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Generalized Useful Fuzzy Inaccuracy Measure

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ABSTRACT

In this paper we present a new class of generalized useful fuzzy inaccuracy measure. This measure is not only new but some known measures are the particular cases of our proposed measure. We also obtained the bounds for this measure.

INTRODUCTION

The main objective of information is to remove uncertainty and fuzziness. In fact, we measure information supplied by the amount of probabilistic uncertainty removed in an experiment and the measure of uncertainty removed is also called as a measure of information, while measure of fuzziness is the measure of vagueness and ambiguity of uncertainties.

Let X is a discrete random variable taking values $x_1, x_2, \dots, x_n$ with respective probabilities

$$P = (p_1, p_2, \dots, p_n), p_i \geq 0 \;\forall\; i = 1,2,\dots,n \text{ and } \sum_{i=1}^{n} p_i = 1.$$

Shannon [11] gives the following measure of information and call it entropy.

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \qquad (1.1)$$

The concept of entropy has been widely used in different areas, e.g. communication theory, statistical mechanics, finance pattern recognition, and neural network etc. Fuzzy set theory developed by Lotfi A. Zadeh [14] has found wide applications in many areas of science and technology, e.g. clustering, image processing, decision making etc. because of its capability to model non-statistical imprecision or vague concepts.

A fuzzy set "A" is characterised by a membership function and is represented as:

$$A = \{x_i / \mu_A(x_i) : i = 1, 2, \dots, n\}$$

Where $\mu_A(x_i)$ gives the degree of belongingness of the element x_i to the set "A" and is defined as follows:

$$\mu_A(x_i) = \begin{cases} 0, & if\, x_i \notin A \text{ and there is no ambiguity,} \\ 1, & if\, x_i \in A \text{ and there is no ambiguity,} \\ 0.5, & if\, x_i \in A \text{ or } x_i \notin A \text{ and there is maximum ambiguity,} \end{cases}$$

In fact $\mu_A(x_i)$ associates with each $x_i \in R^n$ a grade of membership function in the set A. When $\mu_A(x_i)$ takes values only 0 or 1, there is no uncertainty about it and a set is said to be a crisp (i.e. non-fuzzy) set.

A fuzzy set A^* is said to a sharpened version of fuzzy set if the following conditions were satisfied

$\mu_{A^*}(x_i) \le \mu_A(x_i)$ if $\mu_A(x_i) \le 0.5$ for all $x_i, i = 1, 2, \ldots, n$
and
$\mu_{A^*}(x_i) \ge \mu_A(x_i)$ if $\mu_A(x_i) \le 0.5$ for all $x_i, i = 1, 2, \ldots, n$

Since $\mu_A(x_i)$ and $1 - \mu_A(x_i)$ gives the same degree of fuzziness, therefore corresponding to entropy (1.1) due to Shannon [11].De-Luca and Termini [4] suggested the following measure of fuzzy entropy.

$$H(A) = -\sum_{i=1}^{n} \left[\mu_A(x_i)\log\mu_A(x_i) + \left(1 - \mu_A(x_i)\right)\log\left(1 - \mu_A(x_i)\right)\right] \tag{1.2}$$

De-Luca and Termini [4] introduced a set of four properties and these properties are widely accepted as for defining a new fuzzy entropy. In fuzzy set theory, the entropy is a measure of fuzziness which expresses the amount of average ambiguity in making a decision whether an element belongs to a set or not. So, a measure of average fuzziness $H(A)$ in a fuzzy set A should have the following properties to be valid fuzzy entropy:

1. (Sharpness): $H(A)$ is minimum if and only if A is a crisp set, i.e., $\mu_A(x_i) = 0$ or 1; for all $x_i, i = 1, 2, \ldots, n$.
2. (Maximality): $H(A)$ is maximum if and only if A is most fuzzy set,i.e., $\mu_A(x_i) = \frac{1}{2}$; for all $x_i, i = 1, 2, \ldots, n$.
3. (Resolution): $H(A^*) \le H(A)$, where A^* is sharpened version of A.
4. (Symmetry): $H(A) = H(A^c)$, where A^c is the complement of A. i.e, $\mu_{A^c}(x_i) = 1 - \mu_A(x_i)$; for all $x_i, i = 1, 2, \ldots, n$.

