal boundary over a flat plate with a convective surface boundary condition using the symbolic algebra software Maple. Further he has found the similarity solution for existence of the energy equation, if the heat transfer coefficient $h_f$ is proportional to $x^{-\frac{1}{2}}$, where $x$ is the distance from the leading edge of the plate. Gireesha et. al [26, 27] studied boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with non-uniform heat source/sink. Further, they have obtained the solution to boundary layer flow and heat transfer of a dusty fluid over a stretching vertical surface with the help of Maple software. Kishan and Deepa [28] studied the effect of viscous dissipation on stagnation point flow and heat transfer of a micropolar fluid with uniform suction /blowing. Gaikwad and Rahuldev [29] analyzed the viscous dissipation effect of permeable fluid on laminar mixed convection in a vertical double passage channel.

In view of the above discussion, present analysis is envisage to investigate two-dimensional study state incompressible boundary layer flow of a dusty fluid over a stretching sheet. Analysis on heat transfer is also carried out taking into the effect of viscous dissipation and magnetic field. In studying the heat transfer characteristics, two different types of boundary conditions are considered namely, PST and PHF boundary conditions. Highly non-linear momentum and heat transfer equations are solved numerically using RKF45 method. In the present investigation, we analyzed the effect of various physical parameters like fluid particle interaction parameter, Chandrasekher number, Prandtl number and Eckert number.

**Flow analysis of the problem**
Consider a steady two dimensional laminar boundary layer flow of an incompressible viscous dusty fluid over a vertical stretching sheet. The flow is generated by the action of two equal and opposite forces along the $x$-axis and $y$-axis being normal to the flow. The sheet being stretched with the velocity $U_w(x)$ along the $x$-axis, keeping the origin fixed. Further the flow field is exposed to the influence of an external transverse magnetic field of strength $H_0$ (along y-axis). Both the fluid and dust particle clouds are suppose to be static at the beginning. The dust particles are

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assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow.

The momentum equations of the two dimensional boundary layer flow in usual notation are [23]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(u_p - u) - \frac{\sigma H_0^2 u}{\rho}, \tag{2.2}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{K}{m}(u - u_p), \tag{2.3}$$

$$u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y} = \frac{K}{m}(v - v_p), \tag{2.4}$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0, \tag{2.5}$$

where $(u, v)$ and $(u_p, v_p)$ are the velocity components of the fluid and dust particle phases along $x$ and $y$ directions respectively. $\mu$, $\rho, \rho_p$ and $N$ are the co-efficient of viscosity of the fluid, density of the fluid, density of the dust phase, number density of the particle phase, $H_0$ is the strength of applied magnetic field, $K$ is the stokes' resistance (drag co-efficient, $m$ is the mass of the dust particle respectively. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. In deriving these equations, the drag force is considered for the interaction between the fluid and particle phases.

The boundary conditions for the flow problem are given by
$$u = U_w(x), v = 0 \text{ at } y = 0,$$
$$u \to 0, u_p \to 0, \ v_p \to v, \rho_p \to k\rho \text{ as } y \to \infty, \tag{2.6}$$

where $U_w(x) = cx$ is a stretching sheet velocity, $c > 0$ is stretching rate, $k$ is the density ratio.

To convert the governing equations into a set of similarity equations, we now introduce the following transformation as,

$$u = cx f'(\eta), v = -\sqrt{\nu c}\, f(\eta), \eta = \sqrt{\frac{c}{\nu}}\, y, \tag{2.7}$$
$$u_p = cx F(\eta), v_p = \sqrt{\nu c}G(\eta), \rho_r = H(\eta).$$

which are identically satisfies (2.1). Substituting (2.7) into (2.2) to (2.5), we obtain the following non-linear ordinary differential equations,

$$f'''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 - Qf'(\eta) + l^*\beta H(\eta)[F(\eta) - f'(\eta)] = 0 \tag{2.8}$$

$$G(\eta)F'(\eta) + [F(\eta)]^2 + \beta[F(\eta) - f'(\eta)] = 0 \tag{2.9}$$

$$G(\eta)G'(\eta) + \beta[f(\eta) + G(\eta)] = 0 \tag{2.10}$$

$$H(\eta)F(\eta) + H(\eta)G'(\eta) + G(\eta)H'(\eta) = 0 \tag{2.11}$$

where a prime denotes differentiation with respect to $\eta$ and $l^* = mN/\rho$, $\tau = m/k$ is the relaxation time of the particle phase, $\beta = 1/c\tau$ is the fluid particle interaction parameter, $Q = \frac{\sigma H_0^2}{c\rho}$ is the Chandrasekhar number and $\rho_r = \rho_p/\rho$ is the relative density.

