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Fixed point theorem in pseudo compact space

Rajesh Shrivastava*, K. Qureshi and Kiran Rathore***

**Department of Mathematics, Govt. Science & Commerce College, Benazir, Bhopal, (MP)*
***Higher Education Department, Govt. of M. P., Bhopal, (MP)*

ABSTRACT

In this paper, the concepts of metric space and pseudo compact tichonov space has been introduced. We have proved some fixed point theorems for the self mapping satisfying a new contractive condition in compact metric spaces and pseudo compact metric spaces.

Keywords: Fixed point, Compact Metric space, pseudo compact Tichonov space, self mapping.

INTRODUCTION

There are several generalizations of classical contraction mapping theorem in Banach space [1]. In 196 Edelstein [2] established the existence of a unique fixed point of a self map *T* of a compact metric space satisfying the inequality $d(Tx,Ty) < d(x,y)$. Which is generalization of Banach. In the past few years a number of authors such as Iseki [3], fisher [4] Bhardwaj [5] have proved the number of interesting result on compact metric space. We are finding some fixed point theorems in psedo compact tichonov spaces.

Recently, Park [8] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces. Kutukcu[2] introduced the notion of intuitionistic Menger spaces with the help of t-norms and t-conorms as a generalization of Menger space due to Menger [3]. Recently in 2009, using the concept of subcompatible maps, Bouhadjera et. al. [1] proved common fixed point theorems in metric space. Using the concept of weakly compatible maps in intuitionistic Menger space, Pant et. al. [7] proved a common fixed point theorem for six self maps without appeal to continuity.

II. PRELIMINARIES

Definition A. Pseudo-compact tichonov space : *A* topological space *X* is said to be Pseudo-compact space, if every real valued continuous function on *X* is bounded. It may be noted that every compact space is psedo compact, but converges may not be true. Tichonov space , we mean a completely regular Housdroff space.

Now we prove, following theorems.

Definition B. Let *T* be a self continuous mapping. *A* space *X* is called a fixed point space, if every continuous mapping *T* of *X* into itself,

has a fixed point.

Theorem 1. Let *P* be a Psedo compact Tichonov space and *d* be a non negative real valued continuous function such that $d : T \times T \rightarrow R^{+}$, satisfying the condition,

$(i) d(x,x) = 0 \; \forall \; x \in X$

$(ii) d\,(x,z) \leq d(x,y) + d(y,z) \forall \; x,y,z \in X$

$(iii) d\,(Tx,Ty) \leq \alpha \,\{d(x,Tx)\} + d(y,Ty) + \beta \,\{d(x,Ty) + d(y,Tx) + \gamma \, d(x,y)\}$

where $\alpha\,\beta\,\gamma\ \geq 0$ *such that*
$0\ \leq\ \alpha+\beta+\gamma\ <1$ *and* $0\leq 1-\ \alpha-\beta\ <1$
Then *T* has unique fixed point in *P*.

Proof. We define a function $\emptyset : P \to R^+$ by $\emptyset(P) = d(p, Tp)$, for all $p \in P$, where R^+ is the set of positive real numbers. It is clear that $\emptyset$ is continuous generated by the composition of two continuous function *T and d*. Since *P* is psedocompact Tichonove space. Every real valued continuous function over *P* is bounded and attend its bounds.

Thus there exists a point $u \in P$ such that $\emptyset(u) = \inf\,[\emptyset(p) : p \in P]$. Now we suppose that *u* is a fixed point for *T*, if not;

Let us $\emptyset(Tu) = d(Tu, T^2u)$

From above
$d(Tu,T^2u) \leq \alpha\{d(u,Tu)+d(Tu,T^2u)\}+\beta\{d(u,T^2u)+d(Tu,Tu)\}+\gamma\ d(u,Tu)$

$(1-\alpha-\beta)d(Tu,T^2u) \leq (\alpha+\beta+\gamma)d(u,Tu)$

$$d(Tu,T^2u) < \frac{\alpha+\beta+\gamma}{1-\alpha-\beta}\ d(u,Tu)$$

$\emptyset(Tu) \leq \emptyset(u)$

u is a fixed point of T in P.

Uniqueness. Let us assume that *w* is another fixed point different from *u* in *P*, so that

$d(u, w) = d(Tu, Tw)$.

From (3),

$d(Tu,Tw) \leq \alpha\{d(u,Tu)+d(w,Tw)\}+\beta\{d(u,Tw)+d(w,Tu)\}+\gamma\ d(u,w)d(Tu,Tw) \leq (2\beta+\gamma)d(u,w)$

which contradiction;

u is unique fixed point of T

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