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Hall effect on radiating and chemically reacting MHD oscillatory flow in a rotating porous vertical channel in slip flow regime

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ABSTRACT

An investigation of the combined influence of Hall effects and chemical reaction on the flow of an oscillatory convective MHD viscous, incompressible, radiating and electrically conducting fluid in a vertical porous rotating channel in slip flow regime is carried out. The fluid is assumed to be gray, absorbing and emitting radiation out in non scattering medium. The magnetohydrodynamic (MHD) flow is assumed to be laminar and fully developed. A closed form solutions of the equations governing the flow are obtained for the velocity, temperature and concentration profile. The velocity, temperature, and concentration profiles as well as skin friction coefficient and mass transfer rate are evaluated numerically and presented graphically for different value of flow parameters.

Keywords: Hall Effect, radiating and chemically reacting fluid, MHD oscillatory flow, rotating channel, slip flow regime.

INTRODUCTION

The study of flow in rotating porous channel is motivated by its practical applications in geophysics and engineering. Among the applications of rotating flow in a porous media to engineering disciplines, one can find the food processing industry, chemical processing industry, centrifugation filtration processes and rotating machinery. Also the hydrodynamic rotating flow of electrically conducting viscous incompressible fluids has gained considerable attention because of its numerous applications in physics and engineering. In geophysics, it applies to measure and study the position and velocities with respect to fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics now days has become an important branch of fluid dynamics due to the increasing interest to study environment .In Astrophysics; it is applied to study the stellar and solar structure, interplanetary and interstellar matter, solar storms etc. In engineering, it finds its application in MHD generator ion propulsion, MHD bearing, MHD pumps, MHD boundary layer control of re-entry vehicles etc. Several scholars viz. Crammer and Pai[1], Ferraro and Plumpton [2]and Shercliff [3] have studied such flows because of their varied importance and applications. The process of heat transfer is encountered in cooling of nuclear reactors, providing heat sink in turbine blades and aeronautics.

There are numerous important engineering and geophysical applications of channel flows through porous medium, for example in the fields of agriculture engineering for channel irrigation and to study the underground water resources, in petroleum technology to study environment of natural gas, oil and water through the oil channels/ reservoirs. The transient natural convection between to vertical walls with porous material having variable porosity has been studied by Paul et al[4]. In the recent years, the effect of transversely applied magnetic field on the flow of the electrically conducting viscous fluid have been studied extensively owing to their astrophysics, geophysical and engineering application Attia and Kotb [5] have studied MHD flow between the two parallel plates with heat transfer . When the strength of magnetic field is strong, one cannot neglect the effect of Hall current. The rotating flow of an electrically conducting fluid in the presence of magnetic field is encountered in geophysical and comical fluid dynamics .It is important in solar physics involved in sun spot development. Hall effect on unsteady MHD free and forced convection flow in a porous rotating channel has been investigated by several researchers Sivaprasad et al [6], Singh and Kumar[7], Singh and Pathak [8], and Ghosh et al [9]. Radiative convective flows have gained attention of many researchers in recent years. Radiation plays a vital role in many engineering, environment and industrial process for example heating and cooling chamber, fossil fuel combustion energy processes astrophysical flows and space vehicle re-entry. Raptis [10] studied the radiation free convective flow through a porous medium. Alagoa et al [11] has analysed the effect of radiation on MHD flow through the porous medium between infinite parallel plates in the presence of time dependent suction. Singh and Kumar [12] have studied the radiation effect on the exact solution of free convective oscillatory flow through porous medium in a rotating porous channel.

The behaviour of the fluid under extreme confinement is of great interest from both the scientific and technological point of view. One of the great complexities is to discover what type of boundary condition is appropriate for solving the continuum fluid problems. Despite of the wide spread acceptance of no slip assumption, there has been existed for the many years, indirect experimental evidence based on the anomalous flow in capillaries and other systems that in some cases, simple liquid can slip against the solid when walls are sufficiently smooth and the no slip boundary condition is no more valid. The no slip boundary condition is only valid when particle close to the surface do not move along with the flow i.e. when adhesion is stronger than cohesion. However this is only true microscopically. Few other limitations of no slip conditions are; they fail for large contact angle, does not hold at very low pressure, does not work for polyethylene, rubber compounds and suspensions, fail in hydrodynamics for hydrophobic surfaces (Vinogradova[13], Zhu and Granick[14]).

Recently, the slip condition has become much more compelling and it now reasonably certain that viscous fluid can slip against solid surfaces if the surface is very smooth Navier[15]. The slip boundary condition has significant application in lubrication, extrusion, medical sciences, especially in polishing article heart valves, flows through porous media, micro and nanofluids, friction studies and biological fluids, Blake [16] and Pit et al [17]. On the other hand chemical reactions have numerous applications such as manufacturing of ceramic, food processing and polymer production. Muthucumaraswamy [18] has analyzed that the rate of diffusion is affected by chemical reaction. Many researchers have shown interest propulsion engines for aircraft technology Murti et al [19]. Recently Reddy et al [20] has analysed the effect of radiation on the unsteady MHD Convective flow through a non uniform horizontal channel. Motivated by the above researches our purpose to investigate the Hall effect on an unsteady MHD oscillatory convective heat transfer flow of a radiating and chemically reacting fluid through a porous medium in a rotating vertical porous channel.

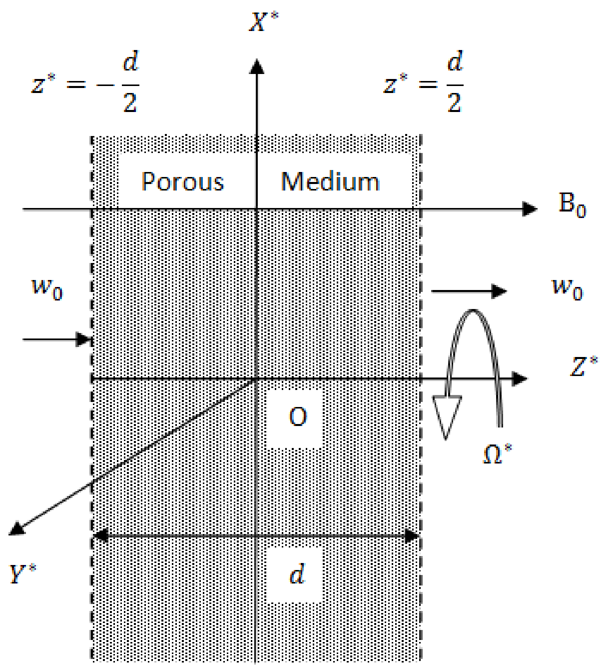

Fig. 1 Geometrical configuration of the problem.

2 FORMULATION OF THE PROBLEM

Consider the flow of a viscous, incompressible and electrically conducting fluid through a porous medium bounded by two infinite vertical insulated plates at 'd' distance apart. We introduce a Cartesian co-ordinate system with X^*-

axis oriented vertically upward along the centreline of this channel and Z^*-axis taken perpendicular to the planes of the plates which is the axis of the rotation and the entire system comprising of the channel and the fluid are rotating as a solid body about this axis with constant angular velocity Ω^*. A constant injection velocity w_0 is applied at the plate $z^* = -\frac{d}{2}$ and the same constant suction velocity, w_0, is applied at the plate $z^* = +\frac{d}{2}$. A uniform magnetic field with magnetic flux density vector B_0 is applied perpendicular to the plane of plates. The schematic diagram of the physical problem is shown in the Fig. 1

Since the plates of the channel occupying the planes $Z^* = \pm\frac{d}{2}$ are of infinite extent, all the physical quantities depend upon only on Z^* and t^*only. Under the Boussinesq approximation the flow of the fluid through the porous medium in a rotating channel is governed by the following equation:

$$\frac{\partial u^*}{\partial t^*} + w_0\frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* + \frac{\sigma {B_0}^2(mv^* - u^*)}{\rho(1+m^2)} + g\beta T^* + g\beta_c C^* - \nu\frac{u^*}{k^*} \quad (1)$$

$$\frac{\partial v^*}{\partial t^*} + w_0\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - \frac{\sigma {B_0}^2(mu^* + v^*)}{\rho(1+m^2)} - \nu\frac{v^*}{K^*} \quad (2)$$

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + w_0\frac{\partial T^*}{\partial z^*}\right) = \kappa\frac{\partial^2 T^*}{\partial z^{*2}} - Q_0 T^* + Q_1 C^* - \frac{\partial q}{\partial z^*} \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} + w_0\frac{\partial C^*}{\partial z^*} = D\frac{\partial^2 C^*}{\partial z^{*2}} - K_1 C^* \quad (4)$$

where ρ is density, P^* is the modified pressure, ν is kinematic viscosity, $t^*$is the time, σ is the electrical conductivity, B_0 is the electromagnetic induction, $m = \omega_e \tau_{e,}$ ($\omega_e$is the electron frequency and τ_e is the electron charge) is hall current parameter, g is the acceleration due to gravity, β is coefficient of volume expansion, $\beta_c$is the coefficient of expansion with concentration, $k^*$is the permeability of the porous medium, $C_p$is the specific heat at the constant pressure, κ is the thermal conductivity, μ is the coefficient of viscosity, Q_0 is the heat absorption, $Q_1$is the radiation absorption, q is the radiative heat, D is the molecular diffusivity , $K_1$is the chemical reaction parameter