The importance of fuzzy set comes from the fact that it can deal with imprecise and inexact information. Its application areas span from design of fuzzy controller to robotics and artificial intelligence.

BASIC CONCEPTS AND METHODS

If $x_1, x_2, \ldots, x_n$ are members of the universe of discourse, with respective membership functions $\mu_A(x_1)$, $\mu_A(x_2)$, $\mu_A(x_3), \ldots, \mu_A(x_n)$, then all $\mu_A(x_1), \mu_A(x_2), \mu_A(x_3), \ldots, \mu_A(x_n)$ lies between 0 and 1 but these are not probabilities because their sum is not unity. $\mu_A(x_i)$ gives the element x_i the degree of belongingness to the set "A". The function $\mu_A(x_i)$ associates with each $x_i \in R^n$ a grade of membership to the set "A" and is known as membership function.

The different elements x_i depends upon the experimenters goal or upon some qualitative characteristics of the physical system taken into account; ascribe to each element x_i a non-negative number (u_i>0) directly proportional to its importance and call u_i the utility of the element x_i.Then the weighted fuzzy entropy [1] of the fuzzy set "A" is defined as:

$$H(A,U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\} \tag{2.1}$$

Now let us suppose that the experimenter asserts that the membership function of the *i*th element is $\mu_B(x_i)$, where the true membership function is $\mu_A(x_i)$,thus we have two utility fuzzy information schemes:

$$F.S = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ \mu_A(x_1) & \mu_A(x_2) & \cdots & \mu_A(x_n) \\ u_1 & u_2 & \cdots & u_n \end{bmatrix}, 0 \le \mu_A(x_i) \le 1 \;\forall\; x_i, u_i > 0 \tag{2.2}$$

Of a set of n elements after an experiment, and

$$F.S^* = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ \mu_B(x_1) & \mu_B(x_2) & \cdots & \mu_B(x_n) \\ u_1 & u_2 & \cdots & u_n \end{bmatrix}, 0 \le \mu_B(x_i) \le 1 \;\forall\; x_i, u_i > 0 \tag{2.3}$$

of the same set of n elements before the experiment.

In both the schemes (2.2) and (2.3) the utility distribution is the same because we assume that the utility u_i of an element x_i is independent of its membership function $\mu_A(x_i)$, or predicted membership function $\mu_B(x_i)$, u_i is only a 'utility' or value of the element x_i for an observer relative to some specified goal (refer to [9]).
The quantitative-qualitative measure of fuzzy inaccuracy corresponding to Taneja and Tuteja measure of inaccuracy [13] with the above schemes is:

$$I(A;B;U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i)\log\mu_B(x_i) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\} \tag{2.4}$$

Guiasu and Picard [5] considered the problem of encoding the letter output by the source (2.2) by means of a single prefix code with code-words $c_1, c_2, \dots, c_n$ having lengths $l_1, l_2, \dots, l_n$ satisfying Kraft [7] inequality:

$$\sum_{i=1}^{n} D^{-l_i} \le 1 \tag{2.5}$$

where D being the size of the code alphabet. Corresponding to Guiasu and picard [5] useful mean code-word length we have the following useful fuzzy mean length of the code

$$L(A;U) = \frac{\sum_{i=1}^{n} u_i\{\mu_A(x_i)+(1-\mu_A(x_i))\}l_i}{\sum_{i=1}^{n} u_i\{\mu_A(x_i)+(1-\mu_A(x_i))\}} \tag{2.6}$$

and obtain bounds for it in terms of (2.4) under the condition:

$$\sum_{i=1}^{n}\{\mu_A(x_i)\mu_B^{-1}(x_i) + (1-\mu_A(x_i))(1-\mu_B(x_i))^{-1}\} D^{l_i} \le 1 \tag{2.7}$$

Where D is the size of code alphabet .Inequality (2.7) is generalized fuzzy Kraft's inequality. A code satisfying generalized fuzzy Kraft's inequality is known as a personal fuzzy code. It is easy to see that for $\mu_A(x_i) = \mu_B(x_i) \ \forall x_i, i = 1,2,3,\dots,n$ (1.9) reduces to Kraft [7] inequality.

In this paper generalized useful fuzzy code-word mean length are considered and bounds have been obtained in terms of generalized useful fuzzy inaccuracy measure of order α and type β .The results obtained here are not only new, but some fuzzy measures are the particular cases of our proposed measure that already exist in the literature.

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