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Fuzzy quantization of Bandlet coefficients for image compression

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ABSTRACT

In this paper, a quantization approach based on fuzzy membership functions is proposed which is applied for image compression. First the original image is decomposed to obtain bandlet coefficients. Then fuzzy quantization is applied to the bandlet coefficients. The proposed approach includes the characteristics of fuzzy sets; thus better compression with lesser loss of data is obtained. Experimental results show that more accurate reconstructed values of bandlet coefficients compared to uniform quantization can be obtained which improves the quality of the reconstructed image.

Keywords: fuzzy quantization, bandlet transform, compression.

INTRODUCTION

In recent years digital imaging techniques have led to a huge increase in the volume of images in various fields like medical imaging [1], remote sensing [2], photography etc. Image compression techniques play a vital role in efficient storage and transmission of images. For decades wavelets have been used in many image processing applications, in particular image compression. Because of their various properties such as multiresolution, localization and critical sampling wavelets are used in various image compression applications and image compression standards like Joint Photographic Experts Group 2000 (JPEG2000) [3].

Wavelets are very good in representing point singularities. In two dimension (2D) wavelets are obtained by a tensor product of one dimensional wavelets, so they are not able to efficiently represent singularities along lines or curves. This is the reason why they are not able to capture the geometrical structures in images. A number of image representations such as curvelets [4], wedgelets [5], beamlets [6], contourlets [7] and bandlets [8] were developed which take advantage of geometrical regularity of image structures which is very useful for image compression.

Quantization is the process of representing a large set of values with a much smaller set. It is one of the important steps in compression and has a significant impact on the compression ratio and on the loss incurred during lossy compression. If quantization is performed on scalar values, the process is called scalar quantization. In uniform quantizer [9] the intervals are the same size except possibly for two outer intervals. The reconstruction values are the midpoints of the intervals. In nonuniform quantizers the position of the reconstruction value is chosen to minimize the total absolute errors within each decision region. This can be done by making the quantization intervals smaller in the regions where the input has high probability distribution. One of the most widely used nonuniform quantizer is Llyod-Max quantizer [10]. Various vector quantization techniques for image compression have also been proposed [11, 12].

The Fuzzy set theory developed by Zadeh [13] has a very interesting feature of partial membership where one object can belong to more than one partition or cluster with various degrees of partition. Several variations of fuzzy clustering techniques have been developed for image compression [14, 15, 16, 17, 18]. In this paper we propose a fuzzy scalar quantization technique based on fuzzy membership functions to ensure minimum distortion. The fuzzy quantization is used to quantize bandlet coefficients.

Review of Bandlet Transform

First bandlet bases were developed by Le Pennec [19]. Later works built on these bandlet bases have made use of the multiscale geometry defined over the coefficients of a wavelet basis [20]. Bandlet decomposition [21, 22, 23, 20] is computed with a geometric orthogonal transform that is applied on orthogonal wavelet coefficients. Wavelet transform, when applied to an image of N pixels, computes the set of N dot products

$$< f, \psi_{jn}^{s} > \quad for\ 2^{-J} \leq 2^{-j} < \sqrt{N}\ and\ 0 \leq n_1, n_2 < 2^{-j}, < f, \phi_{Jn}^{s} > \quad for\ 0 \leq n_1, n_2 < 2^{-J} \text{ --------------- } 1$$

where the projection on ϕ_{Jn}^{s} functions produces a coarse approximation at scale 2^J. The scale 2^J represents the level at which we stop the wavelet transform. These values can be conveniently stored in an array of N pixels. A dyadic square is a square obtained by recursively splitting the original wavelet transformed image f_j^s into four sub-squares of equal size. Let the width of the squares be L pixels with$4 \leq L \leq 2^{-j/2}$. For each dyadic square S at a given scale 2^j and orientation s of the wavelet transform 1D reordering of the grid points is performed. The possible number of 1D reordering may be equal to the number of directions d joining pairs of points in square S of width L . 1D reordering is done by projecting the sampling location along d and sorting the resulting 1D points from left to right. To the resulting 1D discrete signal, f_d, 1D wavelet discrete wavelet transform is performed. For a given threshold T , the direction d , which generated the less approximation error, is selected. Let b_k denote the coefficients of 1D wavelet transform of f_d, and R_B be the number of bits needed to code the quantized coefficients $QT(b_k)$. To select the best geometry, the direction d that minimizes the Lagrangian

$$\xi(f_d, R) = \|f_d - f_{dR}\|^2 - \lambda T_2(R_G - R_B) \qquad \text{--------------------------------2}$$

where f_{dR} is the signal recovered from the quantized coefficients and R_G is the number of bits needed to code the geometric parameter d with an entropy coder. λ is taken as $3/28$ [8].