We assume the flow under the influence of the pressure gradient varying periodically with the time in the X^* -axis of the following form

$$-\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} = A\cos\omega^* t^* \quad (5)$$

where A is the amplitude of the pressure gradient.
Following Street [21] the present problem is subjected to the following appropriate boundary conditions:

$$\left.\begin{aligned} z^* = -\frac{d}{2},\ u^* &= \frac{2-f_1 L}{f_1}\frac{\partial u^*}{\partial z^*} = L_1\frac{\partial u^*}{\partial z^*},\ v^* = \frac{2-f_1 L}{f_1}\frac{\partial v^*}{\partial z^*} = L\frac{\partial v^*}{\partial z^*},\ T^* = 0,\ C^* = 0 \\ &\text{and} \\ z^* = \frac{d}{2},\ u^* &= v^* = 0,\quad T^* = T_0\cos\omega^* t^*,\quad C^* = C_0\cos\omega^* t^* \end{aligned}\right\} \quad (6)$$

where T_0 is the mean temperature, C_0 is the mean concentration, ω^* is the frequency of oscillations, f_1 is Maxwell's reflexion coefficient and $L = \mu\left(\frac{\pi}{2p\rho}\right)^{\frac{1}{2}}$ is the mean free path which is constant for an incompressible fluid.
Following Cogley et al [22] the last term in the energy equation stand for the radiative heat flux which is given by

$$\frac{\partial q}{\partial z^*} = 4\alpha^2 T^* \quad (7)$$

where α is the mean radiation absorption coefficient.
Introducing the following non dimensional quantities

$$\eta = \frac{z^*}{d},\ x = \frac{x^*}{d},\ y = \frac{y^*}{d},\ u = \frac{u^*}{w_0},\ v = \frac{u^*}{w_0},\ T = \frac{T^*}{T_0},\ t = \frac{t^* w_0}{d},\ \omega = \frac{\omega^* d}{w_0},$$
$$P = \frac{P^*}{\rho w_0^2},\ C = \frac{C^*}{C_0} \quad (8)$$

With the help of the non-dimensional parameters (8), equations (1) to (4) reduces

$$R_e\left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \eta}\right) = -R_e\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + 2\Omega v + \frac{H^2}{(1+m^2)}(mv - u) - K^{-1}u + G_r T + G_m C \quad (9)$$

$$R_e\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \eta}\right) = -R_e\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} - 2\Omega u - \frac{H^2}{(1+m^2)}(mu + v) - K^{-1}v \quad (10)$$

$$R_e P_r\left(\frac{\partial T}{\partial t} + \frac{\partial T}{\partial \eta}\right) = \frac{\partial^2 T}{\partial \eta^2} - P_r\phi T + R_e^2 P_r Q C - P_r N^2 T \quad (11)$$

$$S_c R_e \left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial \eta}\right) = \frac{\partial^2 C}{\partial \eta^2} - S_c R_e \chi C \quad (12)$$

where'*' represent the dimensional physical quantities, and

$R_e = \frac{w_0 d}{\nu}$ = Reynolds number, $\Omega = \frac{\Omega^* d^2}{\nu}$ = Rotation parameter,

$K = \frac{K^*}{d^2}$, = Permeability of the porous medium, $G_r = \frac{g\beta d^2 T_0}{\nu w_0}$ = Grashof number,

$G_m = \frac{g\beta_c d^2 C_0}{\nu w_0}$ = Modified Grashof number, $P_r = \frac{\mu C_p}{\kappa}$ = Prandtl number,

$N = \frac{2\alpha d}{\sqrt{\kappa}}$ = Radiation parameter, $\phi = \frac{Q_0 d^2}{\mu C_p}$ = Heat absorption number,

$Q = \frac{Q_1 \nu C_0}{\rho C_p w_0^2 T_0}$ = Radiation absorption number, $H = B_0 d\sqrt{\frac{\sigma}{\mu}}$, = Hartmann number,

$S_c = \frac{\nu}{D}$ = Schmidt number, $\chi = \frac{K_1 d}{w_0}$ = Chemical reaction parameter.

The boundary condition in the non-dimensional form become

$$\left.\begin{array}{l} \eta = -\frac{1}{2}: \quad u = h\frac{\partial u}{\partial \eta}, \ v = h\frac{\partial v}{\partial \eta}, \ T = 0, C = 0 \\ \qquad \text{and} \\ \eta = \frac{1}{2}: \quad u = v = 0, \quad T = \cos\omega t, \ C = \cos\omega t \end{array}\right\} \quad (13)$$

where $h = \frac{L_1}{d}$ is the slip flow parameter.

3. SOLUTION OF THE PROBLEM

Introducing the complex velocity of the form

$$F(\eta, t) = u(\eta, t) + iv(\eta, t), \quad (14)$$

Equation (9) and (10) can be combined to give

$$R_e\left(\frac{\partial F}{\partial t} + \frac{\partial F}{\partial \eta}\right) = R_e A\cos\omega t + \frac{\partial^2 F}{\partial \eta^2} - \left(K^{-1} + 2i\Omega + \frac{H^2}{1+m^2}(1+im)\right)F + G_r T + G_m C \quad (15)$$

The corresponding boundary condition transforms to

$$\left.\begin{array}{l} \eta = -\frac{1}{2}: \quad F = h\frac{\partial F}{\partial \eta}, \ T = 0, \ C = 0 \\ \qquad \text{and} \\ \eta = \frac{1}{2}: \quad F = 0, \ T = \cos\omega t, \ C = \cos\omega t \end{array}\right\} \quad (16)$$

In order to solve equation (11), (12) and (15) under the boundary condition (16) following Chaudhary *et al* [22], we assume the solution of the form:

$$F(\eta, t) = F_0(\eta)e^{i\omega t}, \ T(\eta, t) = \theta_0(\eta)e^{i\omega t}, \ C(\eta, t) = C_0(\eta)e^{i\omega t} \quad (17)$$

The resulting equations are solved under the following boundary conditions

$$\left.\begin{array}{l} \eta = -\frac{1}{2}: \ F_0 = h\frac{\partial F_0}{\partial \eta}, \ T_0 = 0, \ C_0 = 0 \\ \qquad \text{and} \\ \eta = \frac{1}{2}: \ F_0 = 0, \quad T_0 = 1, \ C_0 = 1 \end{array}\right\} \quad (18)$$

and we get the following expression for the velocity, the temperature and the concentration profiles

$$F(\eta, t) = \Big[B_3\cosh q_1\eta + B_4\sinh q_2\eta + \frac{R_e A}{l^2} - \frac{G_r B_1}{2}\left(\frac{e^{n_1\eta}}{n_1^2 - R_e n_1 - l^2} + \frac{e^{-n_1\eta}}{n_1^2 + R_e n_1 - l^2}\right) - \frac{G_r B_2}{2}\left(\frac{e^{n_2\eta}}{n_2^2 - R_e n_2 - l^2} - \frac{e^{-n_2\eta}}{n_2^2 + R_e n_2 - l^2}\right) - A_8 e^{\left(\frac{1}{2}+\eta\right)r_1} + A_9 e^{\left(\frac{1}{2}+\eta\right)r_2}\Big] e^{i\omega t} \quad (19)$$

$$T(\eta, t) = \left[B_1\cosh n_1\eta + B_2\sinh n_2\eta + \frac{P_r R_e^2 Q}{e^{r_1} - e^{r_2}}\left(\frac{e^{\left(\frac{1}{2}+\eta\right)r_1}}{r_1^2 - R_e P_r r_1 - n^2} - \frac{e^{\left(\frac{1}{2}+\eta\right)r_2}}{r_2^2 - R_e P_r r_2 - n^2}\right)\right] e^{i\omega t} \quad (20)$$

$$C(\eta, t) = \frac{1}{e^{r_1} - e^{r_2}}\left(e^{\left(\frac{1}{2}+\eta\right)r_1} - e^{\left(\frac{1}{2}+\eta\right)r_2}\right) \quad (21)$$

where

$$l = \sqrt{K^{-1} + 2i\Omega + i\omega R_e + \frac{H^2}{1+m^2}(1+im)}, \qquad n = \sqrt{N^2 + \phi P_r + i\omega R_e P_r^2}$$

$r = \sqrt{(i\omega + \chi)S_c R_e}$.

The validity and the correctness of the present solution is verified by taking $G_r = G_m = \mathrm{H} = \Omega = \mathrm{h} = 0$ and $K \longrightarrow \infty$ i.e. for the horizontal channel in the absence of rotation, slip flow and the condition of the ordinary medium .In this the case solution reduced to

$$F(\eta, t) = \frac{A}{i\omega}\left(1 - \frac{cosh\left(\frac{R_e+\sqrt{R_e^2+4i\omega R_e}}{2}\right)\eta}{cosh\left(\frac{R_e+\sqrt{R_e^2+4i\omega R_e\eta}}{4}\right)}\right)e^{i\omega t} \tag{22}$$

which is the well known solution reported by Schlichting and Gersten [24]for periodic variation of the pressure gradient along the axis of the channel.