The boundary conditions defined as in (2.6) will becomes,
$$\boldsymbol{f(\eta) = 0, f'(\eta) = 1} \text{ at } \boldsymbol{\eta = 0} \tag{2.12}$$

$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta), H(\eta) = k \text{ as } \eta \to \infty$$

If $\beta = 0$, the analytical solution of (2.8) with boundary condition (2.12) can be written in the form $f(\eta) = \frac{1 - e^{\xi\eta}}{\xi}$, where $\xi = \sqrt{Q + 1}$

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### 2.      Heat Transfer Analysis
The governing dusty boundary layer heat transport equations in the presence of viscous dissipation for two dimensional flow is given by [25]

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = k^* \frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{\tau_T}\left(T_p - T\right) + \frac{N}{\tau_v}\left(u_p - u\right)^2 + \mu \left(\frac{\partial u}{\partial y}\right)^2 \qquad (3.1)$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = -\frac{c_p}{c_m \tau_T}(T_p - T) \qquad (3.2)$$

where $T$ and $T_p$ is the temperature of the fluid and temperature of the dust particle, $c_p$ and $c_m$ are the specific heat of fluid and dust particles, $\tau_T$ is the thermal equilibrium time and is the time required by the dust cloud to adjust its temperature to the fluid, $\tau_v$ is the relaxation time of the of dust particle i.e., the time required by a dust particle to adjust its velocity relative to the fluid, $k^*$ is the thermal conductivity.

The solution of the equations (3.1) and (3.2) depends on the nature of the prescribed boundary conditions. We employ two types of heating process as follows:
(1) PST (Prescribed Power law Surface Temperature),
(2) PHF (Prescribed Power law Heat Flux).

### CASE-1: Prescribed Surface Temperature (PST-Case)
The boundary conditions in case of prescribed power law surface temperature are of the form

$$T = T_w = T_\infty + A \left(\frac{x}{l}\right)^2 \text{ at } y = 0, \qquad (3.3)$$
$$T \to \infty, T_p \to T_\infty \text{ as } y \to \infty,$$

where $T_w$ and $T_\infty$ denote the temperature at the wall and at large distance from the wall respectively, $A$ is a positive constant, $l = \sqrt{\frac{v}{c}}$ is a characteristic length.

Now define the non-dimensional fluid phase temperature $\theta(\eta)$ and dust phase temperature $\theta_p(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \ \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \qquad (3.4)$$

where $T - T_\infty = A \left(\frac{x}{l}\right)^2 \theta(\eta)$.

Using (3.4) and (3.1) to (3.2), we obtain the following non-linear ordinary differential equations

$$\theta''(\eta) + \Pr[f(\eta)\theta'(\eta) - 2f(\eta)'\theta(\eta)] + \frac{NPr}{\rho c \, \tau_T}\Big(\theta_p(\eta) - \theta(\eta)\Big)$$
$$+ \frac{N}{\rho \tau_v} Pr \, Ec\big(F(\eta) - f'(\eta)\big)^2 + PrEcf''^2(\eta) = 0, \quad (3.5)$$

$$2F(\eta)\theta_p(\eta) + G(\eta)\theta_p'(\eta) + \frac{c_p}{c c_m \tau_T}\Big(\theta_p(\eta) - \theta(\eta)\Big) = 0, \qquad (3.6)$$

where $Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $Ec = \frac{c l^2}{A c_p}$ is the Eckert number,

The corresponding boundary conditions for $\theta(\eta)$ and $\theta_p(\eta)$ will becomes

$$\theta(\eta) = 1 \text{ at } \eta = 0, \qquad (3.7)$$
$$\theta(\eta) \to 0, \theta_p(\eta) \to 0 \text{ as } \eta \to \infty.$$

### CASE-2: Prescribed Heat Flux (PHF-Case)
The power law heat flux on the wall surface is considered to be a quadratic power of $x$ in the form

$$-k^* \frac{\partial T}{\partial y} = q_w = D \left(\frac{x}{l}\right)^2 \text{ at } y = 0, \qquad (3.8)$$
$$T \to T_\infty, T_p \to T_\infty \text{ as } y \to \infty,$$

where $D$ is the positive constant. On the other hand define a non-dimensional temperatures g$(\eta)$ and dust phase temperature $g_p(\eta)$ as