Rest of this paper is organized as follows. Section 2 proposes a fuzzy scalar quantization technique and the image compression scheme which uses bandlet transform and the proposed fuzzy scalar quantization scheme. The proposed method is verified through experiments in section 3. Section 4 is the conclusion.

MATERIALS AND METHODS

Proposed Compression Scheme

Fuzzy Quantization: Quantization is very important for efficiently encoding transform coefficients and for improving the performance of the encoder. The design of the quantizer affects the compression ratio and the information loss which occurs during compression. The process of representing a larger set of values with a smaller one is known as quantization. For any given input the range of values is divided into a number of intervals. Each interval is represented by a distinct codeword. All the inputs that fall in an interval is represented by the codeword representing that interval. While reconstructing, the best possible value in the interval is made as the reconstructed value. Since quantization is the step where most of the compression is performed and loss of information occurs, it is one of the important steps in a compression technique.

Let T be the threshold value described in the previous section. Let b_k represent the absolute values of bandlet coefficients rounded off to nearest integer. Let the partition be represented by $\{T, T+1, T+2, \dots \max(b_k + 1)\}$. Let $\{A_i\}$ for $i = 1, 2, \dots n$ be a set membership functions defined in the interval from 0 to $\max(b_k)$ as shown in figure 1. Let x, the input to the fuzzy quantizer, be absolute value of bandlet coefficient.

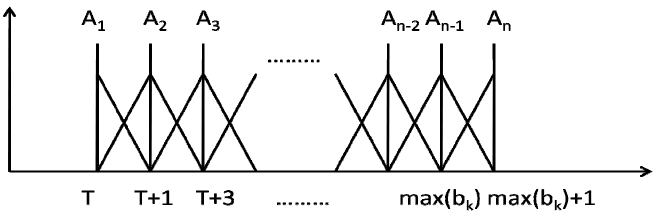

Figure 1. Membership functions defined over an interval of Bandlet coefficients

The first triangular membership function, A_1, has the parameters [0 T T]. The last membership function,A_n, has the parameters [$\max(b_k)$ $\max(b_k)+1$ $\max(b_k)+1$]. All the other membership functions are defined by

$$A_i(x) = \begin{cases} \frac{x-v_{i-1}}{v_i-v_{i-1}} & if\ x \in [v_{i-1}, v_i] \\ \frac{x-v_{i+1}}{v_i-v_{i+1}} & if\ x \in [v_i, v_{i+1}] \end{cases} \quad \text{---3}$$

where $T+1 \leq v_i \leq \max(b_k)$.

There are two possible cases for any given input value x:

Case 1: $abs(x) < T$

This means $0 \leq abs(x) < T$. The absolute value of the input lies in the first partition. In this case none of the membership functions get activated. The quantized value is set as 0.

Case 2: $T \leq abs(x) \leq \max(b_k)+1$

In this case for any given input x, only two neighboring membership functions become activated, that is $A_i(x) = 0, \ldots, A_{i-1}(x) = 0, A_i(x) > 0, A_{i+1}(x) > 0, A_{i+2}(x) = 0, \ldots, A_n(x) = 0$. Let x be any value in the interval $[v_i, v_{i+1}]$, two membership functions, A_i and A_{i+1}, have the membership degrees greater than 0. The quantized value is obtained by using the weighted average method. The quantized value is given by

$$\hat{x} = \frac{v_i * A_i + v_{i+1} * A_{i+1}}{A_i + A_{i+1}} \quad \text{----------------------------------4}$$

Image Compression using Fuzzy Quantization of Bandlet Coefficients

The two dimensional (2D) image,$I(x,y)$, is first decomposed usng bandlet transform to obtain the bandlet coefficients. Let the coefficients of 2D discrete bandelet transform of image $I(x,y)$ be represented as $F(x,y)$. Orthogonal bandelets use an adaptive segmentation and a local geometric flow and is thus able to capture the anisotropic regularity of edge structures.