4. SKIN FRICTION

From the velocity field we can now obtain the skin friction $\tau_L$at the left plate in terms of the it's amplitude and the phase angle as

$$\tau_L = \left(\frac{\partial F}{\partial \eta}\right)_{\eta=-\frac{1}{2}} = |F|cos\omega(t + \varphi), \tag{23}$$

with

$$F_r + iF_i = B_3 q_1 sinhq_1 \frac{1}{2} + B_4 q_2 coshq_2 \frac{1}{2} - \frac{G_r B_1 n_1}{2}\left(\frac{e^{-n_1\frac{1}{2}}}{{n_1}^2 - R_e n_1 - l^2} - \frac{e^{n_1\frac{1}{2}}}{{n_1}^2 + R_e n_1 - l^2}\right) - \frac{G_r B_2 n_2}{2}\left(\frac{e^{-n_2\frac{1}{2}}}{{n_2}^2 - R_e n_2 - l^2} + \frac{e^{n_2\frac{1}{2}}}{{n_2}^2 + R_e n_2 - l^2}\right) - A_8\, r_1 + A_9 r_2 \tag{24}$$

The amplitude is $|F| = \sqrt{{F_r}^2 + {F_i}^2}$ and the phase angle $\varphi = tan^{-1}\frac{F_i}{F_r}$ (25)

5. HEAT TRANSFER

From the temperature field and rate of heat transfer N_u (Nusselt number) in term of its amplitude and the phase angle can be obtained as

$$N_u = \left(\frac{\partial T}{\partial \eta}\right)_{\eta=-\frac{1}{2}} = |H|cos(t + \psi) \ , \tag{26}$$

where

$$H_r + iH_i = -B_1 n_1 sinh\frac{n_1}{2} + B_2 n_2\, cosh\frac{n_2}{2} + \frac{P_r {R_e}^2 Q}{e^{r_1} - e^{r_2}}\left(\frac{r_1}{{r_1}^2 - R_e P_r r_1 - n^2} - \frac{r_2}{{r_2}^2 - R_e P_r r_2 - n^2}\right) \tag{27}$$

The amplitude $|H|$ and the phase angle ψ of the heat transfer are respectively given by

$$|H| = \sqrt{H_r^2 + H_i^2} \quad \text{and} \quad \psi = tan^{-1}\frac{H_i}{H_r} \tag{28}$$

where constants used above have been listed in the appendix.

RESULTS AND DISCUSSION

In order to illustrate the influence of the various parameter on the velocity profile, temperature profile, the concentration profile, the coefficient of skin friction, the rate of heat transfer and the rate of mass transfer in terms of their amplitudes and phase are evaluated numerically for the different value of the flow parameter involved in the governing equations e.g. the rotation parameter Ω, the Reynolds number R_e, the permeability of the porous medium parameter K, the Grashof number G_r, the modified Grashof number G_m, Prandtl number P_r, the radiation parameter N, the heat absorption parameter ϕ, the radiation absorption parameter Q, the magnetic field parameter H, the Schmidt number S_c, the chemical reaction parameter χ, the amplitude of the pressure gradient A, the slip parameter h, the frequency of oscillation ω, the Hall current parameter m and time $t = 0$. The cases of small rotation ($\Omega = 5$) and the large rotation ($\Omega = 10$) are taken to assessed the effect of each parameters. The numerical values obtained are expressed through graphs and tables in order to bring out explicitly the effect of each parameter on the important flow characteristics. The figures for the variation of velocity distribution clearly exhibit the effect of slip parameter at the left plate of the channel otherwise distribution is nearly parabolic in nature with maximum velocity occurring at the middle of the channel.

The effect of the Reynolds number R_e, the porous medium permeability K, the amplitude of the pressure gradient A, the Grashof number G_r, the modified Grashof number G_m,on the velocity profile is shown by the Figs. 2 to 6 respectively for small value of rotation parameter $\Omega = 5$ and a large value of a rotation parameter $\Omega = 10$. It evident from the study of these figures that the velocity profile is enhanced with the increase of these parameters. Figs. 7 to 17 depict the effect of the rotation parameter Ω ,the magnetic field parameter H, Prandtl number P_r, the radiation parameter N, the heat absorption parameter ϕ, chemical reaction parameter χ, the radiation absorption parameter Q, the Schmidt number S_c, the slip parameter h, the Hall current parameter m and the frequency of oscillation ω respectively. It is observed from these figures that the velocity profile is diminished with the increase of all these parameter irrespective of small or large values of rotation parameter Ω except the slip parameter where an interesting feature is observed that the velocity profiles increases up to $\eta = 0.2$ approximately then it start decreasing near the right plate of the channel.

The variation in the temperature profile for t = 0 is presented in the fig 18. It is observed from this Fig.18 that the amplitude of temperature profiles decrease with the increasing Reynolds number R_e, the Prandtl number P_r, the radiation parameter N, the heat absorption parameter ϕ, chemical reaction parameter χ, the radiation absorption parameter Q, the frequency of oscillation ω and it increases with the Schmidt number S_c.

A variation in the concentration profile is plotted in the Fig 19 and it is evident from the study of this figure that the amplitude of the concentration profile decreases with all the parameters effecting the concentration equation.

The amplitude $|H|$ of the rate of heat transfer is presented Fig.20. This figure shows that $|H|$ decreases with the increasing Reynolds number R_e, the radiation parameter N, the heat absorption parameter ϕ, chemical reaction parameter χ, and it increases with the Prandtl numberP_r, the Schmidt number S_c , the radiation absorption parameter Q.

The phase angle ψ of the rate of heat transfer is depicted in Fig. 21. There exist phase lag initially for small values of the frequency of oscillation ω and there after the phase oscillates.

The skin friction at the left plate of the channel is obtained in terms of its amplitude and the phase angle φ. The amplitude $|F|$ is presented in the Table-1 for the different values of parameters involved. The values in this table reveal that $|F|$ decreases with frequency of oscillation ω, the Prandtl number P_r, the radiation parameter N, the magnetic field parameter H, the amplitude of the pressure gradient A, the Grashof number G_r, the modified Grashof number G_m, ϕ the heat absorption parameter , chemical reaction parameter χ, the radiation absorption parameter Q, the Schmidt number S_c, the slip parameter h, the Hall current parameter m and it increases with the rotation parameter Ω , Reynolds number R_e and porous medium permeability K. The phase angle φ of the skin friction is shown in Table-2. The negative values in this table indicate that there is always a phase lag and this phase goes on increasing with increasing frequency of oscillation ω. The phase decreases with increasing rotation parameter Ω, Reynolds number R_e, the porous medium permeability K, the Grashof number G_r, the modified Grashof number G_m, the Schmidt number S_c, the slip parameter h, the Hall current parameter m and it increases with increasing Prandtl number P_r, the radiation parameter N, the magnetic field parameter H, the amplitude of the pressure gradient A, the heat absorption parameter ϕ, chemical reaction parameter χ and radiation absorption parameter Q.

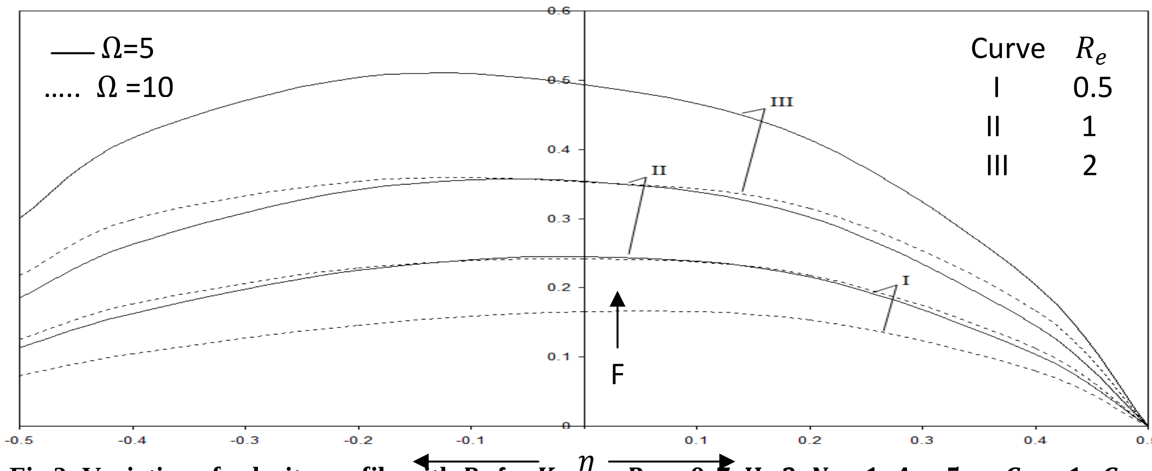

Fig.2: Variation of velocity profile with R_e for K , $P_r = 0.7$, H=2, $N = 1, A = 5,$ $G_r = 1,$ $G_m = 2,$ $\emptyset = 0.10,$ $S_c = 0.22,$ $\chi = 0.2,$ $Q = 1,$ $h = 0.2,$ $m = 0.5,$ $\omega = 5$.

