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Flow of a Casson Fluid Through an Inclined Tube of Non-uniform Cross Section with Multiple Stenoses

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Abstract

A mathematical model is developed to study the steady flow of Casson fluid through an inclined tube of non-uniform cross section with multiple stenoses. Using appropriate boundary conditions, analytical expressions for the velocity, the volumetric flow rate and the flow resistance have been derived. These expressions are computed numerically and the computational results are presented graphically.

Key words: Casson model, multiple stenoses.

INTRODUCTION

The study of blood flow through arteries is of prime importance because it provides an insight into physiological situations. The blood can be viewed as a Newtonian or a non-Newtonian fluid, as the case may be. Under diseased conditions, abnormal and unnatural growth develops in the lumen at various locations of the cardiovascular system. As the growth projects into lumen of the artery, blood flow will be restricted. So there is a coupling between growth of stenosis and the flow of blood in the artery. Hence the detailed knowledge of the flow field in a stenosed tube may help in proper understanding and prevention of arterial diseases. In small vessels, blood exhibits shear-dependent viscosity and requires a finite yield stress before flow can commence, thereby making the non-Newtonian nature of blood an important factor in the modeling. This is confirmed with the experiments of Scott Blair et al. [1] and Bugliarello and Sevilla [2] on the flow of blood in the human body. Based on this fact, Kapur[3] discussed several non-Newtonian models for blood flow in normal and stenosed arteries. Brasseur et al.[4], Usha and Ramachandra Rao[5] and Srinivasacharya et al.[6] modeled biofluid flows through tubes and channels considering the mechanism of peristalsis. One of yield stress models, Viz., the Casson model, assumes the existence of the finite yield stress before flow is possible, leading to the plug flow and introduces a shear dependent viscosity consistent with the results of experiments with human blood. Krishna kumara et al.[7] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field. Ravikumar et al.[8] analyzed the peristaltic transport of a power-law fluid in an asymmetric channel bounded by permeable walls. Kavitha et al.[9] investigated peristaltic flow of a micropolar fluid in a vertical channel with long wave length approximation. Vajravelu et al. [10] studied the peristaltic transport of Herschel-Bulkley fluid through an inclined tube.

To understand the effects of stenosis in an artery many researchers (Forrester and Young[11], Misra and Chakravarthy[12]) investigated the flow of blood through stenosed arteries by treating blood as a Newtonian fluid. Misra and Ghosh [13] formulated and analyzed a model of blood flow in branched arteries.

Ponalagusamy[14] analyzed the blood flow through an artery with mild stenosis , a two-layered model, different shapes of stenoses and slip velocity at the wall. The experimental studies showed that in the vicinity of stenosis, the shear rate of blood is low and therefore blood behaves as a non-Newtonian fluid. Shankar and Hemalatha [15] studied a mathematical model on pulsatile flow of Herschel-Bulkley fluid through stenosed arteries. A theoretical study on the blood flow through an arteries in pathological state is carried out by Misra and Shit[16]. A numerical model for the effect of stenosis shape on blood flow through an artery using power-law fluid was carried out by Sapna Singh and Rajeev Ratan Shah [17].

Maruthi Prasad and Radhakrishnamacharya[18] discussed the blood flow through an artery having multiple stenosis with non-uniform cross-section considering blood as a Herschel-Bulkley fluid. It is known that many ducts in physiological systems are not horizontal but have some inclination to the axis. A mathematical analysis was carried out by Misra and Ghosh [19] with an aim to study the velocity field for the pulsatile flow of blood in a porous elastic vessel of variable cross-section.

The aim of the present investigation is to study the effect of multiple stenoses on the flow of blood. The blood which contains erythrocytes is represented by a Casson fluid depicting the non-Newtonian behavior of the blood. The derived analytical expressions are computed in order to examine the variation of velocity profiles, the volumetric flow rate and the resistance to the blood flow.

