

Heat Transfer in an Ionized Hydro Magnetic Slip-Flo between Parallel Walls in a Rotating System with Hall Effect

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ABSTRACT

An attempt has been made to examine the temperature distribution due to MHD flow of an ionized gas between two parallel walls in a rotating frame of reference under the slip-flow regime by taking Hall currents in to account. Analytical solutions for temperature distribution and rate of heat transfer at the walls by the dimensionless Nusselt numbers have been obtained for two cases, that is, when the walls have been made up of (i) non-conducting and (ii) conducting materials. The temperature profiles have been plotted for different sets of values of the governing parameters involved to discuss the behavior of heat transport. It has been observed that, the temperature distribution decreases as the Hall parameter increases for fixed values of Hartmann number, rotation and slip parameters. Also, the temperature distribution has been found to decrease as Taylor number increases in case of non-conducting walls.

Keywords: MHD, Ionized gas, Hall Effect, Heat Transfer, Rotating fluids, Slip-flow

INTRODUCTION

Magneto Hydrodynamics (MHD) is the science of motion of electrically conducting fluid in presence of magnetic field with numerous applications. Many text books/monographs/papers have been published on the subject of Magneto hydrodynamics. In 1889, Bigelow [1] conjectured that there were magnetic fields in the sun and later Hale [2] confirmed by means of Zeeman effects that the sunspots are seats of powerful magnetic fields. In 1919, Larmor [3] made the suggestion that the magnetic fields of the sun and other heavenly bodies might be due to dynamo action, where the conducting material of the star acted as the armature and stator of the self-exciting dynamo. Later, this subject was tremendously developed mainly due to the pioneer contributions of notable scientists, namely Cowling [4], Hartmann [5], Alfven [6], Shercliff [7], Crammer and Pai [8]. Many authors have studied the problem of MHD under different conditions/geometrical situations and reported useful results in the literature. Alfven [6] reported that, if a conducting fluid is placed in a constant magnetic field, every motion of the liquid generates a force called electromotive force which produces electric currents. A kind of electromotive force induces current, while another tends to generate force known as Lorentz force. Also, he stated that owing to the magnetic field, these currents give mechanical forces which change the state of the liquid, thereby causes a kind of combined electromagnetic and hydrodynamic wave. Moreover, the interest in the study of magnetic field arisen after the invention of electromagnetic pump leading to Hartmann number (which is the ratio of electromagnetic force to viscous force). The MHD phenomenon supports several interesting and useful properties like the propagation of transverse waves called Alfven waves [9]. Ever since, many scientists and researchers have reported several applications in Geophysics, Astrophysics and Astronautically apart from its practical importance in the development of MHD generators, pumps, flow meters and MHD bearings. Engineers employ MHD principles in the design of heat exchanger devices, pumps and flow meters; also in solving space vehicle propulsion, control and re-entry problems; in designing communications and radar systems; in creating novel power generating systems; such as MHD rotating generators, accelerators, space power generators, plasma jets etc.

In 1957, Cowling [10] emphasized that when the strength of magnetic field is sufficiently large, one cannot neglect the effects of Hall currents and hence Ohm's law needs to be modified to include these currents. The Hall effect is due merely to the sideways magnetic force on the drifting free charges. In the presence of a very strong magnetic field, as the motion of charged particles across the magnetic lines is hindered, there arise currents which are perpendicular to both electric and magnetic fields. These currents are known as Hall currents. In the last few decades, the study of Hall effects on hydro magnetic fluid flows and the corresponding heat transfer has been tremendously gained considerable impetus in view of their applications in MHD devices such as MHD power generators etc., and in several geophysical and astrophysical situations. Also, the problem of controlling the skin friction and aerodynamic heat transfer around high speed vehicles has vital importance with the advent of rocketry and supersonic flights. In view of these applications, extensive studies on Hall currents have been dealt by various researchers like Broer [11], Sherman [12], Sato [13], Tani [14], Katagiri [15], Pop [16], Gupta [17], Debnath et al. [18], Raptis and Ram [19], Raju and Rao [20], Ram [21], Raju and Murty [22], Raju and Rao [23], Hara et al. [24].

The problem of flow and heat transfer due to slip flow or stretching at the surface has recently been attracted by many of the investigators, because of their numerous applications in engineering and many industrial manufacturing processes [25-27]. In several practical problems, the particle adjacent to a solid surface no longer takes the velocity of the surface. Navier [28] suggested that, there is a stagnant layer of fluid close to the wall allowing a fluid to slip; also the slip velocity is proportional to the shear stress while the normal velocity remains zero. The existence of slip phenomenon at the boundaries and the interface has been observed in the problems related to the flows of rarefied gases (low density), hypersonic flows of chemically reacting binary mixture, rough surfaces and many such types [29,30]. Partial slips also occur for fluid with particulate such as emulsions, suspensions, polymer solutions etc. In the slip flow regime, the usual no-slip conditions at the boundaries (that is, at a solid boundary, the fluid will have zero velocity relative to the boundary) are to be replaced to incorporate the slip conditions on velocity and temperature. For the slip flow regime the fluid still behaves the Navier-Stokes equation, but the usual no-slip conditions must be replaced by the slip conditions given by $u = \beta \frac{du}{dy}$ and $\theta = \beta \frac{d\theta}{dy}$ at $y = \pm 1$, where the slip coefficient β is a small parameter proportional

to the Knudsen number K (the ratio of the free path of a gas molecule to the length of the plate). The problems of this type have been reported in the literature due to the contributions of many authors, namely Schaaf and Chambre [31], Lance and Rogers [32], Sastry and Bhadram [33], Tamada and Miura [34], Bhatt and Sacheti [35], Miksis and Davis [36] and Matthews and Hill [37], Makinde and Osalusi [38] and Raju [39].