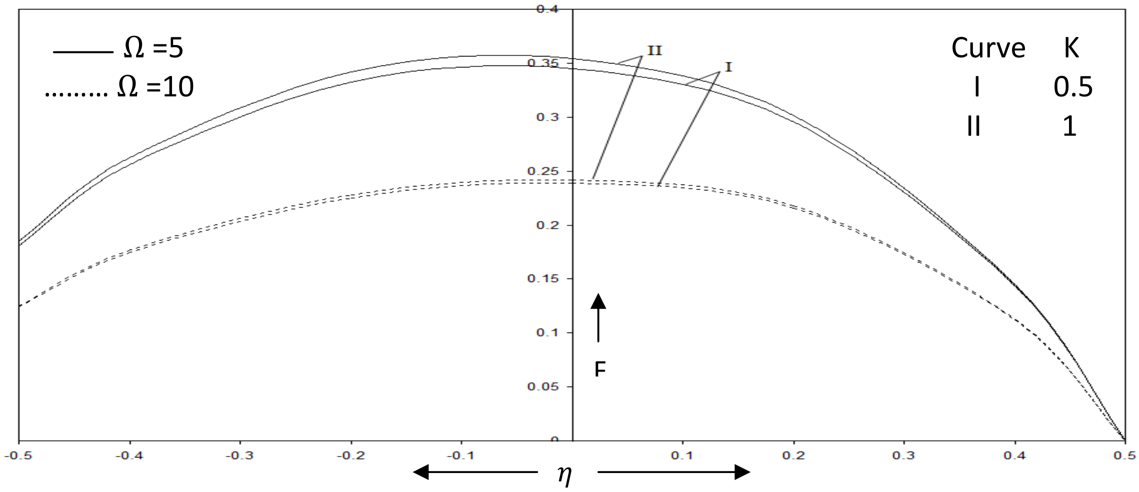

Fig.3: Variation of velocity profile with K for $R_e = 1, H = 2, P_r = 0.7, N = 1, A = 5, G_r = 1, G_m = 2, \emptyset = 0.10, S_c = 0.22, \chi = 0.2, Q = 1, h = 0.2, m = 0.5, \omega = 5.$

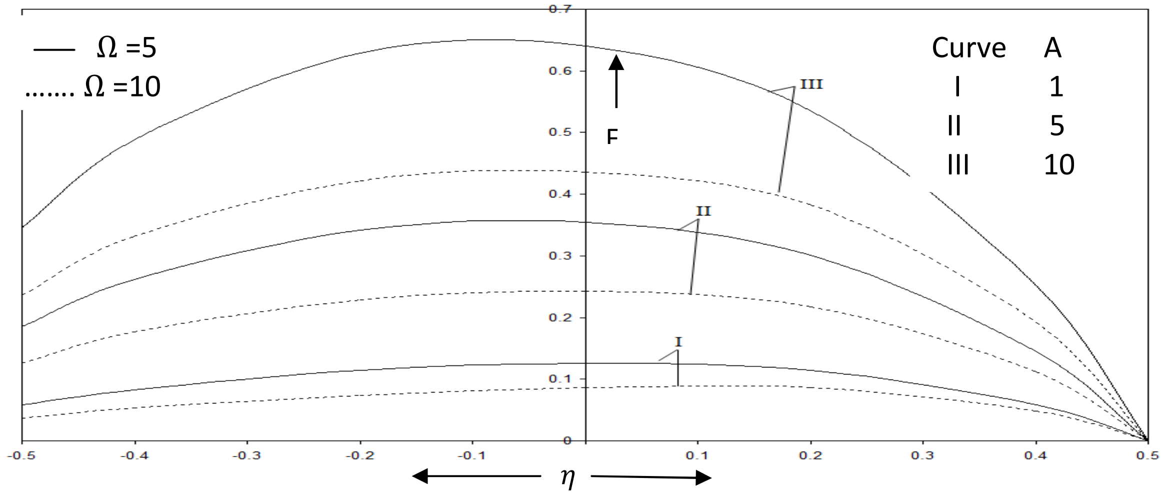

Fig. 4: Variation of velocity profile with A for $R_e = 1, K = 1, P_r = 0.7, H = 2, N = 1, G_r = 1, G_m = 2, \emptyset = 0.10, S_c = 0.22, \chi = 0.2, Q = 1, h = 0.2, m = 0.5, \omega = 5.$

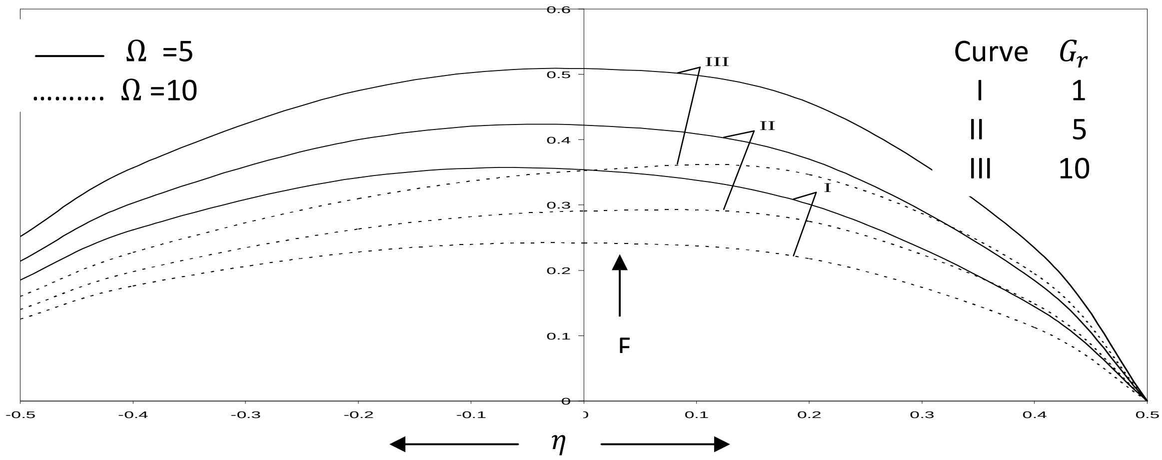

Fig. 5: Variation of velocity profile with G_r for $R_e = 1, K = 1, P_r = 0.7, H = 2, A = 5$, N= 1, $G_m = 2, \emptyset = 0.10, S_c = 0.22, \chi = 0.2, Q = 1, h = 0.2, m = 0.5, \omega = 5.$

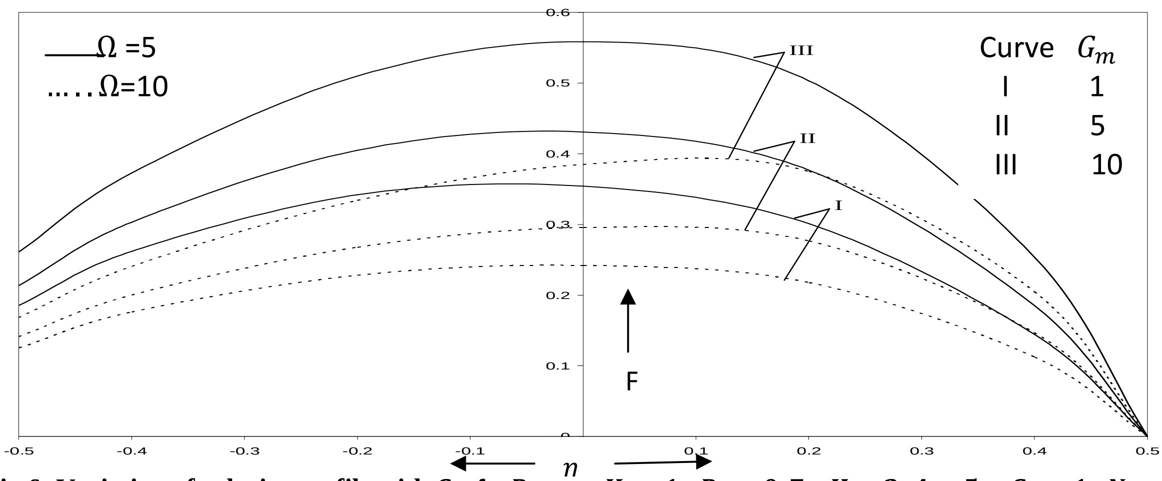

Fig.6: Variation of velocity profile with G_m for $R_e = 1$, $K = 1$, $P_r = 0.7$, $H = 2$, $A = 5$, $G_r = 1$, $N = 1$, $\emptyset = 0.10$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

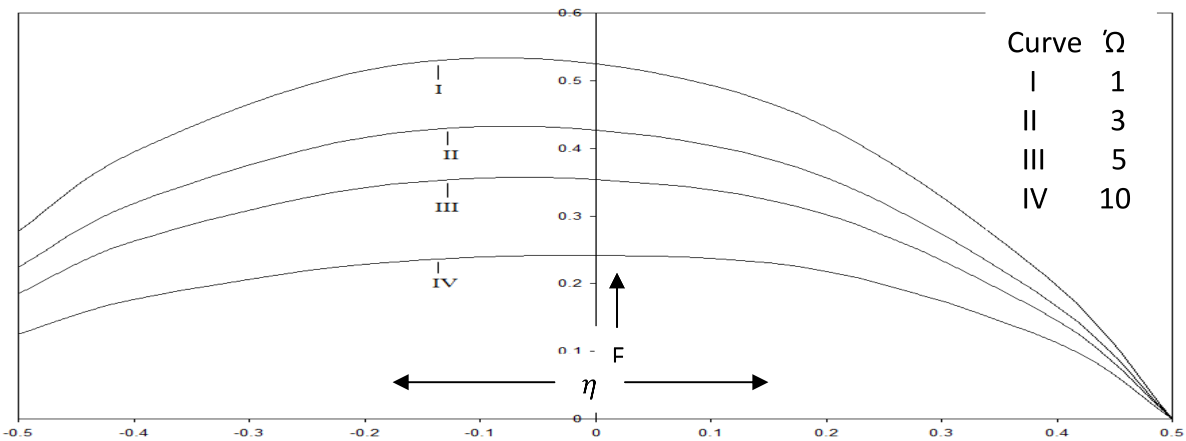

Fig.7: Variation of velocity profile with Ω for $R_e = 1$, $K = 1$, H=2, $P_r = 0.7$, $N = 1$, $A = 5$, $G_r = 1$, $G_m = 2$, $\emptyset = 0.10$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