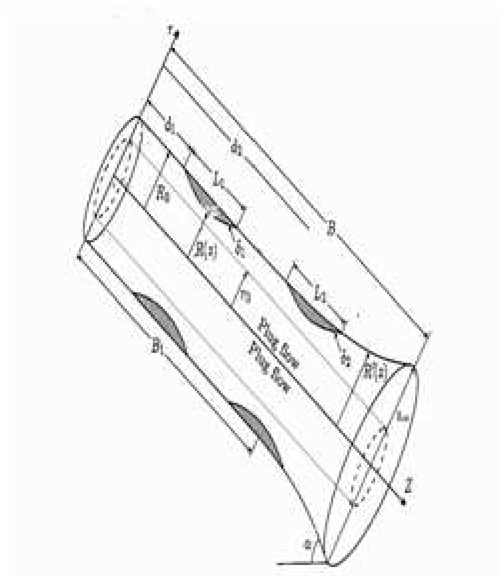

Fig 1 : Physical Model

2. Mathematical formulation

We consider the steady flow of Casson fluid through a tube of non-uniform cross section and with two stenosis. Cylindrical polar co-ordinate system (r, θ, z) is choosen so that the z-axis coincides with the centerline of the channel. It is assumed that the tube is inclined at an angle α to the horizontal direction. The stenoses are supposed to be mild and develop in an axially-symmetric manner. The radius of the tube is taken as

h(z) = R(z)

$$R(z) = \begin{cases} R_0 & 0 \le z \le d_1 \\ R_0 - \dfrac{\delta_1}{2}\left[1 + \cos\dfrac{2\pi}{L_1}\left(z - d_1 - \dfrac{L_1}{2}\right)\right] & d_1 \le z \le d_1 + L_1 \\ R_0 & d_1 + L_1 \le z \le B_1 - \dfrac{L_2}{2} \\ R_0 - \dfrac{\delta_2}{2}\left[1 + \cos\dfrac{2\pi}{L_2}(z - B_1)\right] & B_1 - \dfrac{L_2}{2} \le z \le B_1 \\ R^*(z) - \dfrac{\delta_2}{2}\left[1 + \cos\dfrac{2\pi}{L_2}(z - B_1)\right] & B_1 \le z \le B_1 + \dfrac{L_2}{2} \\ R^*(z) & B_1 + \dfrac{L_2}{2} \le z \le B \end{cases} \quad (1)$$

Here L_i *and* δ_i (i=1,2) lengths and maximum thickness of two stenosis (the suffixes 1 and 2 here indicate the first and second stenosis respectively) and are such that the restrictions for the mild stenosis [Maruthi Prasad and Radhakrishnamacharya[18]] are satisfied.

$$\delta_i \ll \min(R_0, R_{out})$$

$\delta_i \ll L_i \quad ,(i = 1,2)$

where $R_{out} = R(z) \quad at \qquad z = B$

The basic momentum equation governing the fluid is

$$\frac{1}{r}\frac{\partial(r\tau)}{\partial r} + \frac{\partial p}{\partial z} = \rho g \sin\alpha \qquad (2)$$

where τ is the shear stress for the Casson fluid which is given by

$$\tau^{\frac{1}{2}} = \mu^{\frac{1}{2}}\left(-\frac{\partial w}{\partial r}\right)^{\frac{1}{2}} + \tau_0^{\frac{1}{2}} \qquad (3)$$

Here w is the axial velocity, p is the pressure, τ_0 is the yield stress, μ is the fluid viscosity, ρ is the density, g is the acceleration due to gravity, R_o is the radius of the tube.

The boundary conditions are

τ is finite at $r = 0$

$w = 0$ at $r = h(z)$ (4)

Introducing the following non-dimensional variables

$$\bar{r}=\frac{r}{R_0},\bar{\tau}=\frac{\tau}{\mu\left(\frac{U}{R_0}\right)},\bar{\tau}_0=\frac{\tau_0}{\mu\left(\frac{U}{R_0}\right)},\bar{w}=\frac{w}{U},\bar{p}=\frac{pR_0{}^2}{\mu UB},\bar{z}=\frac{z}{B},\overline{d_1}=\frac{d_1}{B},\overline{L_1}=\frac{L_1}{B},$$

$$\overline{L_2}=\frac{L_2}{B}\quad \bar{B}=\frac{B_1}{B},\qquad \overline{R(z)}=\frac{R(z)}{R_0},\quad \bar{Q}=\frac{Q}{\pi R_0{}^2U}\qquad (5)$$

The non-dimensionalised governing equations after dropping bars

$$\frac{1}{r}\frac{\partial(r\tau)}{\partial r}=-\frac{\partial p}{\partial z}+\frac{\sin\alpha}{F}\tag{6}$$

$$\tau^{\frac{1}{2}}=\left(-\frac{\partial w}{\partial r}\right)^{\frac{1}{2}}+\tau_0{}^{\frac{1}{2}}\tag{7}$$

