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Flow of a Jeffrey fluid through a tapered tube with permeable walls

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ABSTRACT

Flow of a Jeffrey fluid through a tapered tube with permeable walls is studied. Beavers – Joseph slip conditions are used to solve the governing equations. The expressions for velocity, volume flow rate and frictional forces are obtained. The effects of various parameters like Darcy number, slip parameter, radius on velocity, frictional force are discussed through graphs.

Keywords: Jeffrey fluid, tapered tube, permeable walls.

INTRODUCTION

Theoretical and experimental studies of the circulatory disorders are the subjects of scientific research, since the investigation of Mann et.al [1]. According to the discovery the cardiovascular disease, such as stenosis or arteroscierosis, is closely associated with the flow conditions in the blood vessels. Scientists have been focusing on this area of Biomechanics. Stenosis medically means narrowing of any body passage (tube). Consequently the flow behavior in the stenosed artery is quite different from the case of the normal artery. Having knowledge on flow parameters in the stenosed artery, such as the velocity pattern, the flow rate and the stresses will help bio-medical engineers in developing bio-medical instruments for treatment (surgical) modalities. Several authors have considered various mathematical models for flows through stenosed/constricted ducts (Young [2], Lee and Fung [3], Shukla et al. [4], Chaturani and Samy [5]. In all these mathematical studies, blood has been characterized as a Newtonian fluid. But Majhi and Nair [6] suggested that blood behaves like a non-Newtonian fluid under certain conditions. Various non-Newtonian fluids like Herschel-Bulkley fluid, Casson fluid, Bingham fluid, Power-law fluid etc are considered by many researchers. Chakravarthhy and Mandal [7],Mandal [8] studied Newtonian and non-Newtonian blood flow through tapered arteries with stenosis.

In view of this, we proposed to study the flow of Jeffrey fluid through a tapered tube with permeable walls. The taper of the tube will be an important factor in pressure development. The steady flow of non-Newtonian fluids is characterized by an arbitrary time independent flow curve through a slightly tapered tube. The velocity field is obtained using Beavers and Joseph [9] slip condition at the permeable wall. The flow of fluid is governed by Darcy's law. Some results are deduced and discussed. The velocity, the mass flow rate and its fractional increase are obtained.

Mathematical Formulation

Consider the flow of a Jeffrey fluid through a tapered tube of length L with permeable walls. The tube has a radius R_0 at the entrance and radius R_L at the exit. The tube radius at any distance Z from the inlet is given by

$$R(Z) = R_0 + \left(\frac{R_0 - R_L}{L}\right) Z \qquad (1)$$

The flow takes place due to pressure gradient. The porous medium is homogeneous with permeability K. The flow in the porous medium is governed by the Darcy's law.

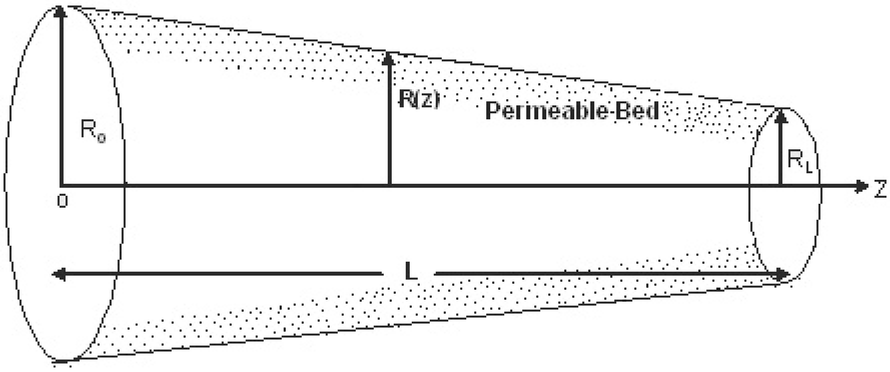

Fig. 1 Physical Model

The flow is axi-symmetric. Cylindrical polar coordinate system is used. The following assumptions are made in deriving the basic equations.

a) The flow is steady and incompressible.
b) The flow is in axial direction.
c) All physical quantities except the pressure are functions of r only.
d) The body forces are negligible.