Then fuzzy bandlet quantization of the bandlet coefficients is performed as discussed in the previous section. For each of the bandlet coefficient it's absolute value is taken. Let the bandlet coefficient be x and its absolute value be $abs(x)$. If $abs(x)$ is less than T (case 1) the quantized value, $\hat{x}$, is set as 0. If $T \leq abs(x) \leq \max(b_k)+1$ (case 2), two membership functions, A_i and A_{i+1}, have the membership degrees greater than 0. The quantized value is given by equation 4. In this case if the bandlet coefficient is positive, the quantized value is $\hat{x}$. If the bandlet coefficient is negative, the quantized value is $-\hat{x}$. The block diagram of the proposed scheme is given in figure 2.

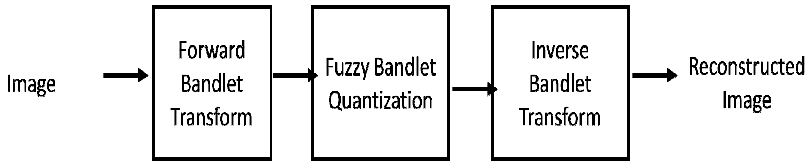

Figure 2. Block diagram of the proposed compression scheme.

RESULTS AND DISCUSSION

We have used four 2D images of dimension 512*512, where each pixel is of eight bits. The bandlet transform is based on the bandlet toolbox available at [24]. We have compared the performance of the proposed compression scheme with scalar uniform quantization of the bandlet coefficients. In this method the quantized value is obtained using

$$\hat{x} = \begin{cases} 0 & if\ |x| \leq T \\ sign(x)\left(1 + \frac{1}{2}\right)T & if\ qT \leq |x| < (q+1)T \end{cases} \qquad \text{---------------------------5}$$

where T is the threshold value, q is $floor(x/T)$ and x is a bandlet coefficient.

The results are given in table 1. The results for lena, barbara, fingerprint and boat images are shown in figures 3, 4, 5 and 6 respectively. It can be interpreted that the proposed fuzzy quantization of bandlet coefficients is better than scalar uniform quantization of bandlet coefficients for various bit rates in terms of Mean-Squared-Error (MSE) and Peak-Signal-to-Noise Ratio (PSNR).

Table 1. Bits-Per-Pixel, MSE and PSNR for Images using Bandlet Transform and Fuzzy Quantization

Image	Threshold (T)	Bits-per-pixel		Bandlet Transform + Scalar Quantization	Bandlet Transform + Fuzzy Quantization
Lena	0.5	5.49	MSE	0.03	0.01
			PSNR	63.87	68.77
	1.0	4.40	MSE	0.12	0.06
			PSNR	57.19	60.21
	3.0	2.57	MSE	1.39	1.05
			PSNR	46.71	47.91
Barbara	0.5	6.03	MSE	0.03	0.01
			PSNR	63.89	68.92
	1.0	4.93	MSE	0.12	0.06
			PSNR	57.25	60.44
	3.0	3.12	MSE	1.29	0.92
			PSNR	47.02	48.48
Fingerprint	1.0	5.58	MSE	0.11	0.03
			PSNR	57.90	63.01
	3.0	3.83	MSE	1.19	0.67
			PSNR	47.39	49.84
	7.0	2.47	MSE	6.92	5.04
			PSNR	39.73	41.11
Boat	1.0	4.66	MSE	0.13	0.06
			PSNR	57.12	60.05
	3.0	2.84	MSE	1.30	0.96
			PSNR	46.99	48.33
	7.0	1.64	MSE	5.41	4.40
			PSNR	40.80	41.70