The rotating flow of an electrically conducting fluid in presence of magnetic field has developed its importance from geophysical problems, solar physics etc. Because of its occurrence in various natural phenomena and for its applications in several technological situations which are directly governed by the actions of the Coriolis forces. The Magneto hydrodynamic heat transfer phenomenon has been encountered in almost all branches of technology and its associated heat transfer from rotating surfaces is of considerable importance in most of the engineering applications [16,40].

The first investigation of magnetohydrodynamic channel flow was carried out by Hartmann in 1937 [5] and solved the problem of flow between two parallel walls, the fluid being virtually infinite in directions perpendicular to the imposed transverse magnetic field. After this famous investigation, Shercliff [7] studied the more realistic problem where the flow was bounded also in these directions by two parallel walls, namely the flow through a rectangular channel in the presence of an imposed transverse magnetic field. Inspired by these investigations, the channel flows of an electrically conducting viscous fluid under the action of transverse magnetic field have been studied by many researchers, notably, Sherman [12], Alpher [41] and Nigam and Singh [42]. However the MHD aspects of heat transfer in channel flows has been studied in a variety of ways by several researchers, namely Siegel [43], Greshuni and Zhukhovitskii [44], Sparrow and Cess [45], Eraslan and Eraslan [46]. Several channel flows have also been carried out in a non-rotating frame of reference by many authors, namely Eraslan [47], Javeri [48], Bharali and Borkakati [49] and Datta and Mazumdar [50]. Of course, Jana et al. [40] studied the thermal distribution of MHD Couette flow in a rotating frame of reference, whereas Mohan [51] discussed the combined effects of free and forced convection on MHD flow in a rotating channel. The Hall effects on the combined free and forced convection of an electrically conducting fluid in a parallel plate channel has been studied by Mazumdar et al. [52]. In all the above studies the plates of the channel are assumed to be perfect electrical insulators. However, the effect of wall conductance on the convective flows has been dealt by Datta and Jana [53]. But, Mazumdar et al. [52] studied the flow and heat transfer in the hydromagnetic Ekman layer on a porous plate taking Hall effects into account and arrived at an interesting result that the asymptotic solution for velocity exists for both suction and blowing at the plate. Krishna and Rao [54] studied the Hall effects on free and forced convective flow in a rotating system. Raju and Rao [55] studied the Hall effect on the viscous flow of

an ionized hydromagnetic flow with rotation in two cases, that is, when both the walls are made up of non-conducting and conducting material. Subsequently, Jana et al. [40] have discussed its corresponding heat transfer problem. Raju and Murty [22] studied the quasi-state solutions of MHD ionized flow and heat transfer between parallel walls in a rotating system with Hall currents. Raju [39] studied magnetohydrodynamic slip flow regime in a rotating channel.

Hence, an attempt has been made to examine the effect of Hall current on temperature distribution due to MHD flow of an ionized gas between two parallel walls in a rotating frame of reference under the slip flow regime. Exact solutions for temperature distribution and the rate of heat transfer at walls by the dimensionless Nusselt numbers have been obtained in cases of non-conducting (insulated) and conducting side walls. The profiles are plotted for different sets of values of the governing parameters involved. The behavior of the heat transport is discussed by analyzing these distributions. The interest in this topic is motivated in part by the fact that the analysis of such flows is essential for the solution of many engineering problems such as in MHD power generators, MHD pumps and meters, plasma jets, spacecraft etc. As another practical application in aerodynamics, the possibility arises of altering the flow and heat transfer around high speed vehicles assuming that the air is sufficiently ionized.

MATHEMATICAL ANALYSIS FOR EQUATIONS OF MOTION, ENERGY AND THE BOUNDARY CONDITIONS

Consider a steady flow of an ionized gas with constant properties bounded by two parallel walls along x- and z-directions under the action of uniform transverse magnetic field. The x-axis is taken in the direction of hydrodynamic pressure gradient in the plane parallel to the channel walls but not in the direction of flow. A parallel uniform magnetic field B_0 is applied in the y-direction. The height of the channel is taken as $2h$ and the width is assumed to be very large in comparison with the channel height $2h$. The whole system is rotated with an angular velocity Ω in a counter clockwise direction about y-axis (that is which is perpendicular to the walls). The fluid is driven by a constant pressure gradient. It is further considered the assumptions as in Spitzer [56], Sato [13], to make the theoretical analysis more simpler form. With these assumptions, the fundamental equations to be solved are the equations of motion and current for the steady flow of neutral fully-ionized gas in a rotating frame of reference, and are expressed as:

$$k_1 + \frac{d^2 u}{dy^2} - \frac{\sigma_1}{\sigma_0} M^2 (m_z + u) + \frac{\sigma_2}{\sigma_0} M^2 (m_x - w) = 2K^2 w, \quad (1)$$

$$k_2 + \frac{d^2 w}{dy^2} + \frac{\sigma_1}{\sigma_0} M^2 (m_x - w) + \frac{\sigma_2}{\sigma_0} M^2 (m_z + u) = -2K^2 u \quad (2)$$

$$\text{The boundary conditions are } u = \beta \frac{du}{dy} \text{ and } w = \beta \frac{dw}{dy} \text{ at } y = \pm 1 \quad (3)$$

