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Finsler Geometry & Cosmological constants

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ABSTRACT

It has been noticed that Finsler Geometry fails to solve the problem of geometrization of the Maxwell Electrodynamics in 4- dimension despite the fact that the Finslerian metric tensor is a function of the local state of the physical system. The interesting example is obviously non-physical Finslerian conclusion about the Lagrangian of the mass point, it may be +1 degree of homogeneity function of the mass point velocity. Here in the present communications we are agree with the results published by Synge, V.wagner, L.Berwald, E.cartan, H. Bssemann, and H.Rund which indicate the deep connection of Riemannian Geometry with Finsler Geometry although their approaches are different but with the help of Finslerian metric tensor we may suggest the slight modification in cosmological term as discussed by us along with other peer researchers.

Key Words : Cosmology, Cosmological constant, Finsler Geometry Classification:083,085.

INTRODUCTION

As we know that Finsler Geometry is a theory of spaces where lengths are measured in small steps and the scale of measurement depends on the point of the space and the choice of direction at the point. While that the infinitesimal distance ds between the adjacent points, whose coordinates in any system are x^i and $x^i + dx^i$ (i= 1,2,3) is connected by the relation $ds^2 = g_{ij}\ dx^i\ dx^j$ (i, j = 1,2,3,....n). Where the coefficients g_{ij} are function of the coordinates x^i. The quadratic differential form in the second member of above relation is called as Riemannian metric and geometry based upon a Riemannian metric is called Riemannian geometry. To investigate the connection of Relativity & Finsler geometry first of all we have to address the problem of Relativity of the Finsler Geometry as suggested by *Synge [1]*. It has been noticed that Finsler Geometry fails to solve the problem of geometrization of the Maxwell Electrodynamics in 4-dimension despite the fact that the Finslerian metric tensor is a function of the local state of the physical system. The interesting example is obviously non- physical Finslerian conclusion about the Lagrangian of the mass point, it may be +1 degree of homogeneity function of the mass point velocity. Here we are agree with the definition that Finsler Geometry is a geometry of metric spaces possessing an intrinsic local space whose matrices do not reduce to a quartic form in the differentials of the coordinates. But after certain investigation we can say that the results published by Synge, V.wagner, L.Berwald, E.cartan, H. Bssemann, and H.Rund indicate the deep connection of Riemannian Geometry with Finsler Geometry although their approaches are different but with the help of Finslerian metric tensor we may suggest the slight modification in the Cosmological term. This modified cosmological term may help to discuss the numerous cosmological consequences with variable cosmological and gravitational constants as published by Rahman, Berman, Pandey, Chandra & Mishra, Johari & Chandra [2-10]

Einstein's Field equations with Lagrangian

The fundamental metric tensor which we used in the Einstein's field equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -\left(\frac{8\pi G}{c^4}\right) T_{ij} \qquad (i, j = 1,2,3,4) \tag{1}$$

May be expressed as $g_{ij} = \dot{\partial}_i \dot{\partial}_j \left(\frac{G^2}{2}\right)$ (2)

And the C- tensor $C_{ijk} = \dot{\partial}_i \dot{\partial}_j \dot{\partial}_k \left(\frac{G^4}{4}\right) of F^n$ (3)

May be written as

$$g_{ij} = (n-1) b_{ij} - (n-2) b_i b_j \ldots\ldots\ldots\ldots \tag{4}$$

$$2GC_{ijk} = (n-1)(n-2)\left[b_{ijk} - b_{ij} b_k - b_{jk} b_i - b_{ki} b_j + 2 b_i b_j b_k\right] \tag{5}$$

Here the metric tensor G is called regular of the basic tensor has the non- vanishing determinant.

& Here the conservation law is same as in General Relativity, we have

$$T^{ij}{}_{;i} = 0 \qquad (i, j = 1,2,3,4) \tag{6}$$

To avoid the creation or vanishing of the energy in the universe. The equation (6) when applied to the field equation will give

$$\frac{8\pi G_{;i}}{c^4} T^{ij} + \Lambda_{;i} g^{ij} = 0 \qquad (i, j = 1,2,3,4) \tag{7}$$

Which governs the variation of G & Λ. The field equations (6) & (7) are not derived from the Lagrangian formulation. If one attempts to derive the field equations via Lagrangian formulation, one has to add the term consists of the first order partial derivative of G and Λ in the Lagrangian density and the resulting field equation contains undetermined constants.

N-DIMENSIONAL EINSTEIN'S EQUATIONS

The metric for an N-dimensional space time with flat $N = n - 1$ dimensional foliation is:

$$ds^2 = g_{ij} dx^i dx^j \tag{8}$$

$$g_{ij} = [\alpha^2(y)\beta_{kl}, \pm 1] \tag{9}$$

where β_{kl} is the n-dimensional metric for flat Euclidian/Minkowski space & y is a space like/time like Gaussian normal coordinate for upper/lower signs so that $ds^2 < 0$ is the time like case.