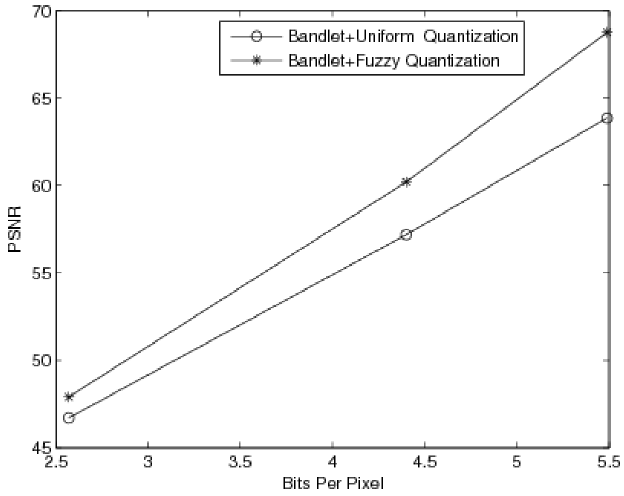

Figure 3. Bits-per-pixel and PSNR for lena image

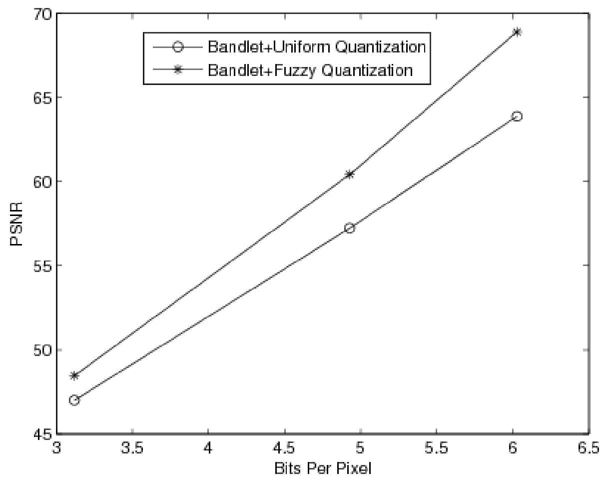

Figure 4. Bits-per-pixel and PSNR for barbara image

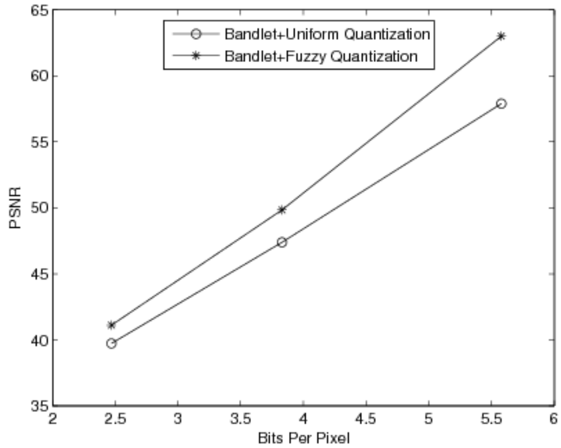

Figure 5. Bits-per-pixel and PSNR for fingerprint image

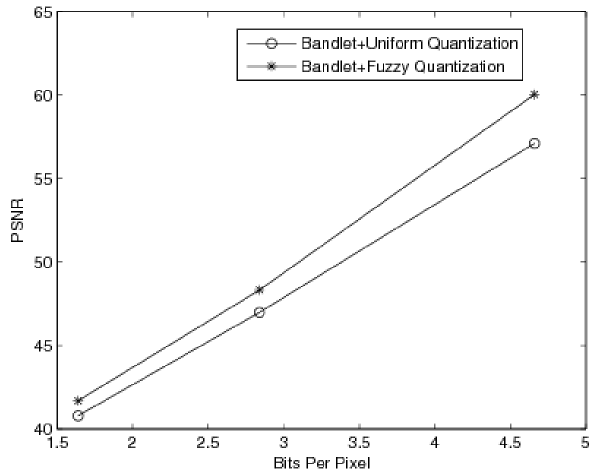

Figure 6. Bits-per-pixel and PSNR for boat image

CONCLUSION

In this paper a fuzzy quantization technique for image compression has been proposed. It has been used to quantize bandlet coefficients. The method is able to more accurately quantize bandlet coefficients and thus improves the quality of the reconstructed image compared to uniform scalar quantization for various images.

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