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# First Axisymmetric Problem of Micropolar Elasticity with Voids

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A R T I C L E I N F O

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A B S T R A C T

The theory of elasticity in its broad aspects deals with a study of the behavior of these substances which possess the property of recovering their size and shape when forces producing deformation are removed. In theory of micropolar elasticity, a body was assumed consisting of interconnected particles in the form of small rigid bodies undergoing translational motion as well as rotational motion. The presence of small pores or voids, in the constituent materials can also be formally introduced in a continuum model.

This paper is an attempt to explain an axisymmetric problem of a homogeneous isotropic micropolar elastic medium with voids subject to a set of normal point sources by employing the eigen value approach. The analytical expression of displacement components, microrotation, force stresses, couple stresses and volume fraction field have been derived for micropolar elastic solid with voids.

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## INTRODUCTION

The classical theory of elasticity successfully explains the behavior of construction materials (various sorts of steel, aluminum, concrete) provided the stresses do not exceed the elastic limit and no stress concentration occurs. The linear theory of elastic materials with voids is one of the generalizations of the classical theory of elasticity. This theory has practical utility to investigate various type of geological, biological and synthetic porous materials for which the elastic theory is inadequate. It is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void's volume is included among the kinematic variables, and in the limiting case when the volume tends to zero this theory reduces to the classical theory of elasticity. The process of voids is known to affect the estimations of the physicalmechanical properties of the composite and also to weaken the bond as these pores (voids) get spread over a wide area.

In this paper the axisymmetric deformation problem in micropolar elastic solid with voids is considered. A general solution of the equation of motion of a micropolar elastic medium with voids, in cylindrical polar co-ordinate system is obtained to the axisymmetric deformation. The Laplace and Hankel transforms are used to solve the equations through eigen value approach.

### Laplace transform in Cartesian co-ordinates

Laplace transform [Debnath (1995)] of a function f((x, z, t) with respect to time variable t, with p as the Laplace transform variable, is defined as

$$\bar{f}(x,z,p) = L\{f(x,z,t))\} = \int_0^{\infty} f(x,z,t)e^{-pt}dt, \tag{1.1}$$

Along with the following basic properties [Debnath (1995)]

$$L\left\{\frac{\partial f}{\partial t}\right\} = p\bar{f}(x,z,p) - f(x,z,0), \tag{1.2}$$

$$L\left\{\frac{\partial^2 f}{\partial t^2}\right\} = p^2\bar{f}(x,z,p) - pf(x,z,0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}, \tag{1.3}$$

Also, the Laplace transform of the Dirac delta function δ(t) is given as

$$L\{\delta(t)\}=1, \tag{1.4}$$

### Laplace transform in polar co-ordinates

The Laplace transform with respect to time variable t, with p as the Laplace transform variable, is defined as

$$\bar{f}(r,z,p) = L\{f(r,z,t))\} = \int_0^{\infty} f(r,z,t)e^{-pt}dt, \tag{1.5}$$

Along with the following basic properties

$$L\left\{\frac{\partial f}{\partial t}\right\} = p\bar{f}(r,z,p) - f(r,z,0), \tag{1.6}$$

$$L\left\{\frac{\partial^2 f}{\partial t^2}\right\} = p^2\bar{f}(r,z,p) - pf(r,z,0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}, \tag{1.7}$$

### The Hankel transform

The Hankel transform [Sneddon (1979)] of order n of $\bar{f}(r,z,p)$ with respect to variable r is defined as

$$\tilde{f}(\xi,z,p) = H_n\{\bar{f}(r,z,p)\} = \int_0^\infty r\,\bar{f}(r,z,p)\,J_n(\xi r)dr, \tag{1.8}$$

Where $\xi$ is the Hankel transform variable, $J_n$(---) is the Bessel function of first kind of order n, alongwith the following basic operational properties [Sneddon (1979)]

$$H_0\left\{\frac{\partial \bar{f}}{\partial r} + \frac{1}{r}\bar{f}\right\} = \xi H_1\{\bar{f}\}, \qquad H_0\left\{\frac{\partial^2 \bar{f}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{f}}{\partial r}\right\} = -\xi^2 H_0\{\bar{f}\}, \tag{1.9}$$

$$H_1\left\{\frac{\partial \bar{f}}{\partial r}\right\} = -\xi H_0\{\bar{f}\}, \qquad H_1\left\{\frac{\partial^2 \bar{f}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{f}}{\partial r} - \frac{1}{r^2}\bar{f}\right\} = -\xi^2 H_1\{\bar{f}\}, \tag{1.10}$$