In view of the above assumptions, the basic equation and boundary conditions of the flow take the following form. The governing equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w}{\partial r}\right) = -\frac{\partial p}{\partial z} \quad (2)$$

Boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (3)$$

$$w = -\frac{\sqrt{Da}}{\alpha}\frac{\partial w}{\partial r} - 1 \quad \text{at} \quad r = R \quad \text{(Beavers and Joseph condition [9])} \quad (4)$$

where Da is the Darcy number and α is the slip parameter

3.Solution of the Problem

Solving equation (2) along with boundary conditions (3) and (4), we obtain the velocity field as

$$w = P\left(\frac{1+\lambda_1}{4}\right)\left(R^2 - r^2\right) + \frac{\sqrt{Da}}{\alpha}(1+\lambda_1)\frac{PR}{2} - 1 \quad (5)$$

where $P = \frac{\partial p}{\partial z}$

The Volume rate of flow for the flow of a Jeffrey fluid through a tapered tube is

$$Q = \int_0^{2\pi}\int_0^{R} wr\,dr\,d\theta \quad (6)$$

$$Q = \left(\frac{(1+\lambda_1)PR^4}{16} + \frac{\sqrt{Da}}{4\alpha}(1+\lambda_1)PR^3 - \frac{R^2}{2} \right) 2\pi \qquad (7)$$

The fractional increase in the volume rate of flow of the Jeffrey fluid through a tapered tube is given by

$$F = \frac{Q - Q_0}{Q_0} = \frac{4PR\sqrt{Da}}{\alpha\left[(1+\lambda_1)PR^2 - 8\right]}(1+\lambda_1) \qquad (8)$$

RESULTS AND DISCUSSION

The velocity profiles are shown in the figures (2) – (10) for different values of Jeffrey parameter λ_1, Darcy number Da, pressure gradient P, slip parameter α and radius R.

We studied the effect of Jeffrey parameter λ_1 on velocity w, as a function of r with $Da = 0.9,\ P = 0.8,\ R = 0.6,\ \alpha = 0.05$ and is shown in figure (2). It is noticed that for fixed λ_1, velocity decreases with increase in r. Also observed that for given r, the increase in λ_1 increases the flow velocity w.

We studied the effect of slip parameter α on velocity w, as a function of r for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $Da = 0.9,\ R = 0.6,\ P = 0.8,$ and is shown in figures (3) and (4). It is noticed that the velocity decreases with increase in r for fixed value of α. Also observed that the velocity decreases as the slip parameter α increases for fixed r. And it is observed that for a given r and α the velocity increases as λ_1 increases, that is the velocity increases as the non-Newtonian behavior of the fluid increases.

From equation (5), we studied the effect of tube radius R on velocity w, as a function of r for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $Da = 0.9,\ P = 0.9,\ \alpha = 0.1$ and is shown in figures is (5) and (6). It is observed that for fixed R, the velocity decreases with increase in r. Also it is noticed that the velocity increases with increase in R for fixed r. And it is observed that for a given r and R the velocity increases as λ_1 increases, that is the velocity increases as the non-Newtonian behavior of the fluid increases.

We studied the effect of pressure gradient P on velocity w, as a function of r for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $Da = 0.9,\ \alpha = 0.05,\ R = 0.5$ and is shown in figures (7) and (8). It is noticed that velocity decreases with increase in r for fixed value of P. Also observed that for fixed r, the increase in P increases the flow velocity w. And it is noticed that for a given r and P the velocity increases as λ_1 increases, that is the velocity increases as the non-Newtonian behavior of the fluid increases.