The non dimensional boundary conditions are

τ is finite at $r=0$
$w=0$ at $r=h(z)$ (8)

3. Solution of the problem
(i) Velocity Distribution

Solving equation (6) and using the condition (8) the axial velocity can be obtained as

$$w=\left(\frac{P+f}{2}\right)\left[\frac{h^2}{2}-\frac{r^2}{2}+\frac{2}{3}r_0{}^{\frac{1}{2}}\left(r^{\frac{3}{2}}-h^{\frac{3}{2}}\right)+r_0(h-r)\right]\tag{9}$$

and the plug flow velocity is

$$w_p=\left(\frac{P+f}{2}\right)\left[\frac{h^2}{2}-\frac{5}{6}r_0{}^2+\frac{2}{3}r_0{}^{\frac{1}{2}}h^{\frac{1}{2}}+r_0h\right]\tag{10}$$

where $P=-\frac{\partial p}{\partial z}$ and $f=\frac{\sin\alpha}{F}$

The volumetric flow rate is obtained as

$$Q=(P+f)F_1\tag{11}$$

where

$$F_1=\left[\frac{r_0{}^3}{6}-\frac{2}{3}r_0{}^{\frac{3}{2}}h^{\frac{3}{2}}+r_0{}^2h+\frac{h^4}{8}+\frac{2}{9}r_0{}^{\frac{1}{2}}h^3-\frac{1}{3}r_0{}^{\frac{1}{2}}h^{\frac{7}{2}}+\frac{r_0h^3}{2}-\frac{r_0{}^2h^2}{4}+\frac{r_0{}^4}{8}-\frac{2}{9}r_0{}^{\frac{7}{2}}-\frac{1}{3}r_0{}^{\frac{5}{2}}h^{\frac{3}{2}}-\frac{r_0{}^3h}{2}\right]$$

(ii) Pressure Difference

The pressure difference ΔP along the total length of the tube as follows

$$\Delta P = \int_0^1 \left(-\frac{Q}{F_1} + f\right) dz \qquad (12)$$

(iii) Resistance of the Flow

Resistance of the flow λ is calculated as

$$\lambda = \frac{\Delta P}{Q} = -\frac{1}{Q}\int_0^1 \left(-\frac{Q}{F_1} + f\right) dz \qquad (13)$$

The pressure drop in the absence of the stenosis $(h = 1)$ denoted by ΔP_N, can be obtained from

$$\Delta P_N = \int_0^1 \left(-\frac{Q}{F_2} + f\right) dz$$

where $F_2 = \frac{{r_0}^3}{6} - \frac{2}{3}r_0^{\frac{3}{2}} + {r_0}^2 + \frac{2}{9}r_0^{\frac{1}{2}} - \frac{1}{3}r_0^{\frac{1}{2}} + \frac{r_0}{2} - \frac{{r_0}^2}{4} + \frac{{r_0}^4}{8} - \frac{2}{9}r_0^{\frac{7}{2}} + \frac{1}{3}r_0^{\frac{5}{2}} - \frac{{r_0}^3}{2}$

The resistance to the flow in absence of the stenosis denoted by λ_N is obtained as

$$\lambda_N = \frac{\Delta P_N}{Q} \qquad (15)$$

The normalized resistance to the flow, denoted by $\bar{\lambda}$ is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} \qquad (16)$$

RESULTS AND DISCUSSION

Velocity profiles are drawn in Figures (2) and (3) for different values of τ_0 in the two stenotic regions (i.e. $d_1 \le z \le d_1 + L_1 \;\; and \quad B_1 - \frac{L_2}{2} \le z \le B_1$)

[During numerical computation we assumed that

$$\frac{R^*(z)}{R_0} = \exp\,[K(z - B_1)^2]_= \exp\,[\overline{K}(\bar{z} - \overline{B_1})^2]$$

where $\overline{K} = KB^2$ and

$\overline{d_1} = 0.2, \overline{L_1} = 0.2, \overline{L_2} = 0.2, \overline{B_1} = 0.8$ (Rajiv Sharma,1985)]

It is found that in the two stenotic regions the velocity increases with the increase in the yield stress $\tau_0$or the index n.