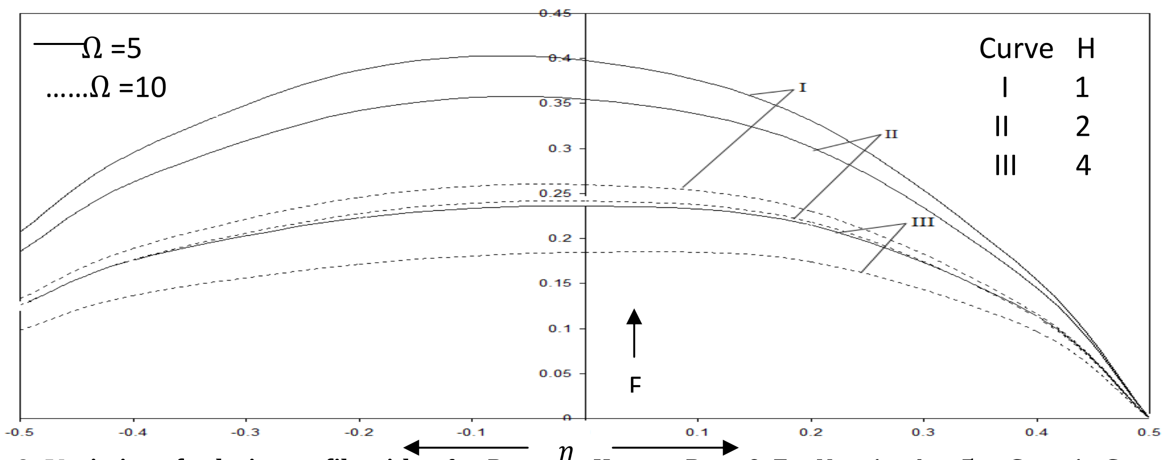

Fig. 8: Variation of velocity profile with H for $R_e = 1$, $K = 1$, $P_r = 0.7$, $N = 1$, $A = 5$, $G_r = 1$, $G_m = 2$, $\emptyset = 0.10$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

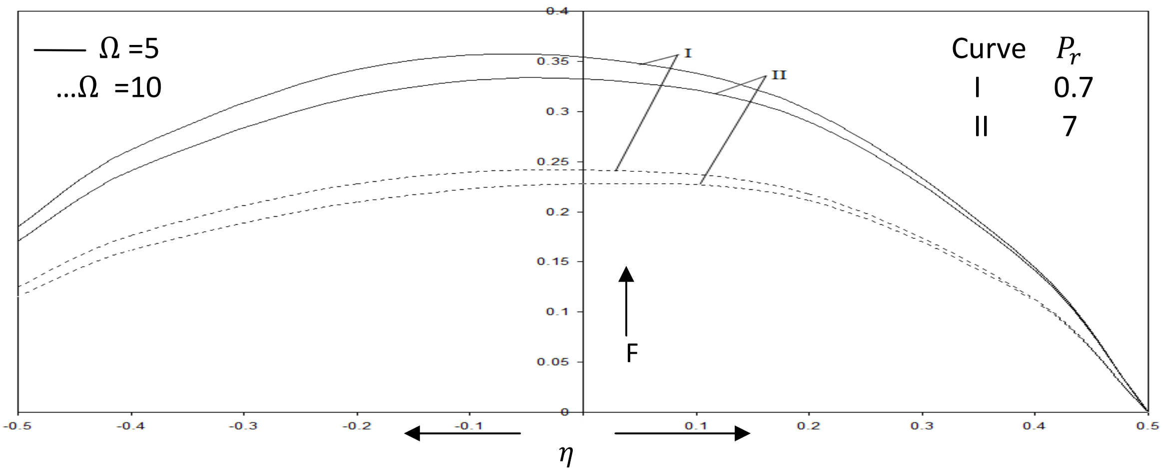

Fig. 9: Variation of velocity profile with P_r for $R_e = 1$, $K = 1$, $H = 2$, $N = 1$, $A = 5$, $G_r = 1$, $G_m = 2$, $\emptyset = 0.10$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

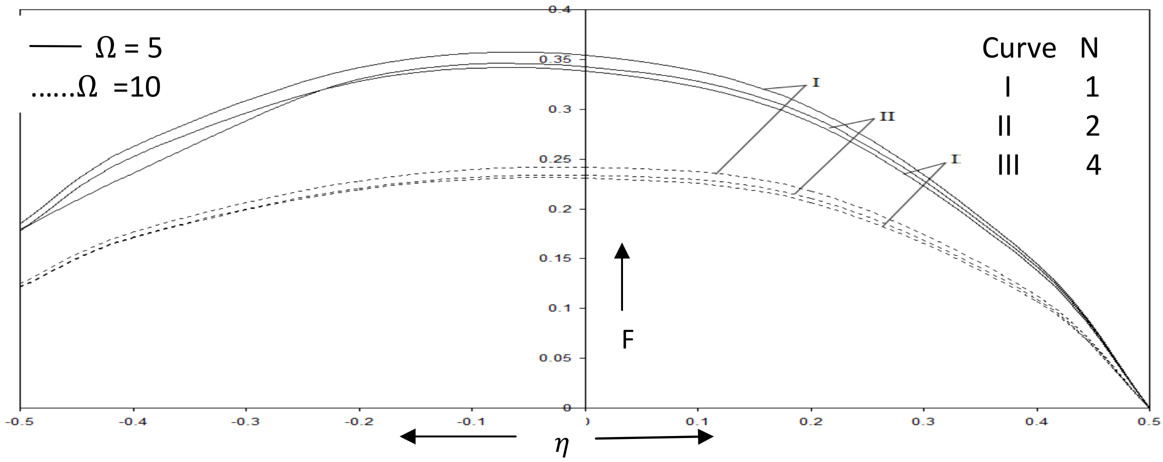

Fig. 10: Variation of velocity profile with N for $R_e = 1$, $K = 1$, $P_r = 0.7$, $H = 2$, $A = 5$, $G_r = 1$, $G_m = 2$, $\emptyset = 0.10$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

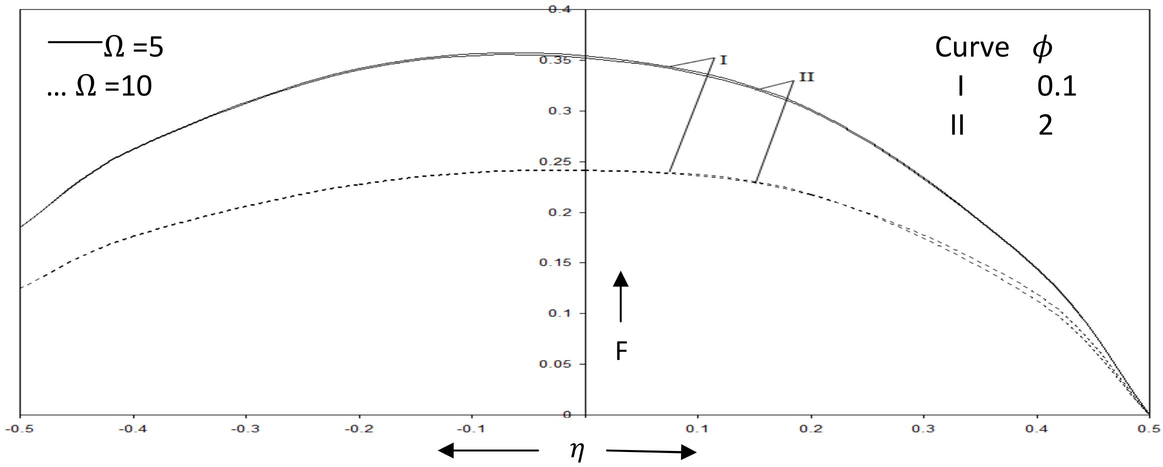

Fig. 11: Variation of velocity profile with $\emptyset$ for $R_e = 1$, $K = 1$, $P_r = 0.7$, $A = 5$, $H = 2$, $G_r = 1$, $N = 1$, $G_m = 2$, $S_c = 0.22$, $\chi = 0.2$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