From equation (5), we studied the effect of Darcy number Da on velocity w, as a function of r for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $R = 0.4,\ P = 0.8,\ \alpha = 0.05$ and is shown in figures (9) and (10). It is observed that for fixed R, the velocity decreases with increase in r. Also it noticed that the velocity w, increases with increase in Da for fixed r. And it is observed that for a given r and Da the velocity increases as λ_1 increases, that is the velocity increases as the non-Newtonian behavior of the fluid increases.

The Volume rate flow profiles are shown in figure (11) – (17) for different values of Jeffrey parameter λ_1, Darcy number Da, pressure gradient P and slip parameter α.

We studied the effect of Jeffrey parameter λ_1 on volume flow rate Q, as a function of R with $Da = 0.5,\ P = 0.8,\ \alpha = 0.05$ and is shown in figure (11). It is noticed that for fixed λ_1, volume flow rate Q, increases with increase in R. And also observed that for given R, the increase in λ_1 increases the volume flow rate Q.

We studied the effect of slip parameter α on volume flow rate Q, as a function of R for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $Da = 0.5$, $P = 0.5$ and is shown in figures (12) and (13). It is noticed that the volume flow rate Q, increases with increase in R for fixed value of α. Also observed that for given R, the increase in α decreases the flow rate Q. And it is observed that for a given R and α, the flow rate increases as λ_1 increases, that is the volume flow rate Q increases as the non-Newtonian behavior of the fluid increases.

We studied the effect of pressure gradient P on the volume flow rate Q, as a function of R for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $Da = 0.5$, $\alpha = 0.07$ and is shown in figures (14) and (15). It is noticed that volume flow rate increases with increase in R for fixed value of P. Also observed that for given R, the increase in P increases the volume flow rate Q. And also it is observed that for a given R and P, the volume flow rate increases as λ_1 increases, that is the volume flow rate Q increases as the non-Newtonian behavior of the fluid increases.

From equation (7), we calculated the effect of Darcy number Da on volume flow rate Q, as a function of R for Newtonian fluid ($\lambda_1 = 0$) and non-Newtonian fluid ($\lambda_1 = 0.2$) with $P = 0.5$, $\alpha = 0.07$ and is shown in figures (16) and (17). It is observed that for fixed Da, the increase in R increases the volume flow rate Q. Also it noticed that the volume flow rate increases with increase in Da for fixed R. And also it is observed that for a given R and Da, the volume flow rate increases as λ_1 increases, that is the volume flow rate Q increases as the non-Newtonian behavior of the fluid increases.

The fractional increase in volume rate of flow is evaluated from equation (8) and is depicted in figures (18) and (19). It is observed that for fixed values of P, α, R and λ_1, increase in Darcy number Da increases the fractional increase in volume flow rate, which is shown in

Fig (18), whereas for fixed values of Da, P, R and λ_1, increase in slip parameter α decreases the fractional increase in volume flow rate, which is shown in figure (19).