We know that [Debnath (1995)], the Dirac delta function δ(- -) having the property

$$\mathrm{H}_0\left\{\frac{\delta(\mathrm{r})}{\mathrm{r}}\right\} = 1, \tag{1.11}$$

## BASIC EQUATIONS

The field equations and the constitutive relations in micropolar elastic solid with voids without body force and body couples are given by Eringen (1968) and Iesan (1985) as

$$\beta^*\nabla q + (\lambda + 2\mu + K)\nabla(\nabla.\vec{u}) - (\mu + K)\nabla\times\nabla\times\vec{u} + K\nabla\times\vec{\varphi} = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{2.1}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla.\vec{\varphi})) - \gamma\nabla\times\nabla\times\vec{\varphi} + K\nabla\times\vec{u} - 2K\vec{\varphi} = \rho j\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2.2}$$

$$\alpha^*\nabla^2 q^* - \zeta^* q^* - \omega^*\frac{\partial q^*}{\partial t} - \beta^*\nabla.\vec{u} = \rho K^*\frac{\partial^2 q^*}{\partial t^2}, \tag{2.3}$$

$$t_{ij} = \left(\beta^* q^* + \lambda u_{r,r}\right)\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right), \tag{2.4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{2.5}$$

Where λ, μ, K, α, β, γ are the material constants, ρ is the density, j is the micro-intertia, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the microrotation vector, $\varphi^*$ is the scalar microstretch, $t_{ij}$ is the force stress tensor, $m_{ij}$ is the couple stress tensor, $q^*$ is the volume fraction field, $\delta_{ij}$ is the Kronecker delta, $\varepsilon_{ijr}$ is the unit anti-symmetric tensor and $\alpha^*$, $\beta^*$, $\zeta^*$, $\omega^*$, $K^*$ are the material constants due to presence of voids.

The dynamic equations (2.1)-(2.3) in cylindrical polar co-ordinate system (r, θ, z) in component form become

$$\beta^*\frac{\partial q^*}{\partial r} + (\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} - \frac{1}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial r} + \frac{\partial^2 u_z}{\partial r\partial z}\right) + (\mu+K)\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2} + \frac{\partial^2 u_r}{\partial z^2}\right) + \frac{K}{r}\left(\frac{\partial\phi_z}{\partial\theta} - r\frac{\partial\phi_\theta}{\partial z}\right) = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{2.6}$$

$$\frac{\beta^*}{r}\frac{\partial q^*}{\partial \theta} + (\lambda+\mu)\frac{1}{r}\left(\frac{\partial^2 u_r}{\partial\theta\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{1}{r}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial\theta\partial z}\right) + K\left(\frac{\partial \phi_r}{\partial z} - \frac{\partial \phi_z}{\partial r}\right) + (\mu+K)\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right) = \rho\frac{\partial^2 u_\theta}{\partial t^2}, \quad (2.7)$$

$$\beta^*\frac{\partial q^*}{\partial z} + (\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) + \frac{K}{r}\left(\frac{\partial (r\phi_\theta)}{\partial r} - \frac{\partial \phi_r}{\partial r}\right) + (\mu+K)\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right) = \rho\frac{\partial^2 u_z}{\partial t^2}, \quad (2.8)$$

$$(\alpha+\beta)\left(\frac{\partial^2 \phi_r}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_r}{\partial r} - \frac{\phi_r}{r^2} - \frac{1}{r^2}\frac{\partial \phi_\theta}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \phi_\theta}{\partial\theta\partial r} + \frac{\partial^2 \phi_z}{\partial r\partial z}\right) + \frac{K}{r}\left(\frac{\partial u_z}{\partial \theta} - r\frac{\partial u_\theta}{\partial z}\right) + \gamma\left(\frac{\partial^2 \phi_r}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_r}{\partial r} - \frac{\phi_r}{r^2} - \frac{2}{r^2}\frac{\partial \phi_\theta}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 \phi_r}{\partial \theta^2} + \frac{\partial^2 \phi_r}{\partial z^2}\right) - 2K\phi_r = \rho j\frac{\partial^2 \phi_r}{\partial t^2}, \quad (2.9)$$

$$(\alpha+\beta)\frac{1}{r}\left(\frac{\partial^2 \phi_r}{\partial\theta\partial r} + \frac{1}{r}\frac{\partial \phi_r}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \phi_\theta}{\partial \theta^2} + \frac{\partial^2 \phi_z}{\partial\theta\partial z}\right) + K\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) - 2K\phi_\theta + \gamma\left(\frac{\partial^2 \phi_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \phi_\theta}{\partial \theta^2} + \frac{1}{r}\frac{\partial \phi_\theta}{\partial r} + \frac{\partial^2 \phi_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial \phi_r}{\partial \theta} - \frac{\phi_\theta}{r^2}\right) = \rho j\frac{\partial^2 \phi_\theta}{\partial t^2}, \quad (2.10)$$