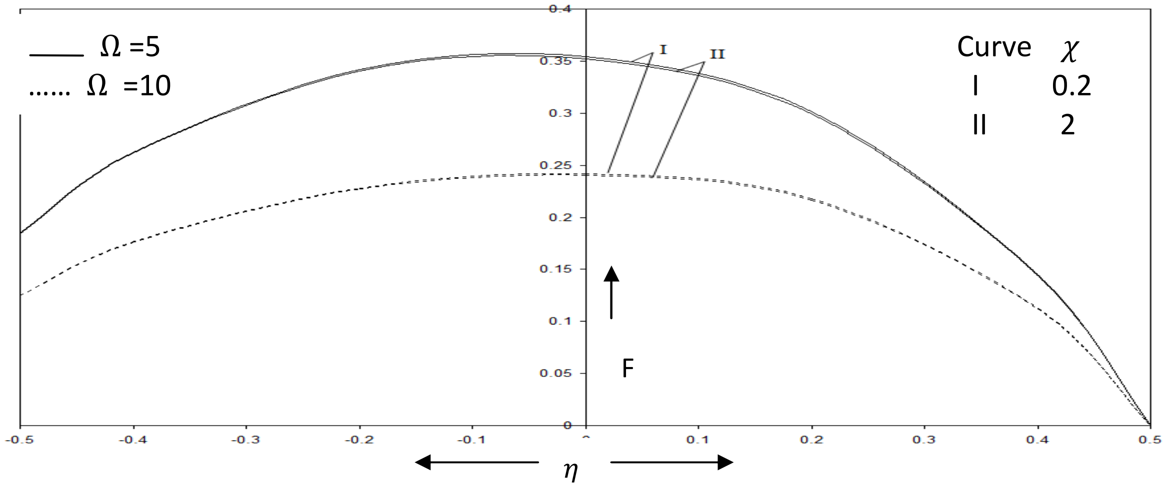

Fig.12: Variation of velocity profile with χ for $R_e = 1$, $K = 1$, $P_r = 0.7$, $H = 2$, $A = 5$, $G_r = 1$, $N = 1$, $G_m = 0.10$, $S_c = 0.22$, $\phi = 0.10$, $Q = 1$, $h = 0.2$, $m = 0.5$, $\omega = 5$.

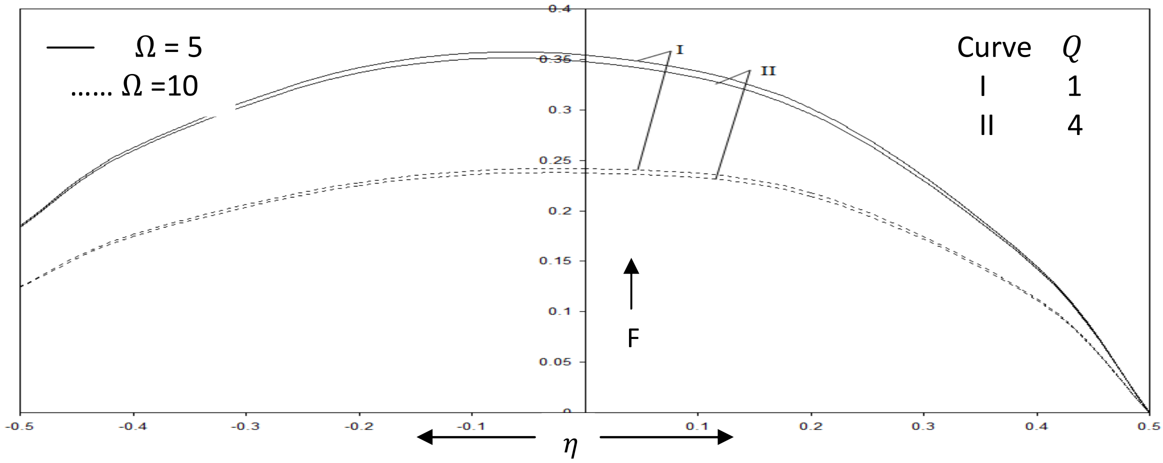

Fig. 13: Variation of velocity profile with Q for $R_e = 1$, $K = 1$, $P_r = 0.7$, $H = 2$, $A = 5$, $G_r = 1$, $N = 1$, $G_m = 0.10$, $S_c = 0.22$, $\phi = 0.10$, $\chi = 0.2$, $h = 0.2$, $m = 0.5$, $\omega = 5$

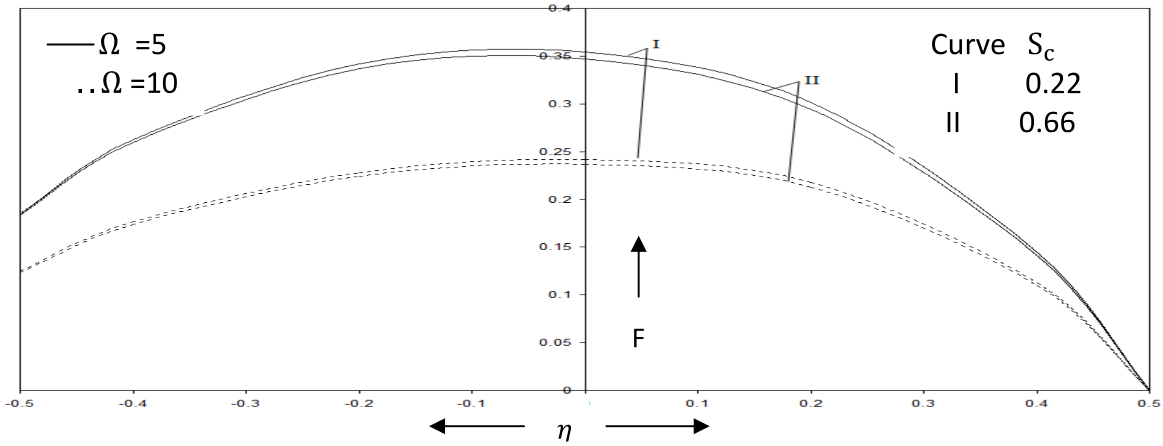

Fig. 14: Variation of velocity profile with S_c for $R_e = 1$, $K = 1$, $P_r = 0.7$, $H = 2$, $A = 5$, $G_r = 1$, $N = 1$, $G_m = 2$, $Q = 1$, $\phi = 0.10$, $\chi = 0.2$, $h = 0.2$, $m = 0.5$, $\omega = 5$

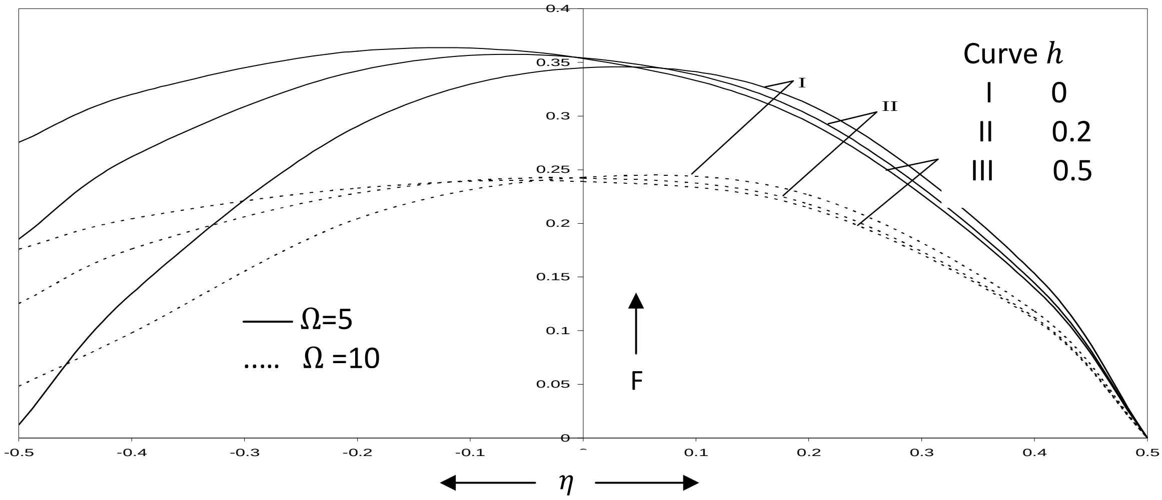

Fig. 15: Variation of velocity profile with h for $R_e = 1, K = 1, P_r = 0.7, H = 2, A = 5, G_r = 1, N = 1, G_m = 2, Q = 1, \phi = 0.10, \chi = 0.2, S_c = 0.22, m = 0.5, \omega = 5.$

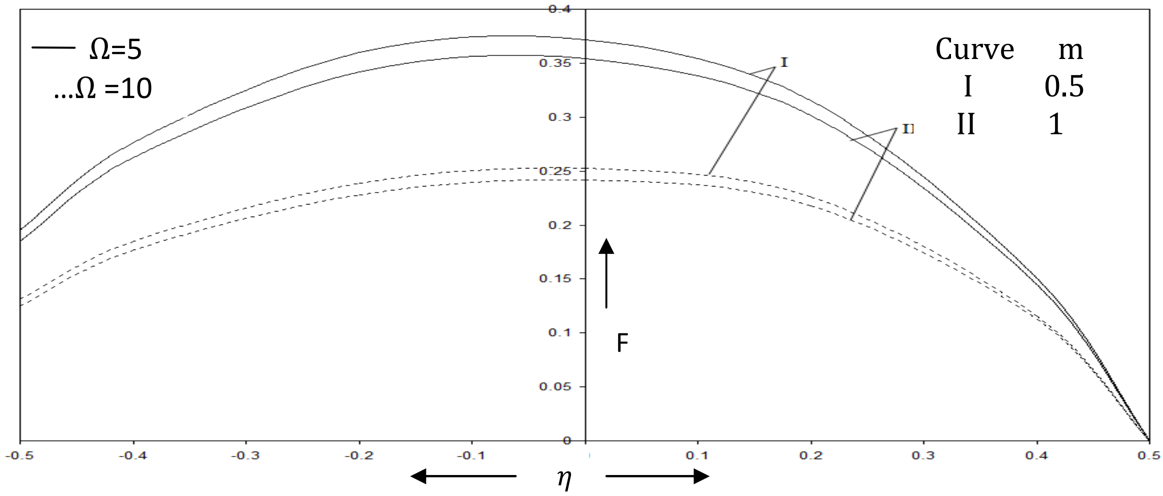

Fig.16: Variation of velocity profile with m **for** $R_e = 1, K = 1, P_r = 0.7, H = 2, A = 5, G_r = 1, N = 1, G_m = 2, S_c = 0.22, \phi = 0.10, \chi = 0.2, h = 0.2, Q = 1, \omega = 5$