In which, $k_1 = 1 - s(1 - \frac{\sigma_1}{\sigma_0})$, $k_2 = -s(\frac{\sigma_2}{\sigma_0})$, $m_x = E_x / (B_0 u_p)$, $m_z = E_z / (B_0 u_p)$, the Hartmann number M is defined as

$$M^2 = \frac{B_0^2 h^2 \sigma_0}{\rho \nu} \text{ and } K^2 \text{ (called as Taylor number)} = \frac{\Omega h^2}{\nu}$$

Writing $q = u + iw$, $k = k_1 + ik_2$, $E = m_x + im_z$; equations (1) and (2) can be written in complex form as:

$$\frac{d^2 q}{dy^2} + \left(\frac{-\sigma_1}{\sigma_0} M^2 + i \frac{\sigma_2}{\sigma_0} M^2 + 2iK^2 \right) q = -k - i \frac{\sigma_1}{\sigma_0} M^2 E - \frac{\sigma_2}{\sigma_0} M^2 E, \quad (4)$$

$$\text{The slip boundary conditions are } q = \beta \frac{dq}{dy} \text{ at } y = \pm 1 \quad (5)$$

Also, I_x and I_z are defined by $J_x / (\sigma_0 B_0 u_p)$ and $J_z / (\sigma_0 B_0 u_p)$ respectively, also these are given in complex notation as

$$I = I_x + iI_z = \frac{\sigma_2 + i\sigma_1}{\sigma_0} \left(q - iE - \frac{s}{M^2} \right) + \frac{is}{M^2} \quad (6)$$

The non-dimensional electric field m_x and m_z are to be determined by boundary conditions at large x and z .

BASIC ENERGY EQUATION with BOUNDARY CONDITIONS

The effect of flow parameters on the behavior of fluid's temperature and the heat transferred between the fluid and the

walls has been considered using the fully developed constant properties flow which can be obtained from the above equations (1-3). Further, assuming that the thermal boundary conditions apply everywhere on the infinite channel walls and neglecting the thermal conduction in the flow direction, the non-dimensionalised form of the governing energy equation is

$$\frac{1}{P_r} \frac{d^2 \theta}{dy^2} = - \left\{ \left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 + M^2 I^2 \right\}, \quad (7)$$

$$\text{where, } \theta = \frac{T - T_w}{(u_p^2 / c_p)}, \quad I_x + iI_z = \frac{J_x + iJ_z}{\sigma_0 B_0 u_p}, \quad I^2 = I_x^2 + I_z^2. \quad (8)$$

$$\text{The slip boundary conditions are } \theta = \beta \frac{d\theta}{dy} \text{ at } y = \pm 1 \quad \text{and} \quad \frac{d\theta}{dy} = 0 \text{ at } y = 0 \quad (9)$$

We use the solutions of the equations (4) to (6) for velocity fields to determine the temperature distribution, mean temperature and the rate of heat transfer coefficient in the fluid flow when both the walls are made up of non-conducting (insulating) and conducting materials.

SOLUTIONS OF THE PROBLEM

The solution of the problem is carried out in the following two cases.

Solutions for non-conducting (insulating) walls

When the side walls are kept at large distance in z-direction and are made up of non-conducting material, then the induced electric current does not go out of the channel but circulates within the fluid. So, an additional condition for the current defined in non-dimensional form is obtained by $\int_0^1 I_z dy = 0$.

If the insulation at large x is also assumed, another relation is obtained as

$$\int_0^1 I_x dy = 0 \quad (10)$$

Making use of the solutions obtained from eqs. (4)-(6) in case of non-conducting walls for u, w and I, we solve :

$$-\frac{1}{P_r} \frac{d^2 \theta}{dy^2} = \frac{dQ}{dy} \frac{d\bar{Q}}{dy} + \frac{H_a^2}{1+m^2} (Q-1)(\bar{Q}-1) \quad (11)$$

$$\text{subject to the slip boundary conditions at the walls as } \theta = \beta \frac{d\theta}{dy} \text{ at } y = \pm 1 \quad (12)$$

$$\text{where, } q = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}, \quad \bar{q} = b_5 e^{b_1 y} + b_6 e^{b_2 y} + \frac{b_3}{b_4}, \quad q_m = \int_0^1 q dy = c_1 a_{26} + c_2 a_{27} + \frac{a_2}{a_1} = b_7, \quad \bar{q}_m = b_8,$$

$$Q = \frac{q}{q_m} = \frac{c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}}{b_7}, \quad \bar{Q} = \frac{\bar{q}}{\bar{q}_m} = \frac{b_5 e^{b_1 y} + b_6 e^{b_2 y} + \frac{b_3}{b_4}}{b_8}, \quad (13)$$

and $q_m = u_m + iw_m$, $\bar{q}$ is the complex conjugate of q and q is the solution of equation (4) subject to (5) and u_m, w_m are the mean velocities. Then using the expressions at equation (13) and solving the resulting differential equation, we obtain

$$\theta = c_3 + c_4 y + b_{53} e^{b_{13} y} + b_{54} e^{b_{14} y} + b_{55} e^{b_{15} y} + b_{56} e^{b_{16} y} + b_{57} e^{m_1 y} + b_{58} e^{m_2 y} + b_{59} e^{b_1 y} + b_{60} e^{b_2 y} + b_{52} y^2 \quad (14)$$