$$(\alpha+\beta)\left(\frac{\partial^2 \phi_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial \phi_r}{\partial z} + \frac{1}{r}\frac{\partial^2 \phi_\theta}{\partial\theta\partial z} + \frac{\partial^2 \phi_z}{\partial z^2}\right) + \frac{K}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial r}\right) - 2K\phi_z + \gamma\left(\frac{\partial^2 \phi_z}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi_z}{\partial \theta^2} + \frac{\partial^2 \phi_z}{\partial z^2}\right) = \rho j\frac{\partial^2 \phi_z}{\partial t^2}, \quad (2.11)$$

$$\alpha^*\left(\frac{\partial^2 q^*}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 q^*}{\partial \theta^2} + \frac{\partial^2 q^*}{\partial z^2} + \frac{1}{r}\frac{\partial q^*}{\partial r}\right) - \zeta^* q^* - \omega^*\frac{\partial q^*}{\partial t} - \beta^*\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right) = \rho K^*\frac{\partial^2 q^*}{\partial t^2}, \quad (2.12)$$

Where ($u_r$, $u_\theta$, $u_z$) and ($\varphi_r$, $\varphi_\theta$, $\varphi_z$) represent the cylindrical polar components of the displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ respectively.

## Formulation and Solution of the Problem

Here, we consider the axisymmetric two dimensional problem in micropolar elastic solid with voids. Since we are considering two-dimensional axisymmetric problem, so we assume the components of displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ of the form

$$\vec{u} = (u_r, 0, u_z), \quad \vec{\phi} = (0, \varphi_\theta, 0), \quad (2.13)$$

And the quantities remain independent of θ, so that $\frac{\partial}{\partial\theta} \equiv 0$. With these considerations and using (2.13), the equations (2.7), (2.9) and (2.11) become identically zero and equations (2.6), (2.8), (2.10) and (2.12) and constitutive relations (2.4)-(2.5) can be written as

$$\beta^*\frac{\partial q^*}{\partial r} + (\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r\partial z}\right) - K\frac{\partial \phi_\theta}{\partial z} + (\mu+K)\left(\nabla^2 u_r - \frac{u_r}{r^2}\right) = \rho\frac{\partial^2 u_r}{\partial t^2}, \quad (2.14)$$

$$\beta^*\frac{\partial q^*}{\partial z} + (\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) + \frac{K}{r}\left(\frac{\partial (r\phi_\theta)}{\partial r}\right) + (\mu+K)\nabla^2 u_z = \rho\frac{\partial^2 u_z}{\partial t^2}, \quad (2.15)$$

$$\gamma\left(\nabla^2 \phi_\theta - \frac{\phi_\theta}{r^2}\right) + K\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) - 2K\phi_\theta = \rho j\frac{\partial^2 \phi_\theta}{\partial t^2}, \quad (2.16)$$

$$\alpha^*\nabla^2 q^* - \zeta^* q^* - \omega^*\frac{\partial q^*}{\partial t} - \beta^*\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) = \rho K^*\frac{\partial^2 q^*}{\partial t^2}, \quad (2.17)$$

$$t_{zr} = (\mu + K)\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r} - K\phi_\theta, \tag{2.18}$$

$$t_{zz} = \beta^* q^* + \lambda(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}) + (\lambda + 2\mu + K)\frac{\partial u_z}{\partial z}, \tag{2.19}$$

$$m_{z\theta} = \gamma\frac{\partial \phi_\theta}{\partial z}, \tag{2.20}$$

Where $\nabla^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)$, (2.21)

We define the non-dimensional quantities as

$$r' = \frac{\omega_1}{c_1}r, \quad z' = \frac{\omega_1}{c_1}z, \quad u_r{}' = \frac{\omega_1}{c_1}u_r,$$
$$u_z{}' = \frac{\omega_1}{c_1}u_z, \quad \phi_\theta' = \frac{j\omega_1^2}{c_1^2}\phi_\theta, \quad t' = \omega_1 t,$$
$$q^{*\prime} = \frac{j\omega_1^2}{c_1^2}q^*, \quad t_{zz}' = \frac{t_{zz}}{\mu}, \quad t_{zr}' = \frac{t_{zr}}{\mu},$$
$$m_{z\theta}' = \frac{j\omega_1}{c_1\gamma}m_{z\theta}, \tag{2.22}$$

Where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \qquad \omega^{*2} = \frac{K}{\rho j}.$$

And h is the constant having dimension of length.