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$$g(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \quad g_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \tag{3.9}$$

where $T_w - T_\infty = \frac{D}{k^*}\left(\frac{x}{l}\right)^2 \sqrt{\frac{v}{c}}.$

Equations (3.1) to (3.2) on using (3.9) can be transformed in terms of $g(\eta)$ and $g_p(\eta)$ as

$$g''(\eta) + \Pr[f(\eta)g'(\eta) - 2f(\eta)'g(\eta)] + \frac{NPr}{\rho c\,\tau_T}\left(g_p(\eta) - g(\eta)\right)$$
$$+ \frac{N}{\rho\tau_v}\,Pr\,Ec\left(F(\eta) - f'(\eta)\right)^2 + PrEcf''^2(\eta) = 0, \tag{3.10}$$

$$2F(\eta)g_p(\eta) + G(\eta)g_p'(\eta) + \frac{c_p}{cc_m\tau_T}\left(g_p(\eta) - g(\eta)\right) = 0, \tag{3.11}$$

where $Ec = (k^*l^2c^{\frac{3}{2}})/(Dc_p v^{\frac{1}{2}})$ is the Eckert number. The corresponding boundary conditions to this case will becomes

$$g(\eta) = -1 \text{ at } \eta = 0, \tag{3.12}$$
$$g(\eta) \to 0, g_p(\eta) \to 0 \text{ as } \eta \to \infty.$$

## 3. Numerical Solution

Equations (2.8) to (2.12), (3.5) to (3.7) and (3.10) to (3.12) are highly non-linear ordinary differential equations. To solve these equations we adopted symbolic algebra software Maple, which was given by Aziz [24].It is very efficient in using the well known Runge Kutta Fehlberg fourth-fifth order method (RKF45 Method) to obtain the numerical solutions of a boundary value problem. In order to verify the accuracy of our present method, a comparison of velocity gradient – $f''(0)$ with those reported by Cortell [11] for various values of Chandrasekhar number is given in Table 1. The comparisons in all the above cases are found to be in excellent agreement.

**Table 1: Comparison results for the function** $-f''(0)$ **for several values of** $Q$ **in the case of** $\beta = 0$, $Pr = 0$, $Ec = 0$ **and** $N = 0$.

| | Cortell[11] | Present Study | |
|---|---|---|---|
| $Q$ | $-f''(0)$ | $-f''(0)$ at $\beta = 0$ | $-f''(0)$ at $\beta = 0.5$ |
| 0.0 | 1.000 | 1.000 | 1.034 |
| 0.2 | 1.095 | 1.095 | 1.126 |
| 0.5 | 1.224 | 1.224 | 1.252 |
| 1.0 | 1.414 | 1.414 | 1.438 |
| 1.2 | 1.483 | 1.483 | 1.506 |
| 1.5 | 1.581 | 1.581 | 1.602 |
| 2.0 | 1.732 | 1.732 | 1.751 |