Further using the boundary conditions (12) and eliminating the arbitrary constants, the expressions for temperature distribution, mean temperature and the rate of heat transfer coefficient at the upper wall are obtained as under:

$$\theta = b_{78} + b_{77} y + b_{53} e^{b_{13} y} + b_{54} e^{b_{14} y} + b_{55} e^{b_{15} y} + b_{56} e^{b_{16} y} + b_{57} e^{m_1 y} + b_{58} e^{m_2 y} + b_{59} e^{b_1 y} + b_{60} e^{b_2 y} + b_{52} y \quad (15)$$

The mean temperature is given by

$$\theta_m = \int_0^1 \theta dy = \frac{b_{53}}{b_{13}}(e^{b_{13}} - 1) + \frac{b_{54}}{b_{14}}(e^{b_{14}} - 1) + \frac{b_{55}}{b_{15}}(e^{b_{15}} - 1) + \frac{b_{56}}{b_{16}}(e^{b_{16}} - 1) + \frac{b_{57}}{m_1}(e^{m_1} - 1) + \frac{b_{58}}{m_2}(e^{m_2} - 1) + \frac{b_{59}}{b_1}(e^{b_1} - 1) + \frac{b_{60}}{b_2}(e^{b_2} - 1) + \frac{b_{52}}{3} + \frac{b_{77}}{2} + b_{78} \quad (16)$$

The rate of heat transfer coefficient or the Nusselt number Nu at the upper wall is given by

$$N_u = -\frac{1}{P_r} \left(\frac{d\theta}{dy} \right)_{y=\pm 1} = -b_{13}b_{53}e^{b_{13}} - b_{14}b_{54}e^{b_{14}} - b_{15}b_{55}e^{b_{15}} - b_{16}b_{56}e^{b_{16}} - m_1b_{57}e^{m_1} - m_2b_{58}e^{m_2} - m_2b_{59}e^{m_2} - b_1b_{59}e^{b_1} - b_2b_{60}e^{b_2} - 2b_{52} - b_{77} \quad (17)$$

For the sake of convenience the symbols involved in the above expressions are shown in Appendix.

Solutions for conducting walls

If the side walls are made up of conducting material and short-circuited by an external conductor, then the induced electric current flows out of the channel. In this case no electric potential exists between the side walls. If we assume zero electric field also in the x- and z- directions, we have $m_x=0$, $m_z=0$. And the constants in the solution are determined by these two conditions.

In view of the above mentioned facts and making use of the solutions obtained in case of conducting walls for u, w and I from equations (4) - (6), we solve the following equation,

$$\frac{1}{P_r} \frac{d^2\theta}{dy^2} = - \left[\frac{dQ}{dy} \frac{d\bar{Q}}{dy} + \frac{H_a^2}{1+m^2} (Q\bar{Q}) + \left(1 - \frac{1}{1+m^2} \right) \frac{s^2}{H_a^4} \frac{1}{q_m \bar{q}_m} + \frac{is}{H_a^2} \frac{m}{1+m^2} \left(\frac{\bar{Q}}{q_m} - \frac{Q}{\bar{q}_m} \right) \right], \quad (18)$$

subject to the boundary conditions as given at equation (9).

In the above equation (18), we write

$$q = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}, \quad \bar{q} = b_5 e^{b_1 y} + b_6 e^{b_2 y} + \frac{b_3}{b_4}, \quad q_m = \int_0^1 q dy = c_1 a_{26} + c_2 a_{27} + \frac{a_2}{a_1} = b_7, \quad \bar{q}_m = \int_0^1 \bar{q} dy = b_8,$$

$$Q = \frac{q}{q_m} = \frac{c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}}{b_7}, \quad \bar{Q} = \frac{\bar{q}}{\bar{q}_m} = \frac{b_5 e^{b_1 y} + b_6 e^{b_2 y} + \frac{b_3}{b_4}}{b_8} \quad (19)$$

Solving the resulting differential equation after using the expressions (19), the solution for temperature distribution is given by

$$\theta = b_{75} + b_{74}y + b_{49}e^{b_{21}y} + b_{50}e^{b_{22}y} + b_{51}e^{b_{23}y} + b_{52}e^{b_{24}y} + b_{53}e^{m_1 y} + b_{54}e^{m_2 y} + b_{55}e^{b_1 y} + b_{56}e^{b_2 y} + b_{57}y^2 \quad (20)$$