The variation of volumetric flow rate Q is calculated from equation (11) for different values of z in the two stenotic regions and shown in Figures (4) and (5).It is observed that for given z, the flux increases with increasing yield stress in the stenosis regions.

The variation of Q with z for different values of K and τ_0 is shown in Figures (6 & 7) in the region $(B_1 + \frac{L_2}{2} \leq z \leq B)$. It is found that the flux increases with increasing $\tau_0$or0 K for a given value of z.

From Fig- 8 & 9, It is observed that, the ratio $\bar{\lambda}$ increases with increase inτ_0.

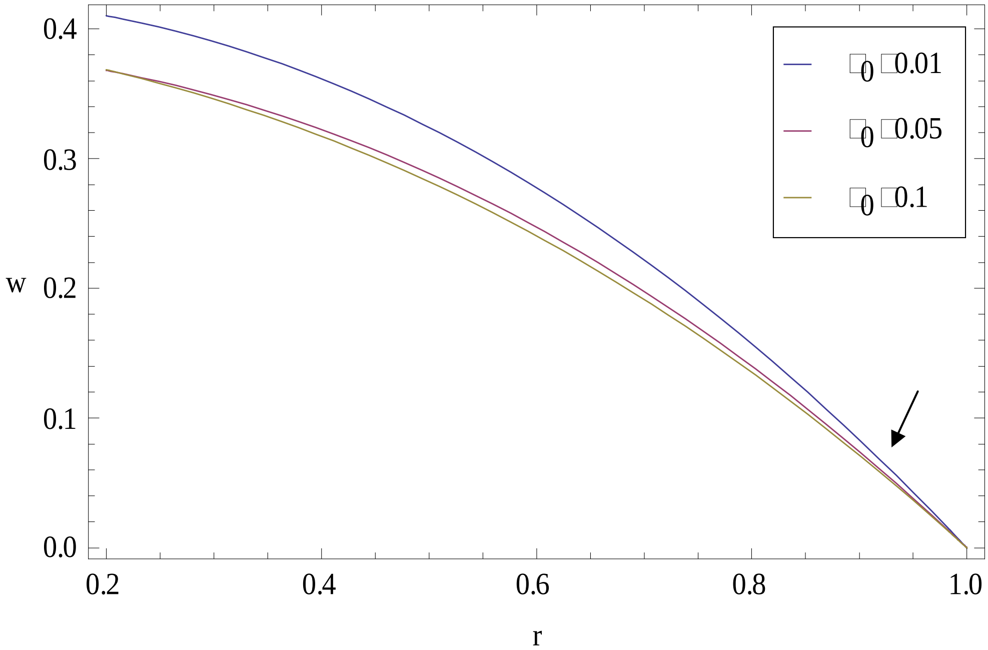

Fig 2 : Velocity Profiles for different τ_0 in the region $d_1 \leq z \leq d_1 + L_1$