## RESULTS AND DISCUSSION

An MHD boundary layer flow and heat transfer of a dusty fluid over a stretching sheet is investigated in presence of viscous dissipation. Numerical solutions are presented for highly non-linear thermal boundary layer equations, where the former is achieved by Runge Kutta Fehlberg fourth-fifth order method. Numerical computation of these solutions have been carried out to study the effect of various physical parameters such as fluid particle interaction parameter $\beta$, Chandrasekhar number $Q$, Prandtl number $Pr$ and Eckert number $Ec$. Further, the impact of some important physical parameters on wall temperature gradient $\theta'(0)$ and wall temperature $g(0)$ may be analyzed from Table 2. Sets of representative numerical results are illustrated graphically.
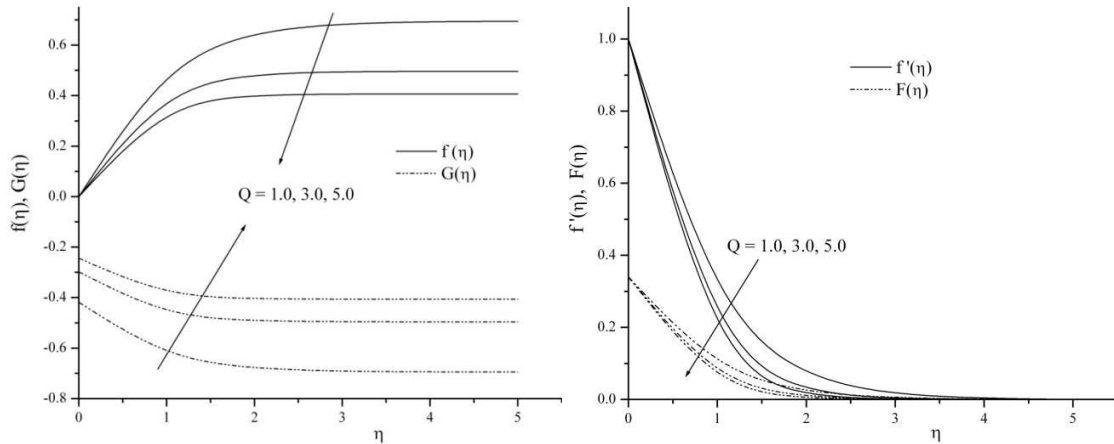
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**Figure-1: Variation of transverse velocity ($f$), non-dimensional particle velocity($G$), fluid velocity ($f'$) and particle velocity ($F$) components for several values of $Q$.**

Figures 1 shows the effect of Chandrashekhar number on velocity components of the fluid velocity $f'(\eta)$, transverse velocity $f(\eta)$, particle velocity $F(\eta)$ and non-dimensional particle velocity $G(\eta)$. From these plots it is observed that the increasing $Q$ clearly escalates the magnitude of the Lorentz retarding hydromagnetic body force which serves to retard the flow considerably but in non-dimensional particle velocity it is contrast. Also we can seen from the Table 1, $f''(0)$ is negative. Physically, negative values of $f''(0)$ means the solid surface exerts a drag force on the fluid. This is not surprising since the development of the velocity boundary layer is caused exclusively on the stretching plate.
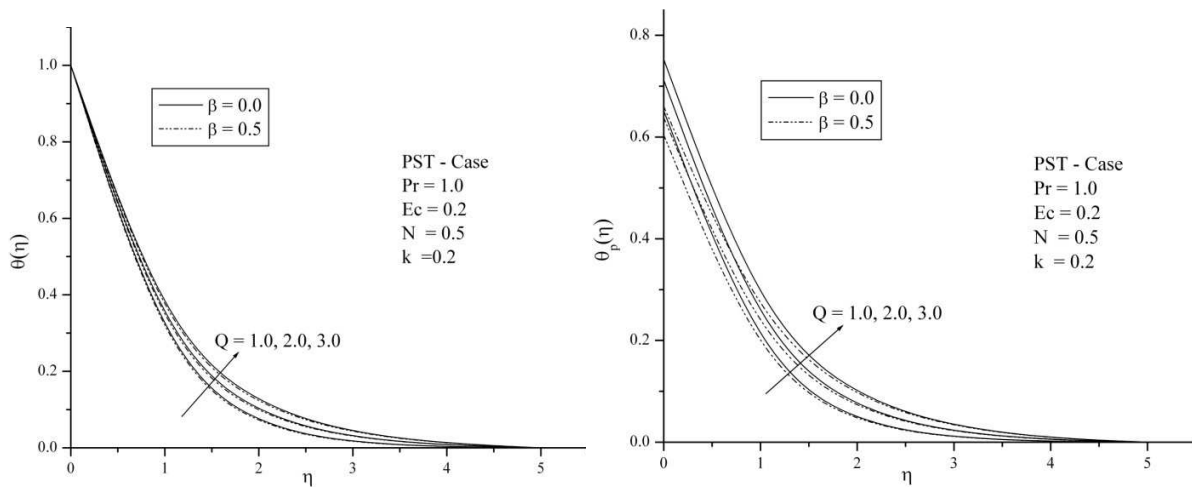