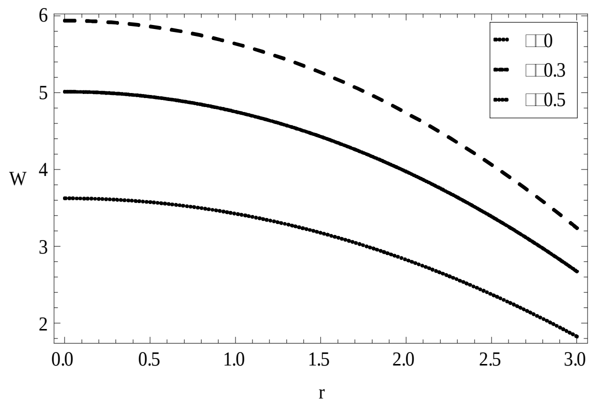

Figure (2). Variation of velocity w with radius r for different values of λ_1

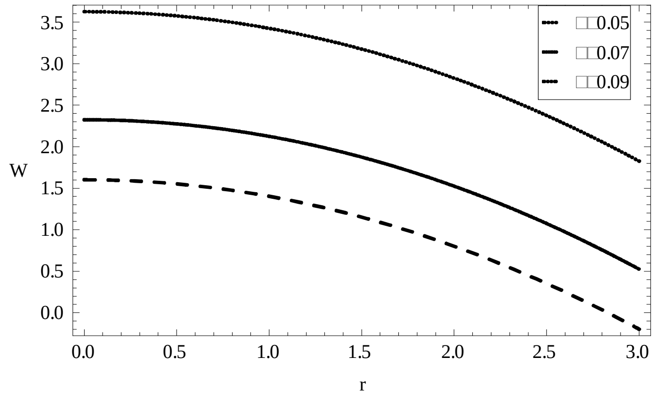

Figure (3). Variation of velocity w with radius r for different values of α with $\lambda_1 = 0$

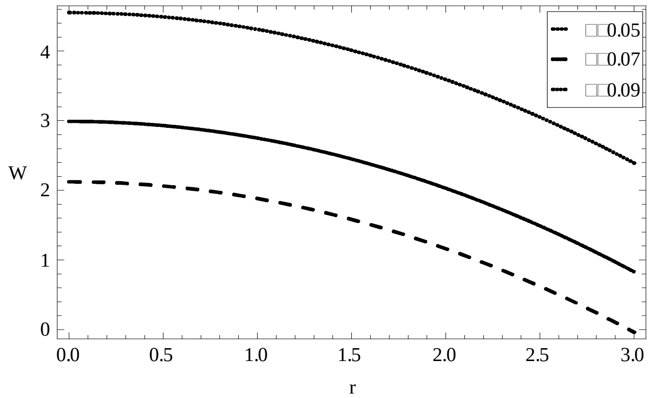

Figure (4). Variation of velocity w with radius r for different values of α with $\lambda_1 = 0.2$

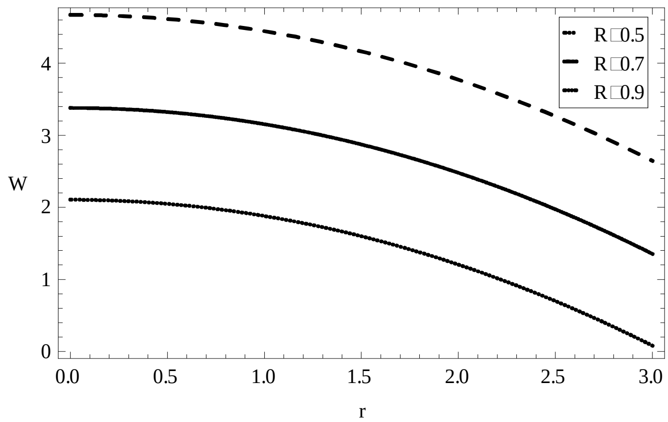

Figure (5). Variation of velocity w with radius r for different values of R with $\lambda_1 = 0$

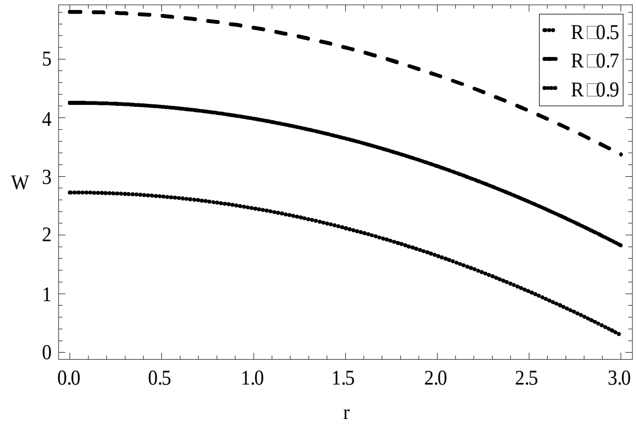

Figure (6). Variation of velocity w with radius r for different values of R with $\lambda_1 = 0.2$