Subsequently, using the boundary conditions (12) to eliminate the arbitrary constants, the expressions for temperature distribution, mean temperature and the rate of heat transfer coefficient at the upper wall are obtained as:

$$\theta = b_{78} + b_{77}y + b_{53}e^{b_{13}y} + b_{54}e^{b_{14}y} + b_{55}e^{b_{15}y} + b_{56}e^{b_{16}y} + b_{57}e^{m_1 y} + b_{58}e^{m_2 y} + b_{59}e^{b_1 y} + b_{60}e^{b_2 y} + b_{52}y \quad (21)$$

The mean temperature is given by

$$\theta_m = \int_0^1 \theta dy = \frac{b_{49}}{b_{21}}(e^{b_{21}} - 1) + \frac{b_{50}}{b_{22}}(e^{b_{22}} - 1) + \frac{b_{51}}{b_{23}}(e^{b_{23}} - 1) + \frac{b_{52}}{b_{24}}(e^{b_{24}} - 1) + \frac{b_{53}}{m_1}(e^{m_1} - 1) + \frac{b_{54}}{m_2}(e^{m_2} - 1) + \frac{b_{55}}{b_1}(e^{b_1} - 1) + \frac{b_{56}}{b_2}(e^{b_2} - 1) + \frac{b_{57}}{3} + \frac{b_{74}}{2} + b_{75} \quad (22)$$

The rate of heat transfer coefficient (the Nusselt number Nu) at the upper wall is

$$N_u = -\frac{1}{P_r} \left(\frac{d\theta}{dy} \right)_{at y=\pm 1} = -b_{21}b_{49}e^{b_{21}} - b_{22}b_{50}e^{b_{22}} - b_{23}b_{51}e^{b_{23}} - b_{24}b_{52}e^{b_{24}} - m_1b_{53}e^{m_1} - m_2b_{54}e^{m_2} - b_{55}b_1e^{b_1} - b_{56}b_2e^{b_2} - 2b_{57} - b_{74} \quad (23)$$

where the symbols/functions involved in the above solutions are shown in Appendix.

RESULTS AND DISCUSSION

Exact solutions of the problem for temperature distributions are obtained in cases of insulated (non-conducting) and conducting side walls. The expressions for mean temperature and the rate of heat transfer coefficient (Nusselt number) at the walls are also determined. The profiles for temperature distribution are presented to discuss the behavior of heat transport, after obtaining the numerical calculations for different sets of values of the governing parameters involved in the study. These profiles are drawn by varying one of the parameters such as Hall parameter m , Hartmann number M , Taylor number K and Slip parameter β by fixing the others. And, these are shown in Figures 1-13. We first note that, $\theta(y)=\theta(-y)$, which implying that the temperature distribution is symmetrical about the channel-center line $y=0$. Also we note that, when $\beta=0$ (no-slip conditions at the walls), these results are in agreement with that of Raju and Rao [39]. While taking $\beta=0$, $K=0$ and $m=0$ (i.e., for without rotation and Hall currents and no-slip at the walls), these results have been coinciding with that of the solution corresponding to $R_E=-1$ (flow meter) as in equation (44) of Cramer and Pai [9] (Figure 1).

Figures 2-5 exhibit the temperature distribution θ for the case when the walls are made up of non-conducting (insulating) material. In this case, the temperature distribution is found to be independent of the partial pressure of electron gas, Figure 2, shows that the temperature ' θ ' increases first for small Hartmann numbers say up to $M=8$, then decreases up to $M=20$ and beyond this number, there is a raise in temperature due to increasing values of the Hartmann number ' M ' with a set of fixed values of Taylor number (rotation parameter) T , Hall parameter ' m ' and slip parameter, β . That is the temperature falls when $8 < M$ and raises at outside this range of values of Hartmann number. The presence of magnetic field (and therefore Hall currents) introduces oscillatory modes in thermal system. Also, from Figure 4, it is noticed that the temperature distribution decreases everywhere except at near to the both walls with an increase in Taylor number for a set of fixed values of M , m and β . But nearer to the walls, the temperature almost remains unchanged for any value of Taylor number. From Figure 5, it is found that the temperature distribution decreases first for the slip parameter $\beta=0.05$, then increases at $=0.01$, there afterwards it again decreases with an increase in for fixed values of M , m and T (Figures 2-5).

Figures 6-9 show the temperature distribution for the case of conducting walls and for weakly ionized gas, that is when $s=0$. From Figure 6, it is observed that the temperature distribution θ increases with an increase in Hartmann number M for fixed values of rotation parameter T , Hall parameter m and slip parameter. That is the influence of magnetic field on temperature is more pronounced for large values of Hartmann number. It is seen from Figure 7 that the temperature diminishes everywhere with an increase in Hall parameter m , when all the remaining governing parameters are fixed. A similar type of behavior reveals as the rotation parameter (Taylor number) increases when all the remaining parameters are fixed, as is evident from Figure 8. While from Figure 9, it is seen that the temperature distribution increases as the slip parameter increases (Figures 6-9).