**Figure-2(a):Effect of Chandrasekhar number ($Q$) on temperature distribution for PST case.**
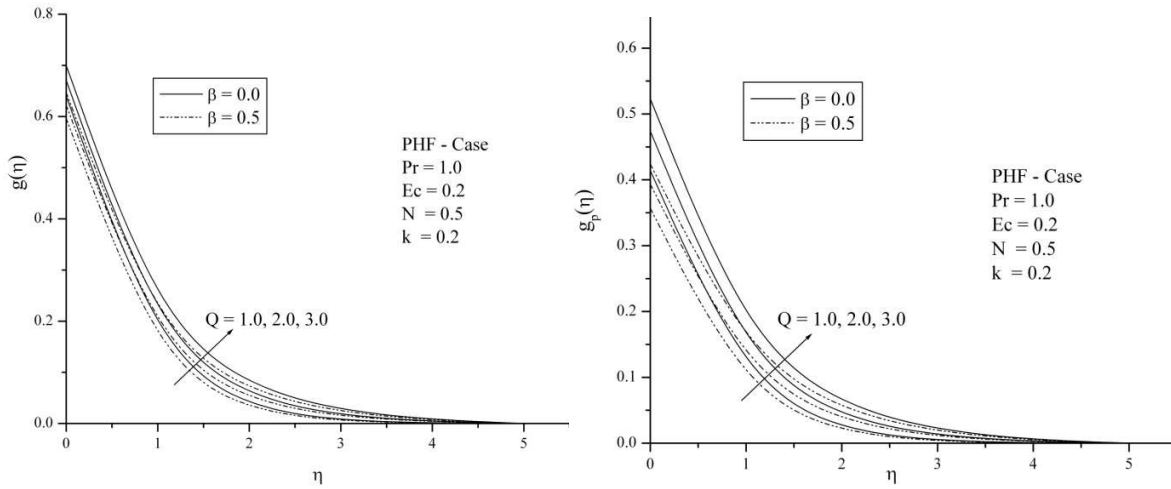
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**Figure-2(b):Effect of Chandrasekhar number ($Q$) on temperature distribution for PHF case.**

Figures 2(a) and 2(b) depict the temperature profiles $\theta(\eta), \theta_p(\eta)$ versus $\eta$ for the PST case and $g(\eta), g_p(\eta)$ versus $\eta$ for the PHF case respectively. From these plots it is observed that the transverse magnetic field contributes to the thickening of thermal boundary layer. It is evident from these graphs that an applied transverse magnetic field produces a body force, called a Lorentz force, which opposes the motion. The resistance offered to the flow is responsible in enhancing the temperature.
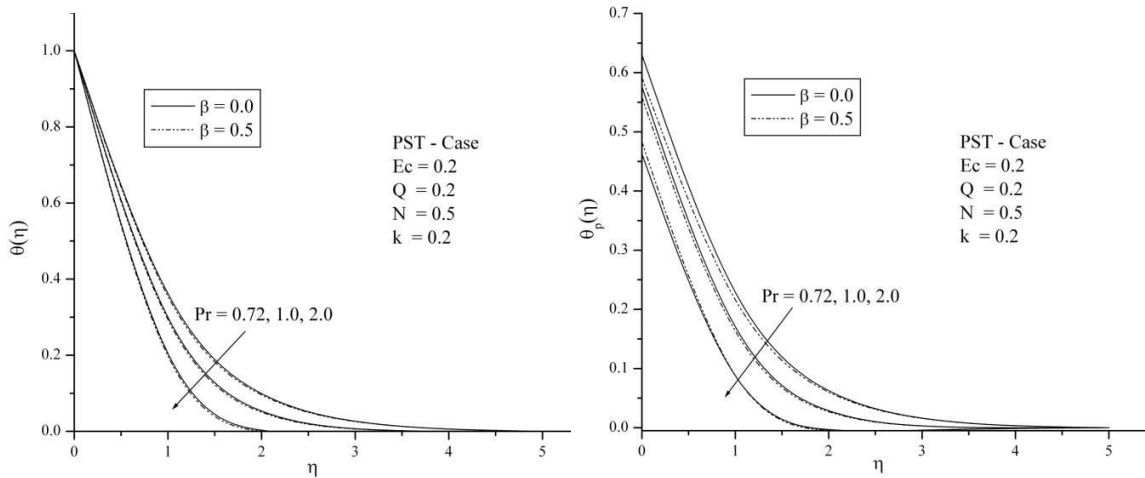