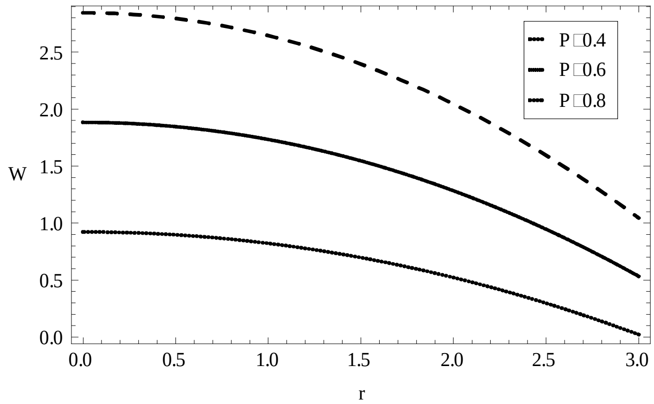

Figure (7). Variation of velocity w with radius r for different values of P with $\lambda_1 = 0$

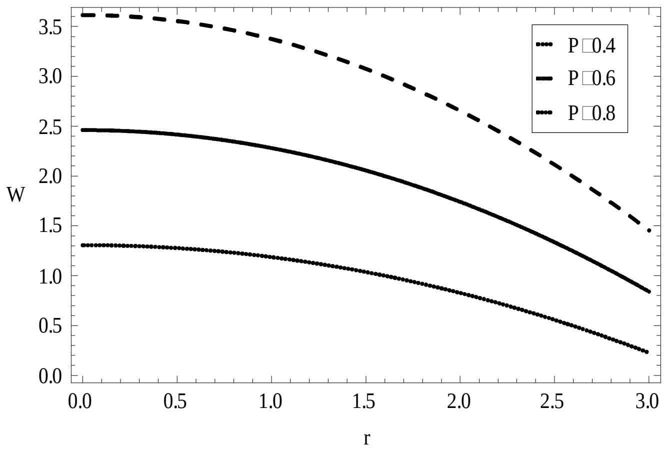

Figure (8). Variation of velocity w with radius r for different values of P with $\lambda_1 = 0.2$

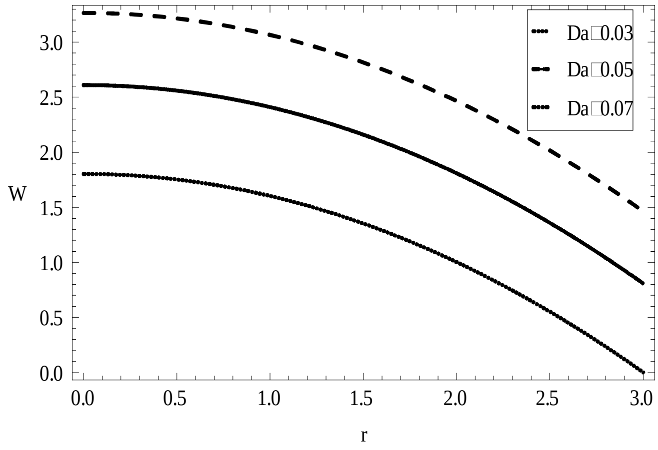

Figure (9). Variation of velocity w with radius r for different values of Da with $\lambda_1 = 0$