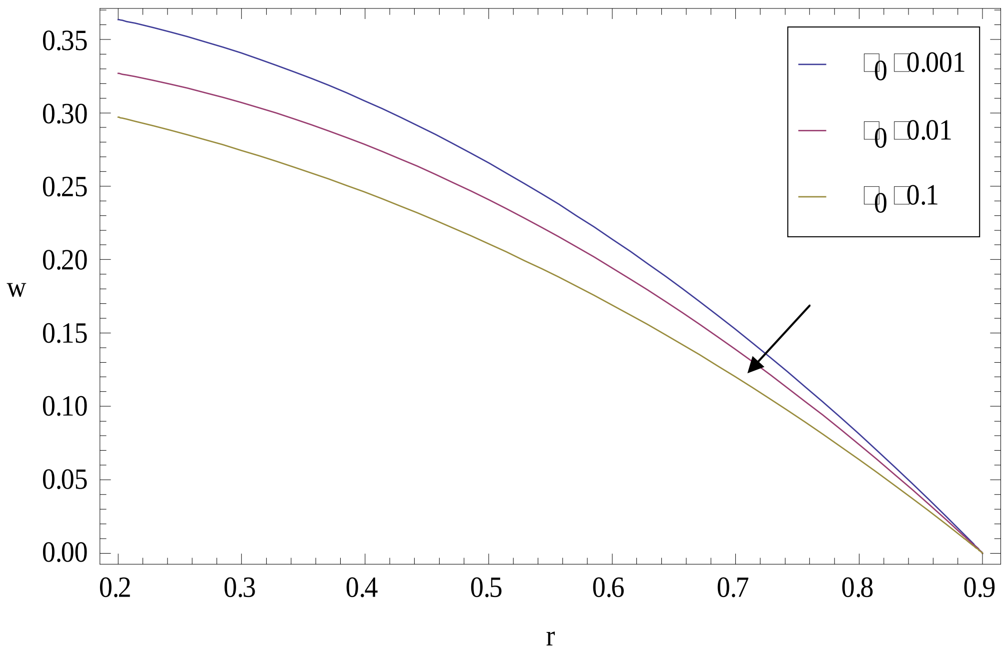

Fig 3 : Velocity Profiles for different τ_0 in the stenotic region $B_1 - \frac{L_2}{2} \leq z \leq B_1$

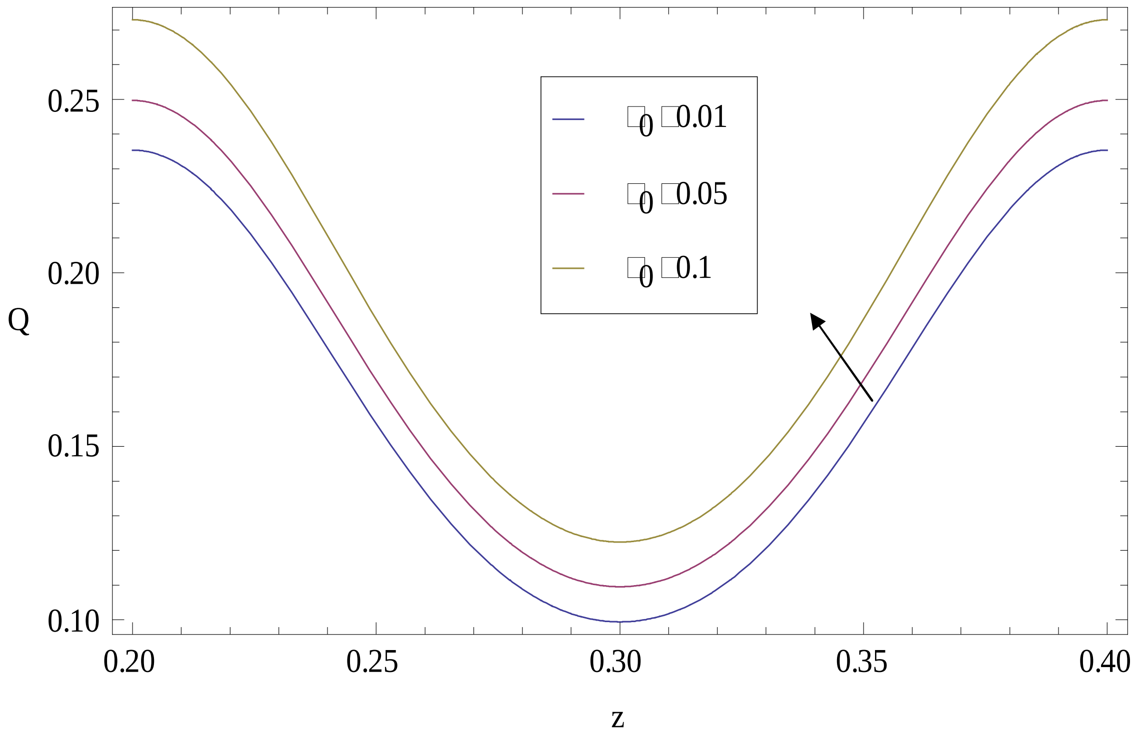

Fig 4: Q for different τ_0 in the first stenotic region $d_1 \le z \le d_1 + L_1$