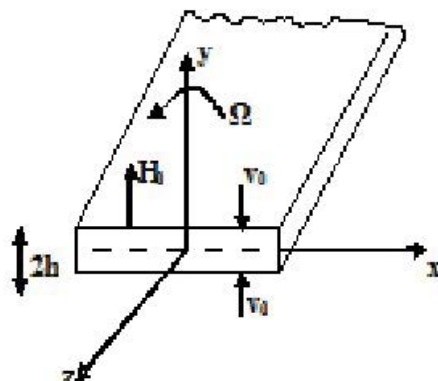


Figure 1: Geometry of the flow problem

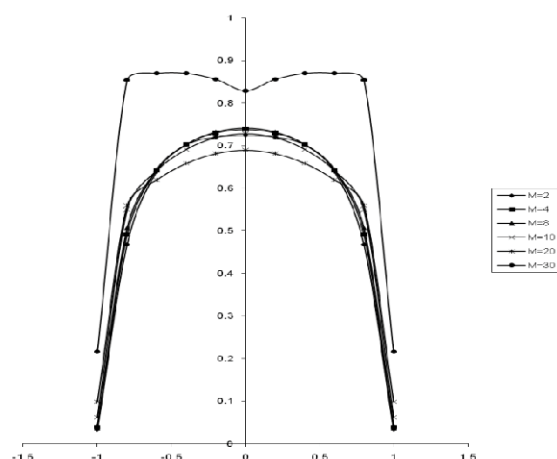


Figure 2: Temperature (Non-conducting) for different M, T=2, m=2 and Beta=0.01

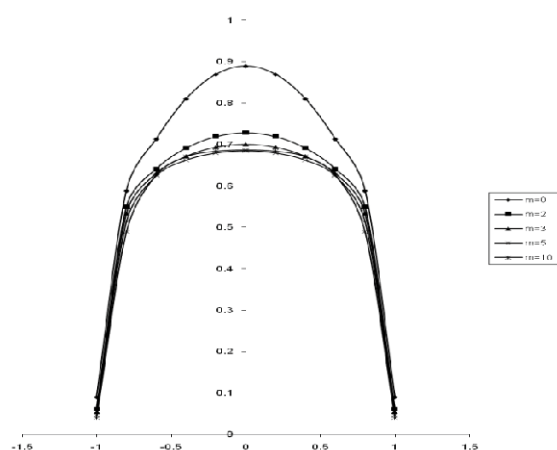


Figure 3: Temperature (Non-conducting) for different m, T=2, m=2 and Beta=0.01

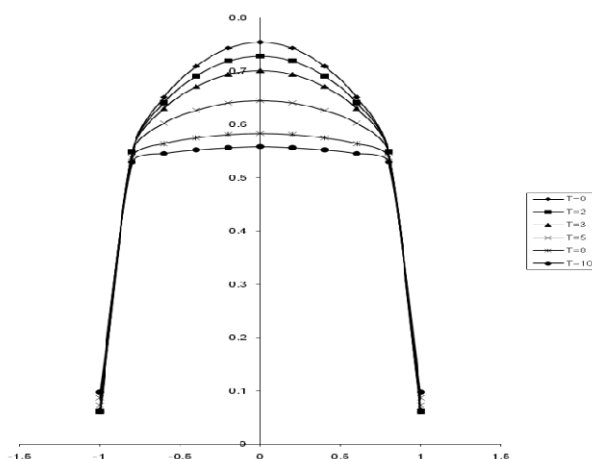


Figure 4: Temperature distribution (Non-conducting) for different T, M=10, m=2 and Beta=0.01

Figures 10–13 show the temperature distribution in case of conducting walls and for $s=\frac{1}{2}$, that is, neutral fully-ionized plasma. From Figure 10, it is observed that as the Hartmann number M increases, the temperature distribution θ increases for fixed values of rotation parameter T , Hall parameter m and slip parameter. From Figure 11, it is found that the temperature distribution decreases as the Hall parameter increases for fixed values of M , T and. But, from Figure 12, it is seen that the temperature distribution θ increases as the rotation parameter increases. It is also seen from Figure 13, that the temperature distribution increases with an increase in slip parameter for fixed values of

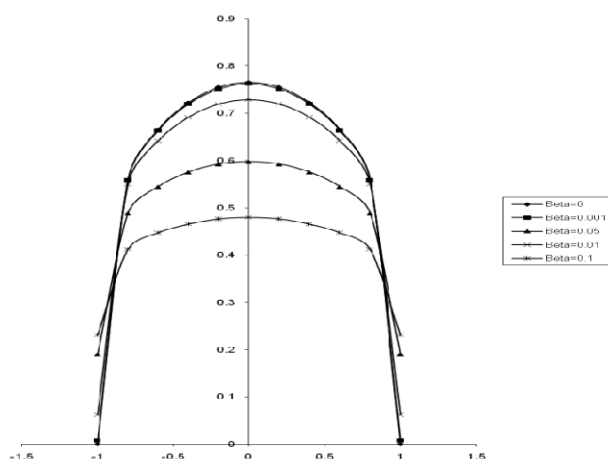


Figure 5: Temperature (Non-conducting) for different M , $T=2$, $m=2$ and $\text{Beta}=0.01$