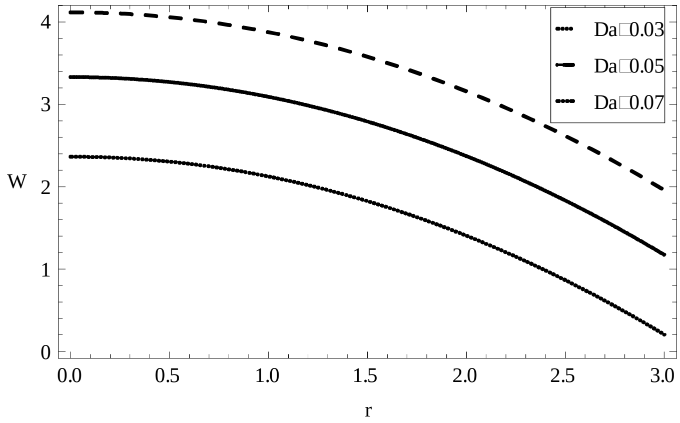

Figure (10). Variation of velocity w with radius r for different values of Da with $\lambda_1 = 0.2$

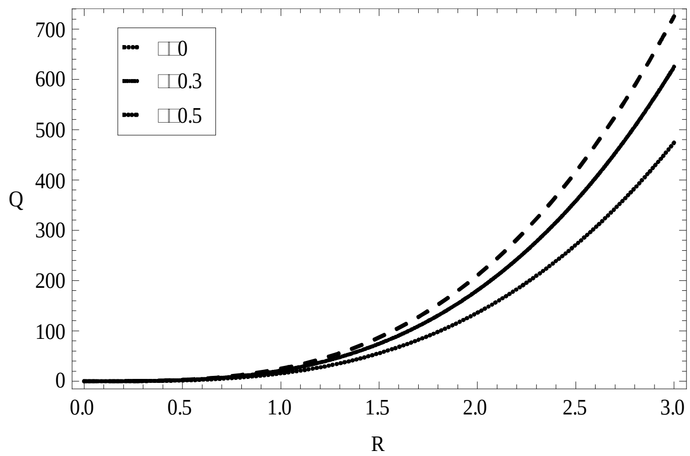

Figure (11). Variation of volume rate flow Q with radius R for different values of λ_1

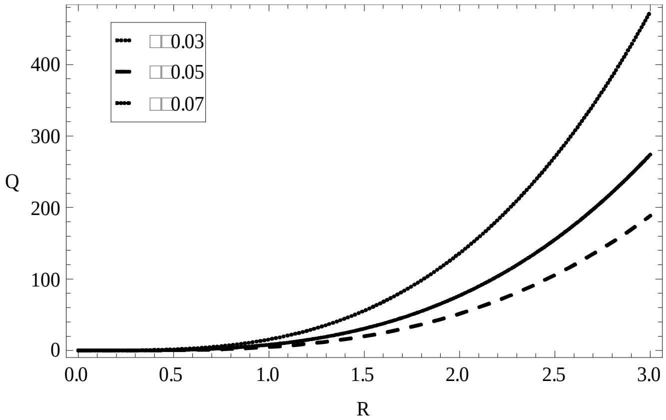

Figure (12). Variation of volume rate flow Q **with radius** R **for different values of** α **with** $\lambda_1 = 0$

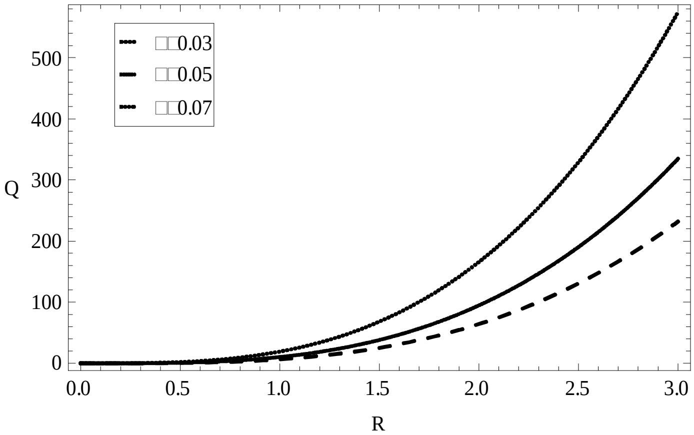

Figure (13). Variation of volume rate flow Q **with radius** R **for different values of** α **with** $\lambda_1 = 0.2$