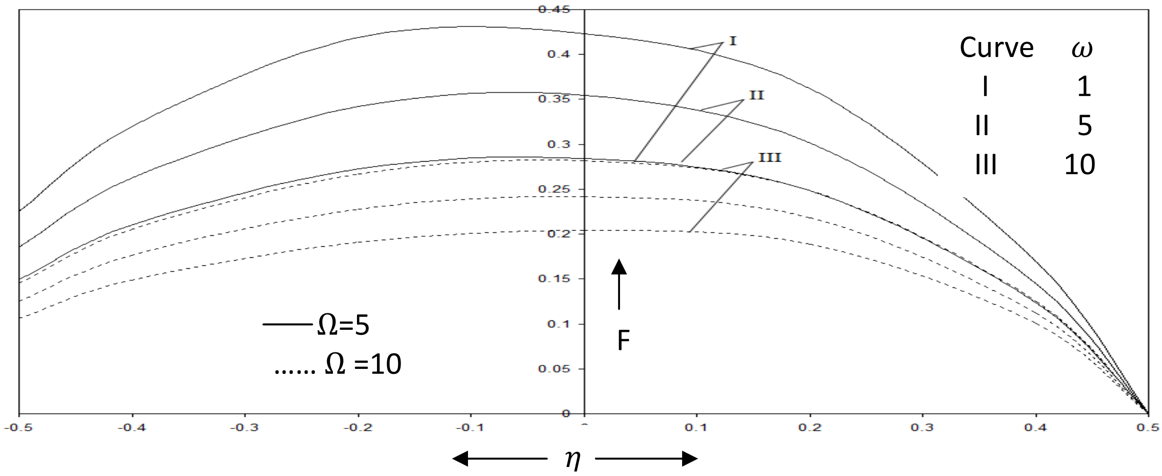

Fig. 17: Variation of velocity profile with ω **for** $R_e = 1, K = 1, P_r = 0.7, H = 2, A = 5, G_r = 1, N = 1, G_m = 2, S_c = 0.22, \phi = 0.10, \chi = 0.2, h = 0.2, m = 0.5, Q = 1$

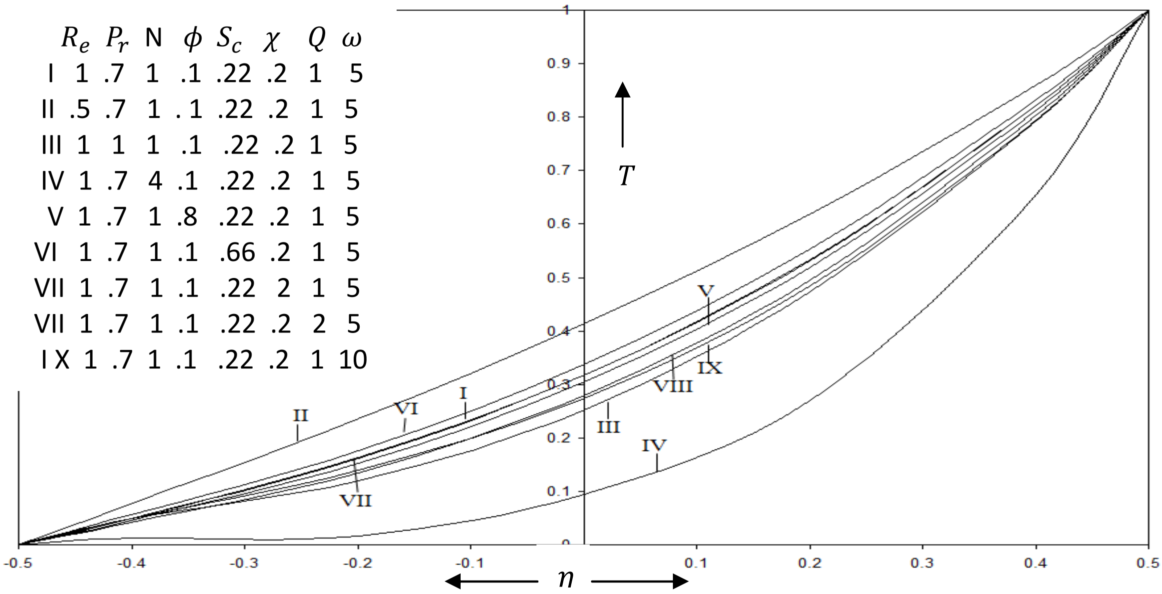

Fig. 18: Variation of the Temperature profile for t=0.

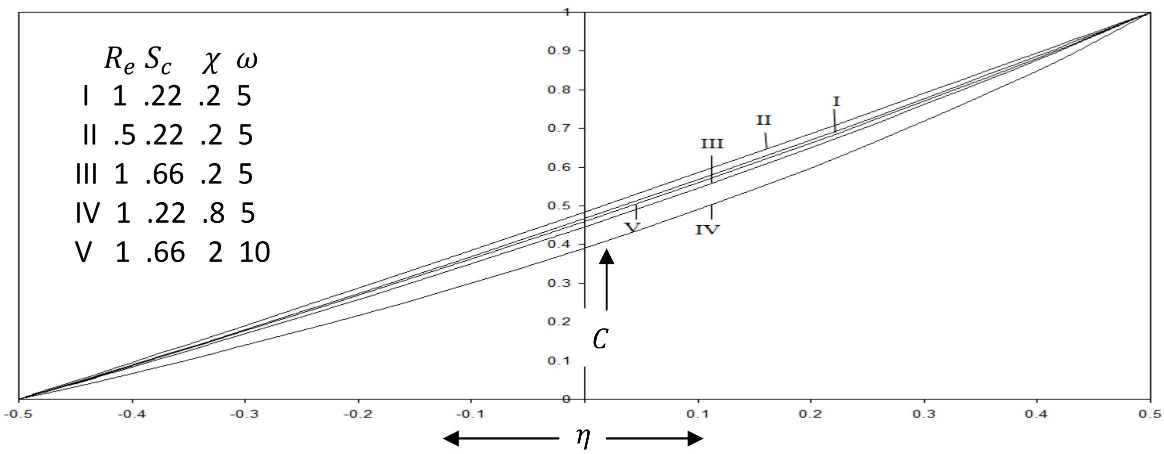

Fig.19: Variation of Concentration profile for t=0

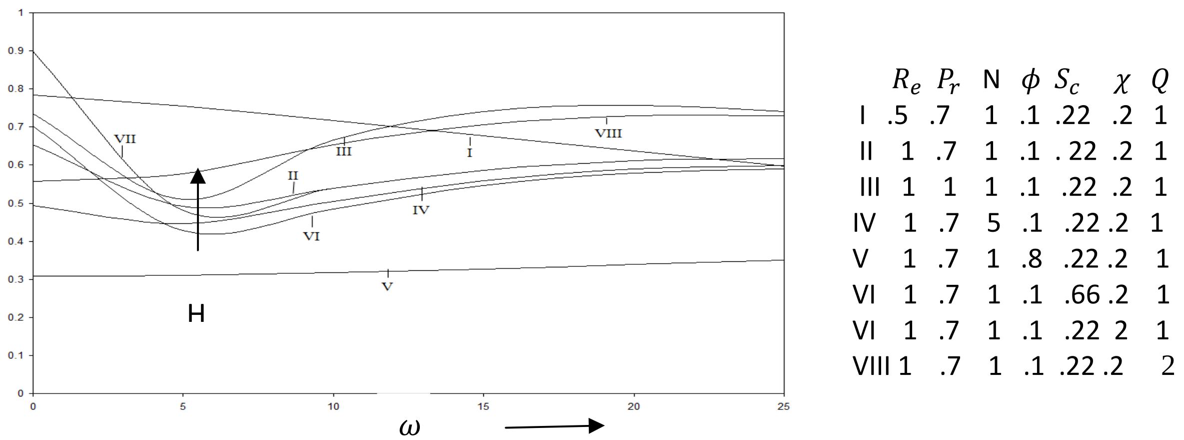

Fig.20: Amplitude $|H|$ of the rate of heat transfer for t = 0.