**Using the quantities given by (2.22), the field equations (2.14) – (2.17) and stress-displacement relations (2.18) – (2.20) may be recast into two dimensionless form (after suppressing the primes for convenience) as**

$$s_0\frac{\partial q^*}{\partial r} + \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r\partial z}\right) + s_1\left(\nabla^2 - \frac{1}{r^2}\right)u_r - s_2\frac{\partial \phi_\theta}{\partial z} = s_3\frac{\partial^2 u_r}{\partial t^2}, \tag{2.23}$$

$$s_0\frac{\partial q^*}{\partial z} + \left(\frac{\partial^2 u_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) + s_1\nabla^2 u_z + \frac{s_2}{r}\frac{\partial(r\phi_\theta)}{\partial r} = s_3\frac{\partial^2 u_z}{\partial t^2}, \tag{2.24}$$

$$\left(\frac{\partial^2 \phi_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_\theta}{\partial r} + \frac{\partial^2 \phi_\theta}{\partial z^2} - \frac{\phi_\theta}{r^2}\right) + s_4\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) - s_5\phi_\theta = s_6\frac{\partial^2 \phi_\theta}{\partial t^2}, \tag{2.25}$$

$$\nabla^2 q^* - s_7 q^* - s_8\frac{\partial q^*}{\partial t} - s_9\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z}\right) = s_{10}\frac{\partial^2 q^*}{\partial t^2}, \tag{2.26}$$

$$t_{zr} = s_{11}\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} - s_{12}\phi_\theta, \tag{2.27}$$

$$t_{zz} = s_{13}q^* + s_{14}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + s_{15}\frac{\partial u_z}{\partial z}, \tag{2.28}$$

$$m_{z\theta} = \frac{\partial \phi_\theta}{\partial z}, \tag{2.29}$$

Where

$$s_0=\frac{\beta^* c_1^2}{(\lambda+\mu) j\omega_1^2}, \quad s_1=\frac{(\mu+K)}{(\lambda+\mu)},$$
$$s_2=\frac{K c_1^2}{(\lambda+\mu) j\omega_1^2}, \quad s_3=\frac{\rho c_1^2}{(\lambda+\mu)},$$
$$s_4=\frac{jK}{\gamma}, \quad s_5=\frac{2K c_1^2}{\gamma\omega_1^2},$$
$$s_6=\frac{j\rho c_1^2}{\gamma}, \quad s_7=\frac{\zeta^* c_1^2}{\alpha^*\omega_1^2},$$
$$s_8=\frac{\omega^* c_1^2}{\alpha^*\omega_1^2}, \quad s_9=\frac{\beta^* j}{\alpha^*},$$
$$s_{10}=\frac{K^*\rho c_1^2}{\alpha^*}, \quad s_{11}=\frac{\mu+K}{\mu},$$
$$s_{12}=\frac{K c_1^2}{j\mu\omega_1^2}, \quad s_{13}=\frac{\beta^* c_1^2}{j\mu\omega_1^2},$$
$$s_{14}=\frac{\lambda}{\mu}, \quad s_{15}=\frac{(\lambda+2\mu+K)}{\mu}, \tag{2.30}$$

Applying Laplace transform defined by expression (1.5), with help of results (1.6) and (1.7) on equations (2.23)-(2.29), we get

$$s_0\frac{\partial\bar{q}^*}{\partial r}+\left(\frac{\partial^2\bar{u}_r}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}_r}{\partial r}-\frac{\bar{u}_r}{r^2}+\frac{\partial^2\bar{u}_z}{\partial r\partial z}\right)+s_1\left(\nabla^2-\frac{1}{r^2}\right)\bar{u}_r-s_2\frac{\partial\bar{\phi}_\theta}{\partial z}=s_3p^2\bar{u}_r, \tag{2.31}$$

$$s_0\frac{\partial\bar{\phi}^*}{\partial z}+\left(\frac{\partial^2\bar{u}_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial\bar{u}_r}{\partial z}+\frac{\partial^2\bar{u}_z}{\partial z^2}\right)+s_1\nabla^2\bar{u}_z+\frac{s_2}{r}\frac{\partial(r\bar{\phi}_\theta)}{\partial r}=s_3p^2\,\bar{u}_z, \tag{2.32}$$

$$\left(\frac{\partial^2\bar{\phi}_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{\phi}_\theta}{\partial r}+\frac{\partial^2\bar{\phi}_\theta}{\partial z^2}-\frac{\bar{\phi}_\theta}{r^2}\right)+s_4\left(\frac{\partial\bar{u}_r}{\partial z}-\frac{\partial\bar{u}_z}{\partial r}\right)-s_5\bar{\phi}_\theta=s_6p^2\,\bar{\phi}_\theta, \tag{2.33}$$