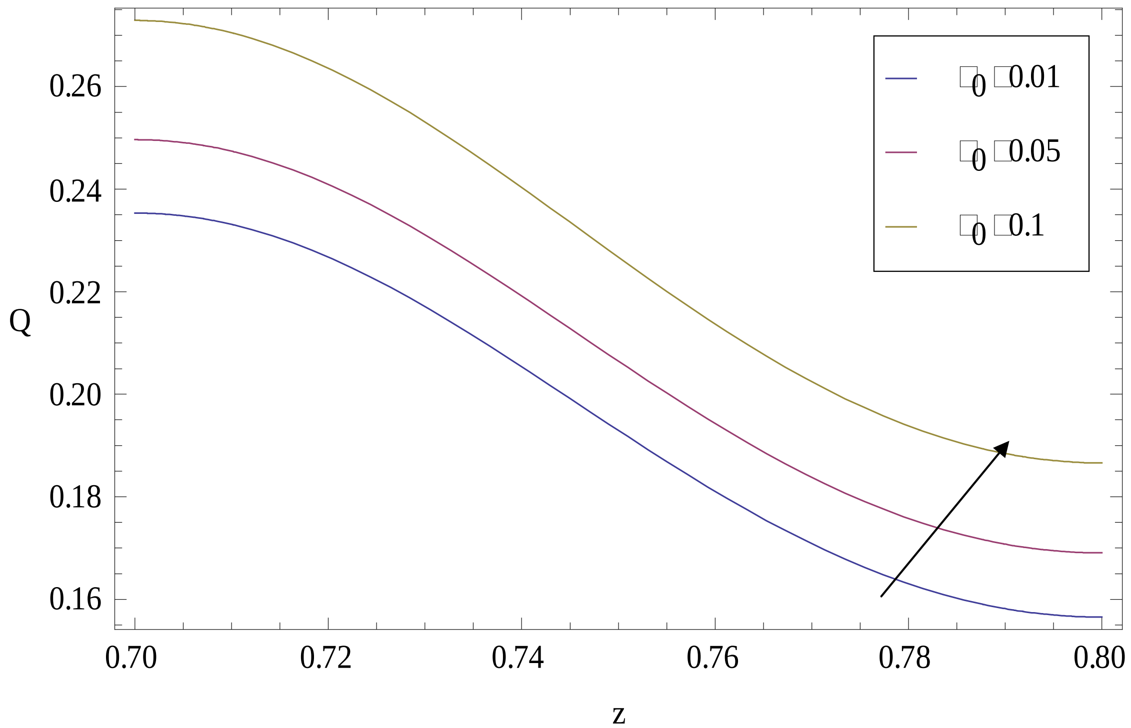

Fig 5 : Q for different τ_0 in the second stenotic region $B_1 - \frac{L_2}{2} \le z \le B_1$

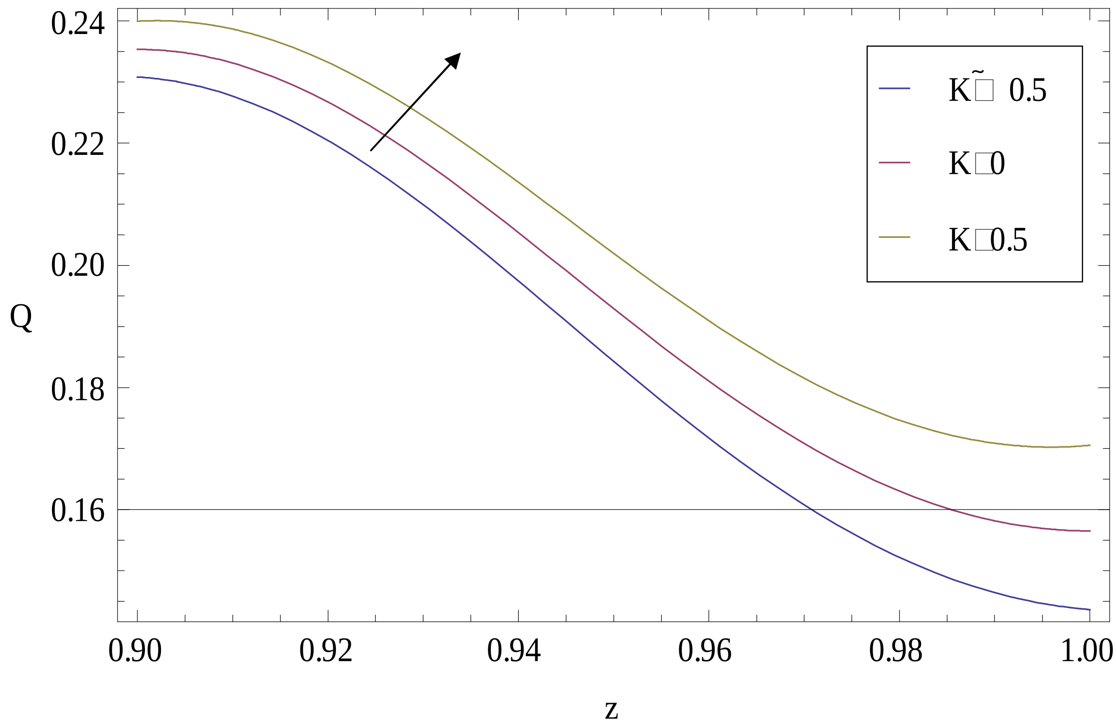

Fig 6: Q for different K in the third stenotic region $B_1 \leq z \leq B_1 + \frac{L_2}{2}$