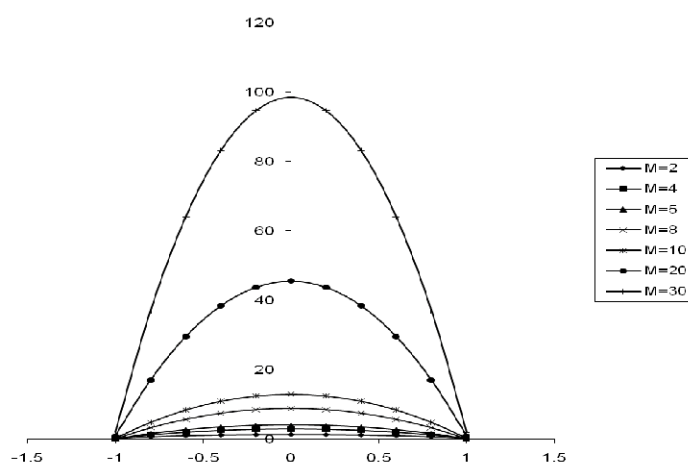


Figure 6: Temperature (conducting) for different M , $m=2$, $T=2$, $\text{Beta}=0.01$ and $s=0$

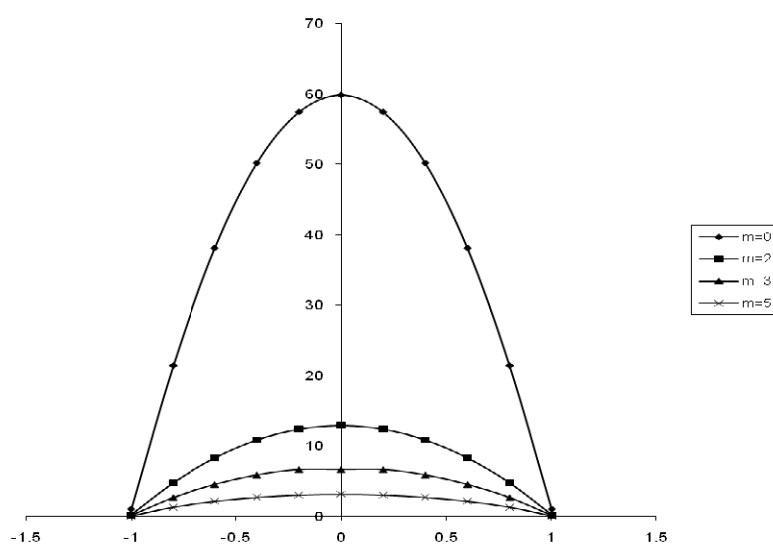


Figure 7: Temperature (conducting) for different m , $M=10$, $T=2$, $\text{Beta}=0.01$ and $s=0$

the remaining parameters M , m and T . It is clear that the increasing Hartmann number enhances the temperature distribution, while increasing Hall parameter diminishes the temperature (Figures 10-13).

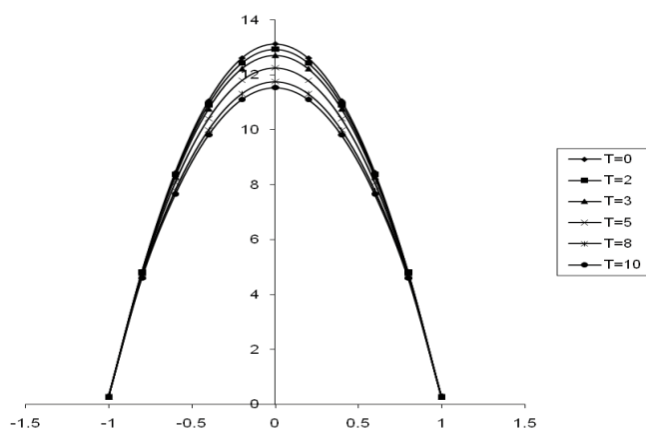


Figure 8: Temperature (conducting) for different T , $M=10$, $m=2$, $\text{Beta}=0.01$ and $s=0$

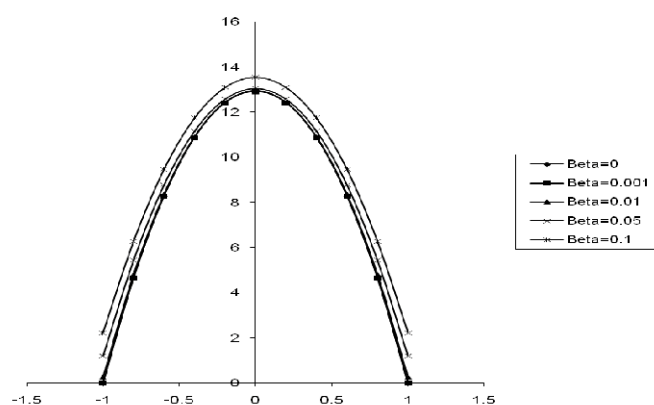


Figure 9: Temperature (conducting) for different Beta , $M=10$, $m=2$, $T=2$ and $s=0$

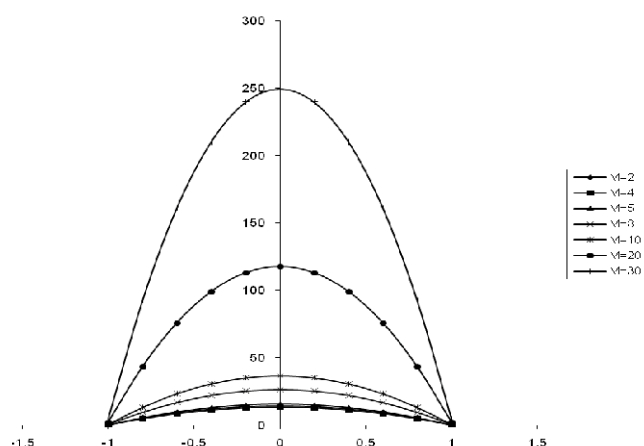


Figure 10: Temperature (conducting) for different M , $m=2$, $T=2$, $\text{Beta}=0.01$ and $s=0.5$