$$\left(\frac{\partial^2\bar{q}^*}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{q}^*}{\partial r}+\frac{\partial^2\bar{q}^*}{\partial z^2}\right)-s_7\bar{q}^*-s_8p\bar{q}^*-s_9\left(\frac{\partial\bar{u}_r}{\partial r}+\frac{\partial\bar{u}_z}{\partial z}\right)=s_{10}\;p^2\bar{q}^*, \tag{2.34}$$

$$\bar{t}_{zr}=s_{11}\frac{\partial\bar{u}_r}{\partial z}+\frac{\partial\bar{u}_z}{\partial r}-s_{12}\bar{\phi}_\theta, \tag{2.35}$$

$$\bar{t}_{zz}=s_{13}\bar{q}^*+s_{14}\left(\frac{\partial\bar{u}_r}{\partial r}+\frac{\bar{u}_r}{r}\right)+s_{15}\frac{\partial\bar{u}_z}{\partial z}, \tag{2.36}$$

$$\bar{m}_{z\theta}=\frac{\partial\bar{\phi}_\theta}{\partial z}, \tag{2.37}$$

Where the initial displacements, microrotation and volume fraction and their corresponding velocities are assumed as zero throughout the medium, that is,

$$u_r(r,z,0)=\left\{\frac{\partial u_r}{\partial t}\right\}_{t=0}=0, \qquad u_z(r,z,0)=\left\{\frac{\partial u_z}{\partial t}\right\}_{t=0}=0,$$

$$\phi_\theta(r,z,0)=\left\{\frac{\partial\phi_\theta}{\partial t}\right\}_{t=0}=0, \qquad q^*(r,z,0)=\left\{\frac{\partial q^*}{\partial t}\right\}_{t=0}=0, \tag{2.38}$$

Applying the Hankel transform defined by (1.8), with help of results (1.9) and (1.10) on equations (2.31) – (2.37), we obtain,

$$\frac{d^2\tilde{u}_r}{dz^2} = \frac{(\xi^2+s_1\xi^2+s_3p^2)}{s_1}\tilde{u}_r + \frac{\xi}{s_1}\frac{d\tilde{u}_z}{dz} + \frac{s_2}{s_1}\frac{d\tilde{\phi}_\theta}{dz} + \frac{\xi s_0}{s_1}\tilde{q}^*, \tag{2.39}$$

$$\frac{d^2\tilde{u}_z}{dz^2} = \left(\frac{s_1\xi^2+s_3p^2}{1+s_1}\right)\tilde{u}_z - \frac{\xi}{1+s_1}\frac{d\tilde{u}_r}{dz} - \frac{\xi s_2}{1+s_1}\tilde{\phi}_\theta - \frac{s_0}{1+s_1}\frac{d\tilde{q}^*}{dz}, \tag{2.40}$$

$$\frac{d^2\tilde{\phi}_\theta}{dz^2} = -s_4\frac{d\tilde{u}_r}{dz} - \xi s_4\tilde{u}_z + (\xi^2 + s_5 + p^2 s_6)\tilde{\phi}_\theta, \tag{2.41}$$

$$\frac{d^2\tilde{q}^*}{dz^2} = -s_9\xi\tilde{u}_r + s_9\frac{d\tilde{u}_z}{dz} + (\xi^2 + s_7 + ps_8 + p^2 s_{10})\tilde{q}^*, \tag{2.42}$$

$$\tilde{t}_{zr} = s_{11}\frac{d\tilde{u}_r}{dz} - \xi\tilde{u}_z - s_{12}\tilde{\phi}_\theta, \tag{2.43}$$

$$\tilde{t}_{zz} = s_{13}\tilde{q}^* + \xi s_{14}\tilde{u}_r + s_{15}\frac{d\tilde{u}_z}{dz}, \tag{2.44}$$

$$\tilde{m}_{z\theta} = \frac{d\tilde{\phi}_\theta}{dz}, \tag{2.45}$$

Where

$$\tilde{u}_r(\xi,z,p) = H_1\{\bar{u}_r(r,z,p)\}, \qquad \tilde{\phi}_\theta(\xi,z,p) = H_1\{\bar{\phi}_\theta(r,z,p)\}, \tag{2.46}$$

$$\tilde{t}_{zr}(\xi,z,p) = H_1\{\bar{t}_{zr}(r,z,p)\}, \qquad \tilde{m}_{z\theta}(\xi,z,p) = H_1\{\bar{m}_{z\theta}(r,z,p)\}, \tag{2.47}$$