The Christoffel symbols are given by

$$\Gamma^a_{bi} = \frac{1}{2} g^{ac}(g_{cb,i} + g_{ci,b} - g_{bi,c}) \tag{10}$$

So that using metric in equation we have

$$\frac{\dot{R}}{R} = \frac{\dot{r}}{r}, \frac{k}{R^2} = -2\frac{c_1}{c^2 r^2}, k = 0, \pm 1 \tag{11}$$

We have

$$\left.\begin{array}{lll} \Gamma^n_{nn} = 0, & \Gamma^n_{nk} = 0, & \Gamma^k_{nn} = 0 \\ \Gamma^k_{\ln} = \frac{\alpha'}{\alpha}\beta^k_l & , & \Gamma^k_{lm} = 0 \\ \Gamma^n_{kl} = \pm\alpha'\alpha\beta_{kl} & & \end{array}\right\} \tag{12}$$

And $\Gamma^n_{kl,n} = [\pm\alpha'\alpha \pm \alpha'^2]\beta_{kl}$ (13)

$$\Gamma^k_{\ln, n} = \left[\frac{\alpha''}{\alpha} - \frac{\alpha'^2}{\alpha^2}\right]\beta^k_l \tag{14}$$

Where dash represents derivative w. r. t. 'Y'

Here Riemann curvature tensor is defined by:

$$R^\sigma_{ia\nu} = \Gamma^\sigma_{i\nu,a} - \Gamma^\sigma_{ja,\nu} + \Gamma^\sigma_{ba}\Gamma^b_{ij} - \Gamma^\sigma_{bj}\Gamma^b_{ia} \tag{15}$$

So that

$$R^n_{knl} = N[\pm\alpha'\alpha \pm \alpha'^2]\beta_{kl} \pm N\alpha'^2\beta_{ab} \pm 2\alpha'^2\beta_{kl}$$

(16)

$$R^k_{njn} = -N\left[\frac{\alpha''}{\alpha} - \frac{\alpha'^2}{\alpha^2}\right]\beta^k_l - N\left(\frac{\alpha'}{\alpha}\right)^2\beta^k_l = -N\frac{\alpha''}{\alpha}\beta^k_l \tag{17}$$

The Ricci tensor is the contraction of the curvature tensor on the first and third indices, derived as

$$R_{ij} = R^a_{iaj} = R_{ji} \tag{18}$$

Using these relations we have

$$R_l^k = g^{km} R_{mj} = \overline{\alpha}^{2} \beta^{km} R_{ml} = \left[\pm \left(\frac{\alpha'}{\alpha} \right)' \pm N \left(\frac{\alpha'}{\alpha} \right)^2 \right] \delta_l^k \tag{19}$$

$$R_n^n = R_{nn} g^{nn} = \pm N \frac{\alpha''}{\alpha} \tag{20}$$

The Ricci scalar is defined as:

$$R = g^{ij} R_{ij} \tag{21}$$

Using above relation we have

$$R = \pm 2N \left(\frac{\alpha'}{\alpha} \right)' \pm N(N+1) \left(\frac{\alpha'}{\alpha} \right)^2 \tag{22}$$

Since the Einstein tensor is defined as:

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R \tag{23}$$

So, $$G_l^k = R_l^k - \frac{1}{2} g_l^k R \tag{24}$$

Due to symmetry,

$$G_1^1 = G_2^2 = - - - - - - - - = G_n^n \tag{25}$$

The energy momentum tensor for a scalar field ϕ and potential $V(\phi)$ is given by

$$T_j^i = \nabla^i \phi \nabla_j \phi - \frac{1}{2} [\nabla^p \phi \nabla_p \phi + 2V] \delta_j^i \tag{26}$$

So that for $\phi = \phi(y)$,

$$T_l^k = -\frac{1}{2} [\pm \phi'^2 + 2V] \delta_l^k \tag{27}$$

RESULTS AND DISCUSSION

(a)The hyper surfaces with constant cosmic time are maximally symmetric subspaces of the whole space-time.

(b)-Not only the metric g_{ij} but all the cosmic tensors such as T_{ij} are form in variant with respect to isometrics of the subspaces. With the help of the result discussed above.

we may also conclude that the coordinate transformation must leave the metric $ds^2 = g_{ij}\ dx^i\ dx^j$ (i, j = 1,2,3,....n) form invariant. Therefore the coordinate transformations are purely the spatial coordinate's transformation as: $t = t'$, $x^i = x^{i'}(x^i)$(i = 1, 2, 3) In fact the spatial coordinate's transformation we may take the origin $x^i = 0$ into another point of the space.

c) An important conclusion of the above equation (7) is that the cosmological constant 'Λ' depends upon '*G*' as well as energy momentum contained in the universe. Contracting the equation (7) we will obtain

$$\Lambda_{;\,l} = -\frac{8\pi}{c^4} G_{ij} T_l^{\,j} \quad , \qquad (i\,,\,l = 1,2,3,4)$$

Putting the value of $T_l^{\,j}$ from (6), we get Under different assumptions as published in the papers [4-9] the authors have already constructed suitable cosmological models with different assumptions of variable cosmological constants. According to the assumptions flat model of the universe is possible for different positive values of n and open model & closed models are possible only for $n = 1$. we have also calculated the numerical values of constants constants from the observation and estimated data.

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