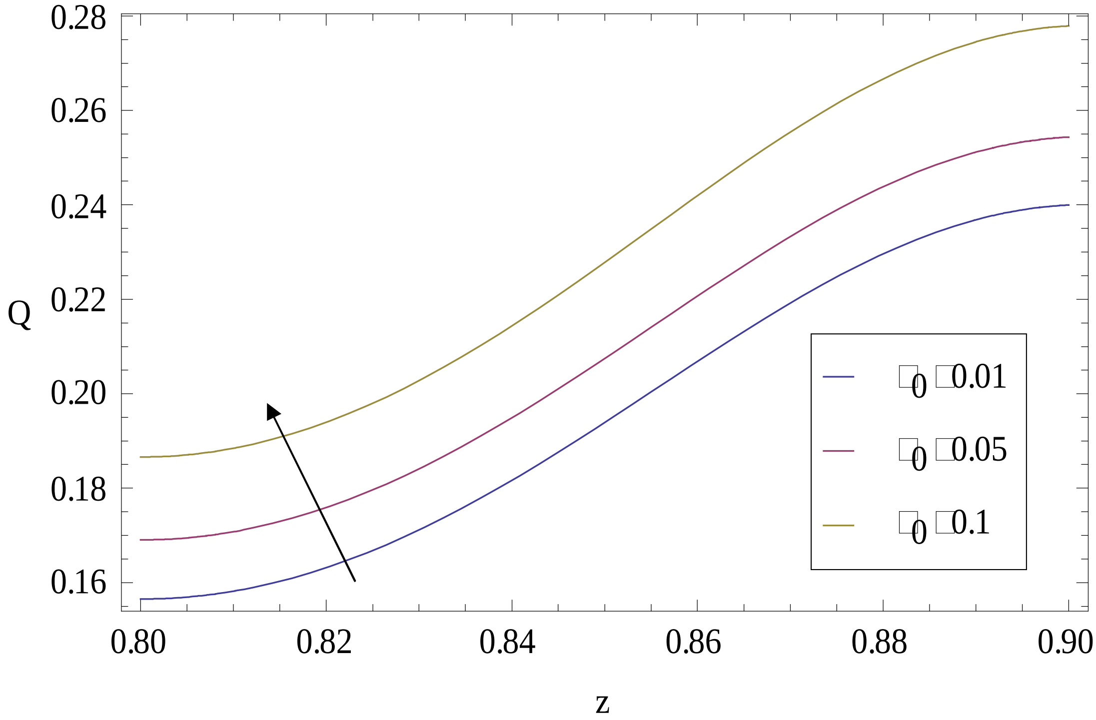

Fig. 7 : Q for different τ_0 in the third stenotic region $B_1 \leq z \leq B_1 + \frac{L_2}{2}$