$$\tilde{u}_z(\xi,z,p) = H_0\{\bar{u}_z(r,z,p)\}, \qquad \tilde{q}^*(\xi,z,p) = H_0\{\bar{q}^*(r,z,p)\}, \tag{2.48}$$

$$\tilde{t}_{zz}(\xi,z,p) = H_0\{\bar{t}_{zz}(r,z,p)\}, \tag{2.49}$$

The system of equations (2.39)-(2.42) can be written in the vector matrix differential equation form as

$$\frac{d}{dz}W(\xi,z,p) = A(\xi,p)W(\xi,z,p), \tag{2.50}$$

Where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \qquad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix},$$

$$U = \begin{bmatrix} \tilde{u}_r \\ \tilde{u}_z \\ \tilde{\phi}_\theta \\ \tilde{q}^* \end{bmatrix}, \qquad O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & a_{12} & a_{13} & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & 0 \\ 0 & a_{42} & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} b_{11} & 0 & 0 & b_{14} \\ 0 & b_{22} & b_{23} & 0 \\ 0 & b_{32} & b_{33} & 0 \\ b_{41} & 0 & 0 & b_{44} \end{bmatrix},$$

With

$a_{12} = \frac{\xi}{s_1},$ $a_{13} = \frac{s_2}{s_1},$

$a_{21} = \frac{-\xi}{1+s_1},$ $a_{24} = \frac{-s_0}{1+s_1},$

$a_{31} = -s_4,$ $a_{42} = s_9,$

$b_{11} = \frac{(\xi^2+s_1\xi^2+s_3p^2)}{s_1},$ $b_{14} = \frac{\xi s_0}{s_1},$

$b_{22} = \frac{(s_1\xi^2+s_3p^2)}{1+s_1},$ $b_{23} = \frac{-\xi s_2}{1+s_1},$

$b_{32} = -\xi s_4,$ $b_{33} = \xi^2 + s_5 + p^2 s_6,$

$b_{41} = -\xi s_9,$ $b_{44} = \xi^2 + s_7 + p s_8 + p^2 s_{10},$

$$D = \frac{d}{dz}. \tag{2.51}$$

To solve the equation (2.50), we take

$$W(\xi, z, p) = X(\xi, p)e^{qz}, \tag{2.52}$$

For some parameter q, so that

$$A(\xi, p)W(\xi, z, p) = qW(\xi, z, p), \tag{2.53}$$

Which leads to eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$\det(A - qI) = 0, \tag{2.54}$$

Which on expansion provides us

$$q^8 - \sigma_1 q^6 + \sigma_2 q^4 - \sigma_3 q^2 + \sigma_4 = 0, \tag{2.55}$$

Where

$$\sigma_1 = b_{11} + b_{22} + b_{33} + b_{44} + a_{24}a_{42} + a_{12}a_{21} + a_{13}a_{31}, \tag{2.56}$$

$$\sigma_2 = b_{11}b_{22} + b_{22}b_{33} + b_{33}b_{11} + b_{44}b_{11} + b_{44}b_{22} + b_{44}b_{33} - a_{12}a_{24}b_{41} + a_{24}a_{42}b_{11} + a_{24}a_{42}b_{33} + a_{24}a_{42}a_{13}a_{31} - b_{32}b_{23} + a_{12}a_{21}b_{33} + a_{12}a_{21}b_{44} - a_{12}a_{31}b_{23} - a_{13}a_{21}b_{32} + a_{13}a_{31}b_{22} + a_{13}a_{31}b_{44} - b_{14}b_{41} - b_{14}a_{21}a_{42}, \tag{2.57}$$

$\sigma_3 = b_{11}b_{22}b_{33} + b_{22}b_{33}b_{44} + b_{33}b_{44}b_{11} + b_{44}b_{11}b_{22} - b_{11}b_{23}b_{32} - b_{44}b_{23}b_{32} + a_{24}a_{42}b_{11}b_{33} + a_{13}a_{24}b_{41}b_{32} + a_{31}a_{42}b_{14}b_{23} + a_{12}a_{21}b_{33}b_{44} - a_{13}a_{21}b_{32}b_{44} - a_{12}a_{31}b_{23}b_{44} - a_{12}a_{24}b_{41}b_{33} - a_{42}a_{21}b_{14}b_{33} + a_{13}a_{31}b_{22}b_{44} - b_{14}b_{41}b_{22} - b_{14}b_{41}b_{33},$ (2.58)