CONCLUSION

The problem of heat transfer in an ionized hydromagnetic slip-flow between two parallel walls in a rotating system with Hall effect is examined. Analytical expressions for temperature distribution as well as the rate of heat transfer at the walls by the dimensionless Nusselt numbers are obtained in cases of non-conducting (insulated) and conducting side walls. The profiles are plotted to discuss the behavior of heat transport after obtaining the numerical calculations for different sets of values of the governing parameters involved in the study. And the conclusions of the present study are as follows:

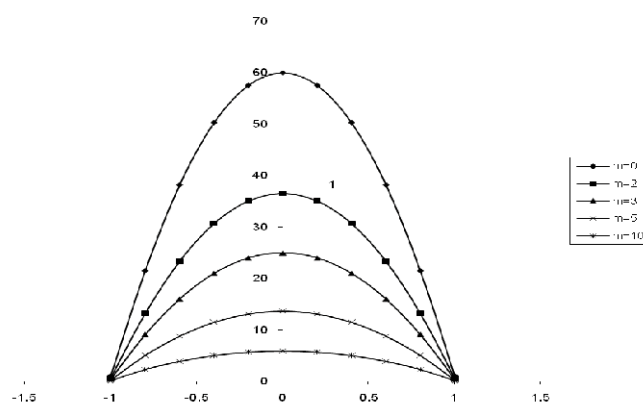


Figure 11: Temperature (conducting) for different m , $M=10$, $T=2$, $Beta=0.01$ and $s=0.5$

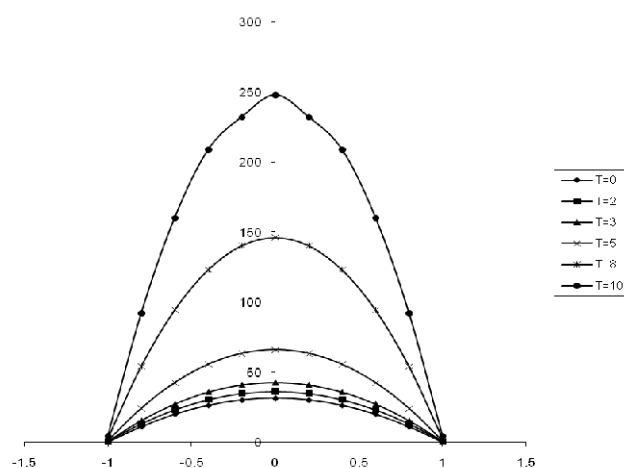


Figure 12: Temperature (conducting) for different T , $M=10$, $m=2$, $Beta=0.01$ and $s=0.5$

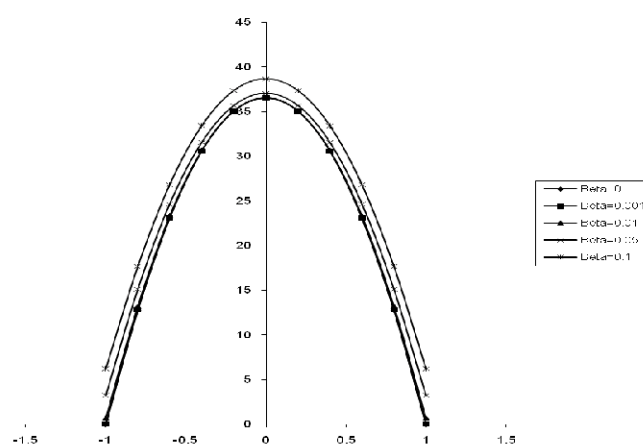


Figure 13: Temperature (conducting) for different $Beta$, $M=10$, $m=2$, $T=2$ and $s=0.5$

In case of non-conducting side walls, the temperature distribution is independent of the ratio of electron pressure to the total pressure. But, the same is depending on this ratio for the case of conducting side walls.

The temperature distribution is fluctuating for smaller values of the Hartmann number and slip parameter and raising for higher values.

The temperature distribution decreases as the Hall parameter increases.

The temperature distribution decreases everywhere except at near to the side walls with an increase in Taylor number. But, at near to the side walls, the temperature almost remains invariant for any value of Taylor number.

In case of conducting side walls and for weakly ionized gas, the temperature distribution increases with an increase in Hartmann number and slip parameter. But, the temperature diminishes with an increase in Hall parameter and rotation parameter, when all the remaining governing parameters are fixed.

In case of conducting side walls and for the neutral fully-ionized plasma, the temperature distribution increases as the Hartmann number, rotation parameter and slip parameter increases. But, temperature distribution decreases as the Hall parameter increases for fixed values of Hartmann number, Taylor number and slip parameter.

Hence, it is concluded that the temperature can be raised or lowered with suitable values of the governing parameters like Hartmann number, Hall parameter, Taylor number and slip parameter.

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