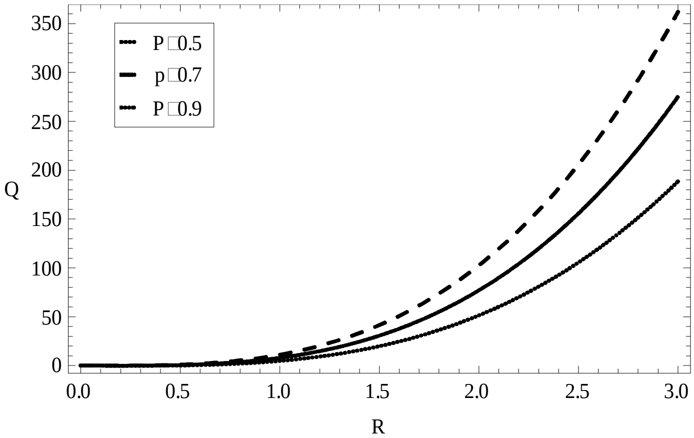

Figure (14). Variation of volume rate flow Q **with radius** R **for different values of** P **with** $\lambda_1 = 0$

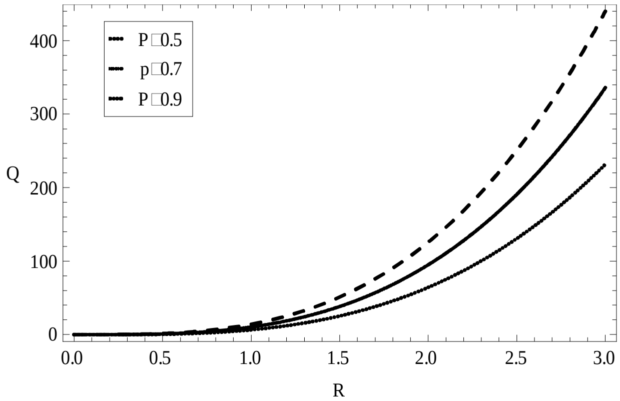

Figure (15). Variation of volume rate flow Q with radius R for different values of P with $\lambda_1 = 0.2$

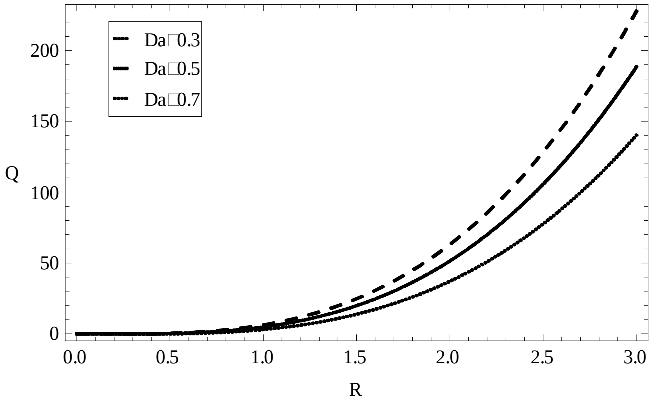

Figure (16). Variation of volume rate flow Q with radius R for different values of Da with $\lambda_1 = 0$

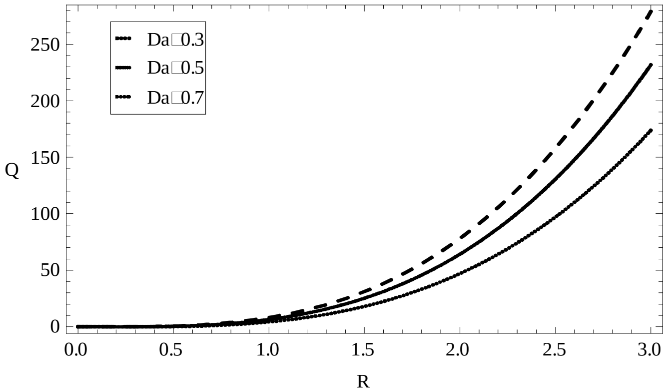

Figure (17). Variation of volume rate flow Q with radius R for different values of Da with $\lambda_1 = 0.2$