$\sigma_4 = b_{11}b_{22}b_{33}b_{44} - b_{23}b_{32}b_{11}b_{44} - b_{14}b_{41}b_{22}b_{33} + b_{23}b_{32}b_{14}b_{41},$ (2.59)

The eigen values of the matrix A are characteristic roots of the equation (2.55). The vectors X(ξ, p) corresponding to the eigen values $q_s$ can be determined by solving the homogeneous equations

$$[A - qI]X(\xi,p) = 0 \tag{2.60}$$

The set of eigen vectors $X_s(\xi,p); s = 1,2,3,\ldots\ldots,8$ may be defined as

$$X_s(\xi,p) = \begin{bmatrix} X_{s1}(\xi,p) \\ X_{s2}(\xi,p) \end{bmatrix} \tag{2.61}$$

Where

$$X_{s1}(\xi,p) = \begin{bmatrix} a_s q_s \\ b_s \\ -\xi \\ c_s \end{bmatrix}, \qquad X_{s2}(\xi,p) = \begin{bmatrix} a_s q^2{}_s \\ b_s q_s \\ -\xi q_s \\ c_s q_s \end{bmatrix}, \qquad q=q_s, \quad s=1,2,3,4$$

(2.62)

$l=s+4$, $q=-q_s$ ;s=1,2,3,4 (2.63)

$a_s = \frac{\xi}{\Delta_s}\{(\xi^2 + s_7 + ps_8 + p^2 s_{10})(\xi^2 + s_5 + p^2 s_6 - {q_s}^2 + s_2 s_4) - s_2 s_4 {q_s}^2 - (\xi^2 + s_5 + p^2 s_6 - {q_s}^2)(s_0 s_9 + {q_s}^2)\}$ (2.64)

$b_s = \frac{-1}{\Delta_s}[(\xi^2 + s_7 + ps_8 + p^2 s_{10} - {q_s}^2)\{(\xi^2 + s_1\xi^2 + s_3 p^2 - s_1 {q_s}^2)(\xi^2 + s_5 + p^2 s_6 - {q_s}^2) + s_2 s_4 {q_s}^2\} - \xi^2 s_0 s_9(\xi^2 + s_5 + p^2 s_6 - {q_s}^2)],$ (2.65)

$$c_s = \frac{q_s s_9(b_s + \xi a_s)}{\{{q_s}^2 - (\xi^2 + s_7 + ps_8 + p^2 s_{10})\}}, \tag{2.66}$$

$\Delta_s = s_4[({q_s}^2 - \xi^2)s_0 s_9 - (\xi^2 + s_7 + ps_8 + p^2 s_{10} - {q_s}^2)\{{q_s}^2 - (\xi^2 + s_1\xi^2 + s_3 p^2 - s_1 {q_s}^2)\}]; \quad s = 1,2,3,4$ (2.67)

Thus, a solution of equation (2.50) as given by (2.52) become

$$W(\xi,z,p) = \sum_{s=1}^{4}\{E_s(\xi,p)e^{q_s z} + E_{s+4}X_{s+4}(\xi,p)e^{-q_s z}\}, \tag{2.68}$$

Where $E_1,E_2,E_3,E_4,E_5,E_6,E_7$ and $E_8$ are eight arbitrary constants.

The equation (2.68) represents the solution of the general problem in the axisymmetric case of micropolar elastic medium with voids and gives displacement, microrotation and volume fraction components in the transformed domains, which are obtained as

$$\tilde{u}_r(\xi,z,p) = a_1q_1E_1e^{q_1z} - a_1q_1E_5e^{-q_1z} + a_2q_2E_2e^{q_2z} - a_2q_2E_6e^{-q_2z} + a_3q_3E_3e^{q_3z} - a_3q_3E_7e^{-q_3z} + a_4q_4E_4e^{q_4z} - a_4q_4E_8e^{-q_4z}, \quad (2.69)$$

$$\tilde{u}_z(\xi,z,p) = b_1E_1e^{q_1z} + b_1E_5e^{-q_1z} + b_2E_2e^{q_2z} + b_2E_6e^{-q_2z} + b_3E_3e^{q_3z} + b_3E_7e^{-q_3z} + b_4E_4e^{q_4z} + b_4E_8e^{-q_4z}, \quad (2.70)$$

$$\tilde{\phi}_\theta(\xi,z,p) = -\xi\{E_1e^{q_1z} + E_5e^{-q_1z} + E_2e^{q_2z} + E_6e^{-q_2z} + E_3e^{q_3z} + E_7e^{-q_3z} + E_4e^{q_4z} + E_8e^{-q_4z}\}, \quad (2.71)$$

$$\tilde{q}^*(\xi,z,p) = c_1E_1e^{q_1z} + c_1E_5e^{-q_1z} + c_2E_2e^{q_2z} + c_2E_6e^{-q_2z} + c_3E_3e^{q_3z} + c_3E_7e^{-q_3z} + c_4E_4e^{q_4z} + c_4E_8e^{-q_4z}, \quad (2.72)$$