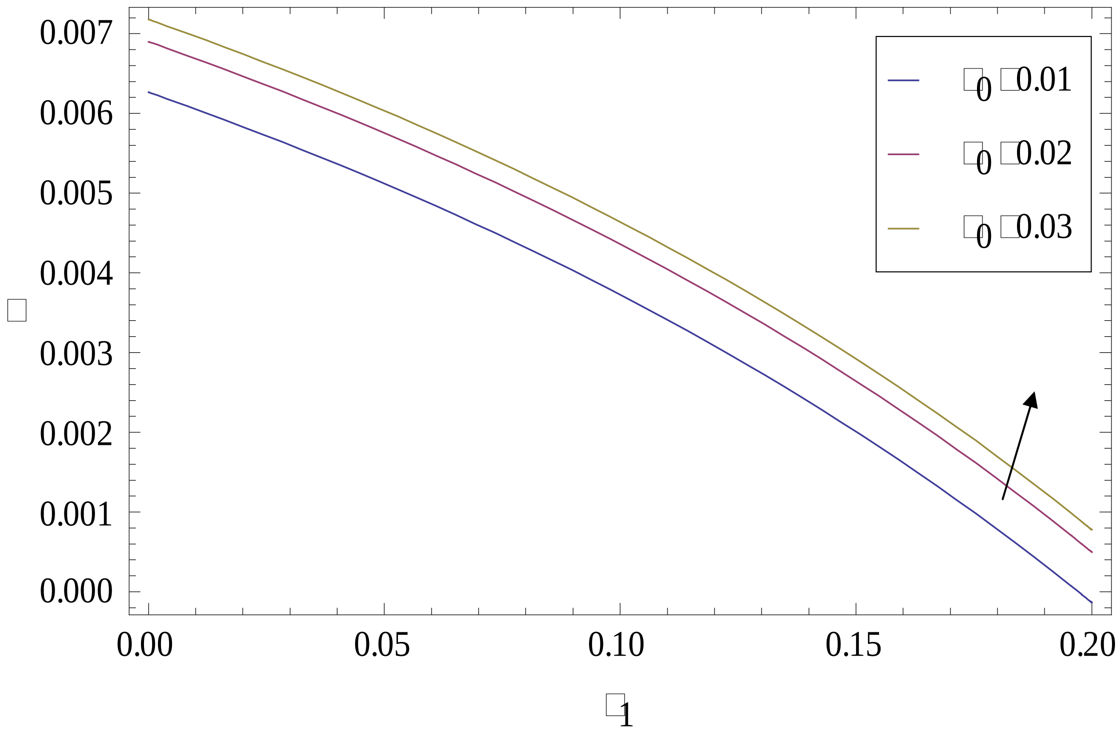

Fig 8: Flow Resistance for different yield stress

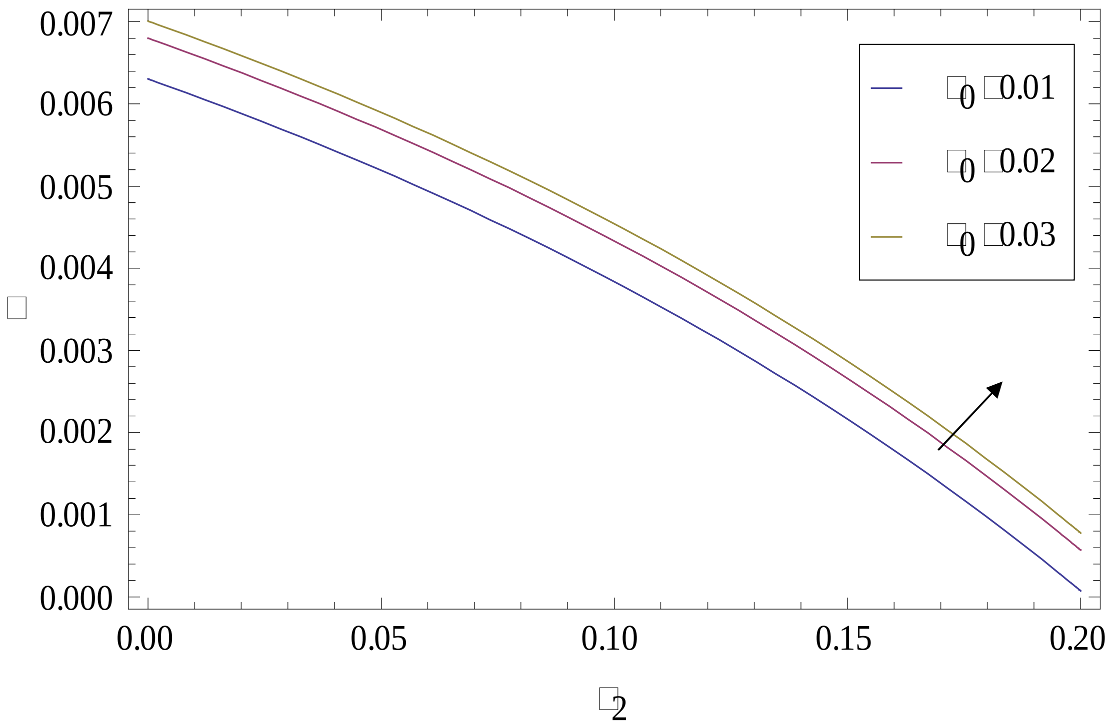

Fig 9 : Flow Resistance for different yield stress

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