**Figure-3(a): Effect of Prandtl number ($Pr$) on temperature distribution for PST case.**
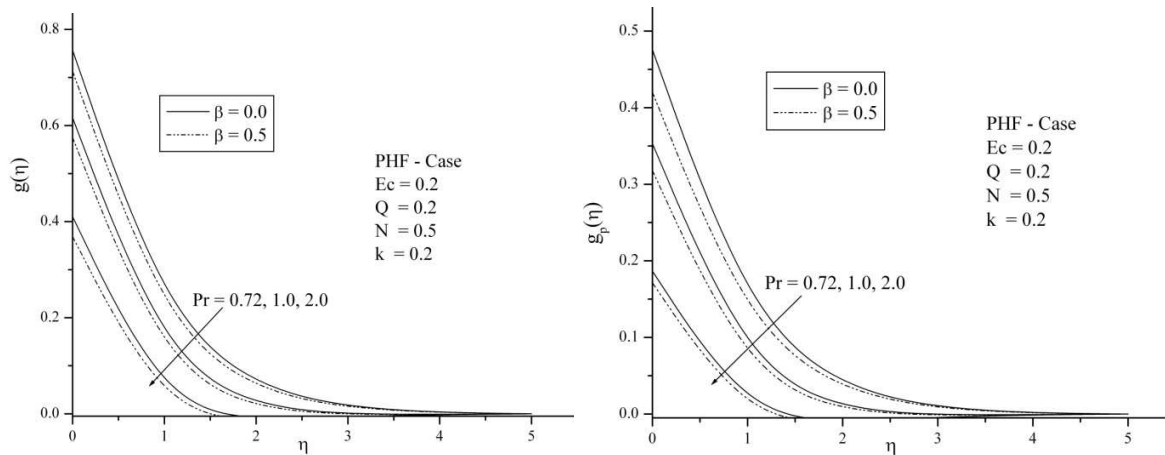
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**Figure-3(b): Effect of Prandtl number ($Pr$) on temperature distribution for PHF case**.

The effect of Prandtl number on the heat transfer is shown in Figures 3(a) and 3(b). By analyzing these graphs it reveals that the effect of increasing the Pr is to decreases the temperature distribution in the flow region in both PST and PHF cases, it is evident that large values of Prandtl number results in thinning of thermal boundary layer. This is in contrast to the effects of other parameters on heat transfer. We have used throughout our thermal analysis the following values for different parameters, like $\tau_T = \tau_v = 0.5$ and $c_p = c_m = 0.2, \rho = 0.5, c = 1$.
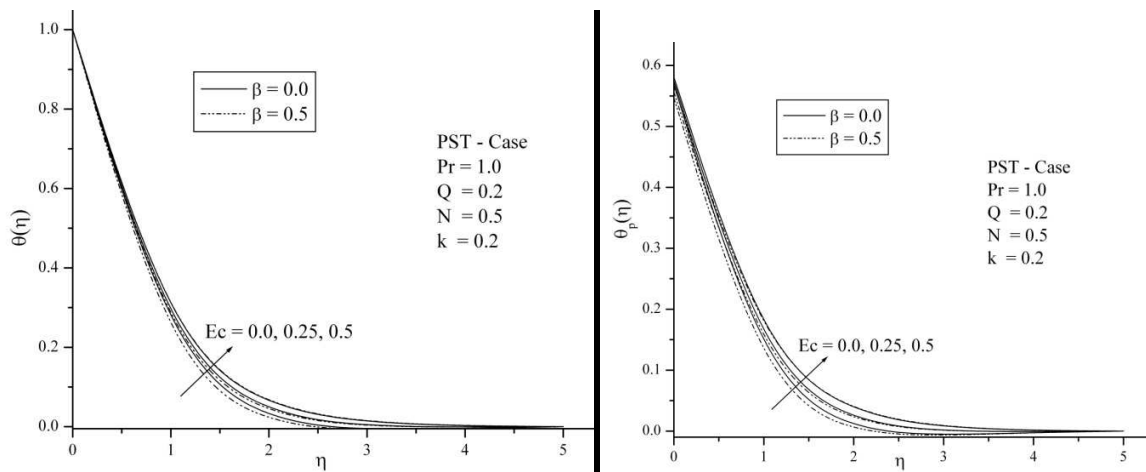