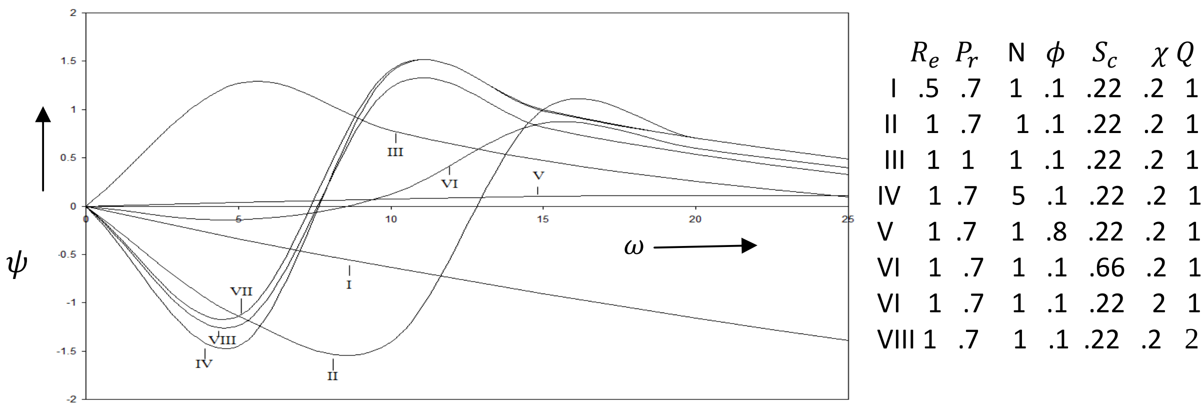

Fig.21: Phase angle ψ of the rate of heat transfer for $t = 0$.

Table-1. Amplitude $|F|$ of skin friction for $t = 0$.

Ω	R_e	K	P_r	N	H	A	G_r	G_m	ϕ	S_c	χ	Q	h	m	$\omega = 5$	$\omega = 10$
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	0.9265	0.0589
1	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	0.19468	01335
5	.5	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	0.61613	0.0462
5	1	.5	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	0.09547	0.6317
5	1	1	7	1	2	5	1	2	.1	.22	.2	1	.2	.5	0.08896	0.0635
5	1	1	.7	5	2	5	1	2	.1	.22	.2	1	.2	.5	0.0888	0.0587
5	1	1	.7	1	4	5	1	2	.1	.22	.2	1	.2	.5	0.8449	0.0627
5	1	1	.7	1	2	10	1	2	.1	.22	.2	1	.2	.5	0.17819	0.1215
5	1	1	.7	1	2	5	5	2	.1	.22	.2	1	.2	.5	0.87293	0.0480
5	1	1	.7	1	2	5	1	5	.1	.22	.2	1	.2	.5	0.09676	0.0572
5	1	1	.7	1	2	5	1	2	.8	.22	.2	1	.2	.5	0.90693	0.0589
5	1	1	.7	1	2	5	1	2	.1	.66	.2	1	.2	.5	0.87489	0.0558
5	1	1	.7	1	2	5	1	2	.1	.22	2	1	2	.5	0.09047	0.0589
5	1	1	.7	1	2	5	1	2	.1	.22	.2	2	.2	.5	0.09034	0.0589
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.5	.5	0.09863	0.0539
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	1	0.08074	0.0515

Table-2. Phase angle ψ of the skin friction for $t = 0$

Ω	R_e	K	P_r	N	H	A	G_r	G_m	ϕ	S_c	χ	Q	h	m	$\omega = 5$	$\omega = 10$
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	-1.0593	-1.1649
1	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	-0.7946	-1.0056
5	.5	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	-0.9948	-1.0887
5	1	.5	.7	1	2	5	1	2	.1	.22	.2	1	.2	.5	-1.0150	-1.1261
5	1	1	7	1	2	5	1	2	.1	.22	.2	1	.2	.5	-1.0227	-1.1062
5	1	1	.7	5	2	5	1	2	.1	.22	.2	1	.2	.5	-1.0519	-1.1546
5	1	1	.7	1	4	5	1	2	.1	.22	.2	1	.2	.5	-0.8352	-0.9362
5	1	1	.7	1	2	10	1	2	.1	.22	.2	1	.2	.5	-1.0284	-1.1228
5	1	1	.7	1	2	5	5	2	.1	.22	.2	1	.2	.5	-1.1506	-1.2797
5	1	1	.7	1	2	5	1	5	.1	.22	.2	1	.2	.5	-1.1001	-1.2269
5	1	1	.7	1	2	5	1	2	.8	.22	.2	1	.2	.5	-1.0588	-1.1644
5	1	1	.7	1	2	5	1	2	.1	.66	.2	1	.2	.5	-1.0734	-1.1785
5	1	1	.7	1	2	5	1	2	.1	.22	2	1	2	.5	-1.059	-1.1638
5	1	1	.7	1	2	5	1	2	.1	.22	.2	2	.2	.5	-1.0597	-1.1637
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.5	.5	-1.2046	-1.3172
5	1	1	.7	1	2	5	1	2	.1	.22	.2	1	.2	1	-1.1237	-1.2182

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Appendix

$$A_1 = \frac{P_r R_e^2 Q_1}{e^{r_1}-e^{r_2}}\left(\frac{1}{r_1^2-P_rR_er_1-n^2}-\frac{1}{r_2^2-P_rR_er_2-n^2}\right),$$

$$A_2 = \frac{P_r R_e^2 Q_1}{e^{r_1}-e^{r_2}}\left(\frac{e^{r_1}}{r_1^2-P_rR_er_1-n^2}-\frac{e^{r_2}}{r_2^2-P_rR_er_2-n^2}\right),\ A_3 = \frac{P_r R_e^2 Q_1}{e^{r_1}-e^{r_2}}\left(\frac{1}{r_1^2-P_rR_er_1-n^2}\right),\ A_4 = \frac{P_r R_e^2 Q_1}{e^{r_1}-e^{r_2}}\left(\frac{1}{r_2^2-P_rR_er_2-n^2}\right),$$

$$A_5 = \frac{1}{e^{r_1}-e^{r_2}},\quad A_6 = G_rA_3+G_mA_5,\ A_7 = G_rA_4+G_mA_5,\quad A_8 = \left(\frac{A_6}{r_1^2-R_er_1-l^2}\right),\ A_9 = \left(\frac{A_7}{r_2^2-R_er_2-l^2}\right),$$

$$A_{10} = -\frac{R_eA}{q^2}+\frac{G_rB_1}{2}\left(\frac{e^{-\frac{n_1}{2}}}{{n_1}^2-R_en_1-l^2}+\frac{e^{\frac{n_1}{2}}}{{n_1}^2+R_en_1-l^2}\right)+\frac{G_rB_2}{2}\left(\frac{e^{-\frac{n_2}{2}}}{{n_2}^2-R_en_2-l^2}-\frac{e^{\frac{n_2}{2}}}{{n_2}^2+R_en_2-l^2}\right)-h\left[\frac{G_rB_1n_1}{2}\left(\frac{e^{-\frac{n_1}{2}}}{{n_1}^2-R_en_1-l^2}-\frac{e^{\frac{n_1}{2}}}{{n_1}^2+R_en_1-l^2}\right)+\frac{G_rB_2n_2}{2}\left(\frac{e^{-\frac{n_2}{2}}}{{n_2}^2-R_en_2-l^2}+\frac{e^{\frac{n_2}{2}}}{{n_2}^2+R_en_2-l^2}\right)\right]+A_8(1-hr_1)+A_9(1-hr_2)$$

$$A_{11} = -\frac{R_eA}{l^2}+\frac{G_rB_1}{2}\left(\frac{e^{\frac{n_1}{2}}}{{n_1}^2-R_en_1-l^2}+\frac{e^{-\frac{n_1}{2}}}{{n_1}^2+R_en_1-l^2}\right)+\frac{G_rB_2}{2}\left(\frac{e^{\frac{n_2}{2}}}{{n_2}^2-R_en_2-l^2}-\frac{e^{-\frac{n_2}{2}}}{{n_2}^2+R_en_2-l^2}\right)+A_8e^{r_1}-A_9e^{r_2},$$

$$B_1 = \frac{1-A_1-A_2}{2\cosh\frac{n_1}{2}},\ B_2 = \frac{1+A_1-A_2}{2\sinh\frac{n_2}{2}},\quad B_3 = \frac{(A_{10}+A_{11})\sinh\frac{q_2}{2}+A_{11}hq_2\cosh\frac{q_2}{2}}{2\cosh\frac{q_1}{2}\sinh\frac{q_2}{2}+h\left(q_2\cosh\frac{q_1}{2}\cosh\frac{q_2}{2}+q_1\sinh\frac{q_1}{2}\sinh\frac{q_2}{2}\right)},$$

$$B_4 = \frac{(A_{11}-A_{10})\cosh\frac{q_1}{2}+A_{11}hq_1\sinh\frac{q_1}{2}}{2\cosh\frac{q_1}{2}\sinh\frac{q_2}{2}+h\left(q_2\cosh\frac{q_1}{2}\cosh\frac{q_2}{2}+q_1\sinh\frac{q_1}{2}\sinh\frac{q_2}{2}\right)},\ q_1 = \frac{R_e+\sqrt{R_e^2+4l^2}}{2},\ q_2 = \frac{R_e-\sqrt{R_e^2+4l^2}}{2},\ n_1 = \frac{P_rR_e+\sqrt{(P_rR_e)^2+4n^2}}{2},$$

$$n_2 = \frac{P_rR_e-\sqrt{(P_rR_e)^2+4n^2}}{2},\quad r_1 = \frac{S_cR_e+\sqrt{(S_cR_e)^2+4r^2}}{2},\quad r_2 = \frac{S_cR_e-\sqrt{(S_cR_e)^2+4r^2}}{2}.$$