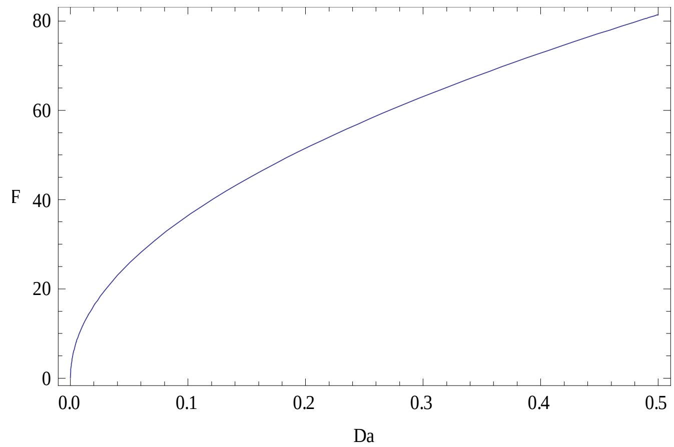

Figure (18). Variation of fractional increase in volume rate of flow with Darcy number Da

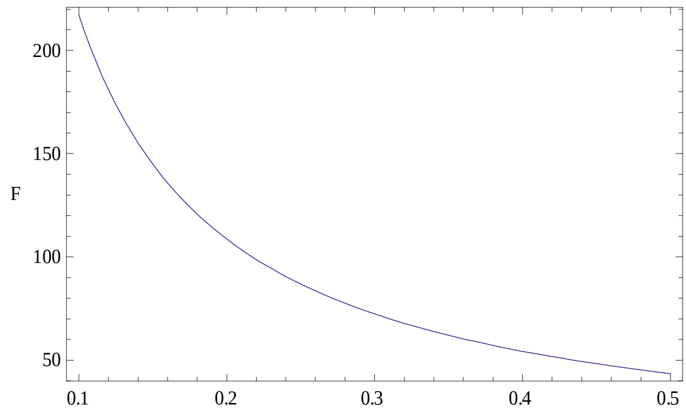

Figure (19). Variation of fractional increase in volume rate of flow with slip parameter α

CONCLUSION

1. It is observed that for a fixed value of λ_1, the velocity W increases as the parameters R, P and Da increases, whereas for the slip parameter α, the velocity W decreases as α increases.
2. Also it is observed that for a fixed value of λ_1, the volume rate flow Q increases as the parameters P and Da increases, whereas the opposite behavior is observed for the varying slip parameter α.

REFERENCES

[1]. Mann, F.G., Herrick, J.F., Essex, H., Blades, E.J., *Surgery*, **1938**, **4**, 249–252
[2]. D.F. Young, *J. Engrg. Ind.*, *Trans ASME*, **1968**, **90**, 248-254.
[3]. J.S. Lee, Y.C. Fung, *J. Appl. Mech., Trans ASME*, **1970**, **37**, 9-16.
[4]. J.B. Shukla, R.S. Parihar, B.R.P. Rao, *Bull. Math. Biol.*, **1980**, **42**, 283-294
[5]. P. Chaturani, R. Ponnalagar Samy, *Biorheol*., **1986**, **23**, 499-511
[6]. S.N. Majhi, V.R. Nair, *Int. J. Engg. Sci*., **1996**, **32**, 5, 839-846
[7]. Chakravarty, S., Mandal, P.K., *Int. J. Nonlin. Mech*. **2000**, **35**, 779–793
[8]. Mandal, P.K., *Int. J. Nonlin. Mech*. **2005**, **40**, 151–164
[9]. Beavers, G.S and Joseph, D.D., J. *Fluid Mech.***1967**,**30**, 197-207.