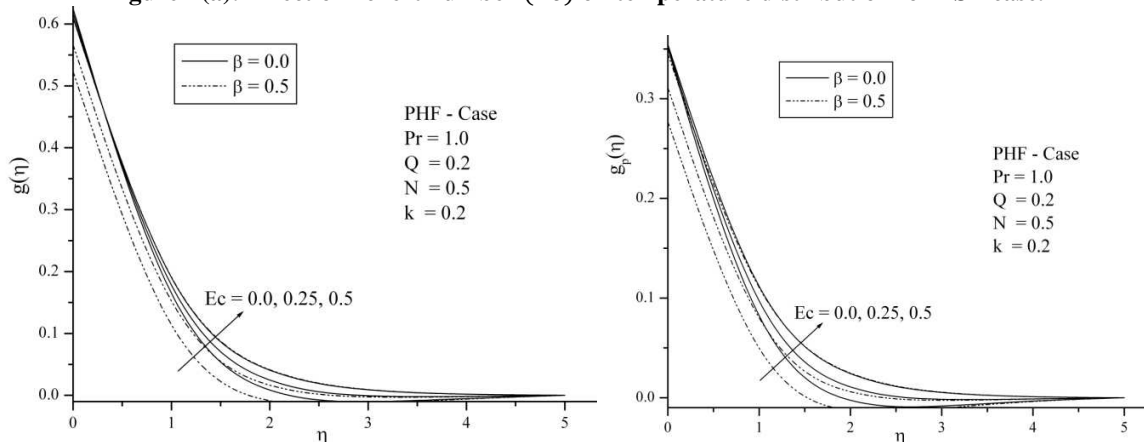**Figure-4(a): Effect of Eckert number ($Ec$) on temperature distribution for PST case.**



**Figure-4(b): Effect of Eckert number ($Ec$) on temperature distribution for PHFcase.**

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Figures 4(a) and 4(b) is plotted for the temperature profiles for PST and PHF cases respectively, for different values of Ec. One can observe that the effect of increasing values of Eckert number is to enhance the temperature at a point which is true for both the PST and PHF cases of the fluid phase as well as dust phase particle. It is observed that the effect of viscous dissipation is to amplify the temperature both in PST and PHF cases. Also it is observed that the fluid phase temperature is higher than the dust phase temperature and also it indicates that the fluid particle temperature is parallel to that of dust particle.

**Table 2: Values of wall temperature gradient $-\theta'(0)$ for different values of $\beta, Gr, Pr, Nr, f_0$ and $Ec$.**

| $\beta$ | $Ec$ | $Pr$ | $Q$ | PST case $-\theta'(0)$ | PHF case $g0)$ |
|---|---|---|---|---|---|
| 0.0 | 0.2 | 1.0 | 3.0 | 1.3966 | 0.7014 |
| 0.2 | | | | 1.4434 | 0.6720 |
| 0.4 | | | | 1.4729 | 0.6542 |
| 0.5 | 0.0 | 1.0 | 3.0 | 1.6486 | 0.5279 |
| | 0.25 | | | 1.5113 | 0.6278 |
| | 0.5 | | | 1.3741 | 0.7277 |
| 0.5 | 0.2 | 1.0 | 3.0 | 1.4839 | 0.6478 |
| | | 1.2 | | 1.9619 | 0.6478 |
| | | 2.0 | | 2.3713 | 0.3730 |
| 0.5 | 0.2 | 1.0 | 0.0 | 1.6227 | 0.5964 |
| | | | 2.0 | 1.5470 | 0.6226 |
| | | | 3.0 | 1.4839 | 0.6478 |

**CONCLUSION**

Mathematical analysis has been carried out to study the Heat transfer in MHD flow of a dusty fluid over stretching sheet with viscous dissipation. The governing partial differential equations are converted into ordinary differential equations by using similarity transformations. The effect of several parameters controlling the velocity and temperature profiles is shown graphically and discussed briefly. The influence of the parameters $\beta, Q, Ec$ and $Pr$ on dimensionless temperature profiles were examined. Some of the important observations of our analysis obtained by the graphical representation are reported as follows.

1) Effect of Chandrasekhar number is to increase temperature distributions in the flow region in both the cases of PST and PHF for both the phases.
2) The strength of external magnetic field should be as mild as possible for effective cooling of the stretching sheet.
3) Fluid phase temperature is higher than the dust phase temperature.
4) The rate of heat transfer $-\theta(0)$ and $g(0)$ decreases with increasing the Prandtl number and fluid-particle interaction parameter. While it increases with increasing the Eckert number.
5) The effect of Prandtl number is to decreases the thermal boundary layer thickness.
6) The PHF boundary condition is better suited for effective cooling of the stretching sheet.
7) The limit $\beta \rightarrow 0$ our results are coincide with the results of Cortell [11].

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