Substituting the values of $\tilde{u}_r, \tilde{u}_z, \tilde{\phi}_\theta$ *and* $\tilde{q}^*$ from equations (2.69)-(2.72) in equations (2.43)-(2.45), we obtain the tangential force stress, normal force stress and tangential couple stress as

$$\tilde{t}_{zr}(\xi,z,p) = (a_1s_{11}q_1^2 - \xi b_1 + \xi s_{12})E_1e^{q_1z} + (a_1s_{11}q_1^2 - \xi b_1 + \xi s_{12})E_5e^{-q_1z} + (a_2s_{11}q_2^2 - \xi b_2 + \xi s_{12})E_2e^{q_2z} + (a_2s_{11}q_2^2 - \xi b_2 + \xi s_{12})E_6e^{-q_2z} + (a_3s_{11}q_3^2 - \xi b_3 + \xi s_{12})E_3e^{q_3z} + (a_3s_{11}q_3^2 - \xi b_3 + \xi s_{12})E_7e^{-q_3z} + (a_4s_{11}q_4^2 - \xi b_4 + \xi s_{12})E_4e^{q_4z} + (a_4s_{11}q_4^2 - \xi b_4 + \xi s_{12})E_8e^{-q_4z}, \quad (2.73)$$

$$\tilde{t}_{zz}(\xi,z,p) = (c_1s_{13} - \xi a_1q_1s_{14} - b_1q_1s_{15})E_5e^{-q_1z} + (c_1s_{13} + \xi a_1q_1s_{14} + b_1q_1s_{15})E_1e^{q_1z} + (c_2s_{13} - \xi a_2q_2s_{14} - b_2q_2s_{15})E_6e^{-q_2z} + (c_2s_{13} + \xi a_2q_2s_{14} + b_2q_2s_{15})E_2e^{q_2z} + (c_3s_{13} - \xi a_3q_3s_{14} - b_3q_3s_{15})E_7e^{-q_3z} + (c_3s_{13} + \xi a_3q_3s_{14} + b_3q_3s_{15})E_3e^{q_3z} + (c_4s_{13} - \xi a_4q_4s_{14} - b_4q_4s_{15})E_8e^{-q_4z} + (c_4s_{13} + \xi a_4q_4s_{14} + b_4q_4s_{15})E_4e^{q_4z}, \quad (2.74)$$

$$\tilde{m}_{z\theta} = -\xi\{q_1E_1e^{q_1z} - q_1E_5e^{-q_1z} + q_2E_2e^{q_2z} - q_2E_6e^{-q_2z} + q_3E_3e^{q_3z} - q_3E_7e^{-q_3z} + q_4E_4e^{q_4z} - q_4E_8e^{-q_4z}\}, \quad (2.75)$$

## NUMERICAL RESULTS AND DISCUSSION

Following Eringen (1984), we take the following values of the relevant parameters for the case of Magnesium crystal as

$$\lambda = 9.4 \times 10^{11} dyne/cm^2, \mu = 4 \times 10^{11} dyne/cm^2$$

$$K = 1 \times 10^{11} dyne/cm^2, \rho = 1.74 \times \frac{gm}{cm^2}, \gamma = 0.779 \times 10^{-4} dyne$$

$$j = 0.2 \times 10^{-15} cm^2$$

And the void parameters are

$$a^* = 3.688 \times 10^{-4} dyne, b^* = 1.13849 \times 10^{11} \frac{dyne^2}{cm},$$

$$\zeta^* = 1.475 \times 10^{11} \frac{dyne}{cm^2}, \omega^* = 0.0787 dyne \frac{sec}{cm^2}, K^* = 1.753 \times 10^{-15} cm^2,$$

The variation of non-dimensional normal displacement, non-dimensional normal force stress, non-dimensional tangential couple stress and volume fraction with distance 'r' may be shown graphically.

## CONCLUSION

The goal of this paper is to find a general solution of the equation of motion of a micropolar elastic medium with voids, in cylindrical polar co-ordinate system to the axisymmetric deformation problem. The solution is obtained through eigen value approach and the Laplace and Hankel transform are used to solve the equations. The expression of displacement, microrotation, volume fraction components in the transformed domain, tangential force stress, normal force stress and tangential couple stress are obtained.

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