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Estimation of apparent fraction defective: A mathematical approach

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ABSTRACT

In this article, an attempt was made to estimate the apparent fraction defective when the inspection risks are unknown using Beta distribution of first kind truncated at point b. This study uses to identify the performance of sampling plans like single sampling plan and Double sampling plan.

Key words: Beta distribution, Truncation, Acceptance Quality Level, Gamma distribution.

INTRODUCTION

Acceptance Inspection is a part of Quality Assurance through which Product Control is exercised. This is in contrast with Process Control in which Control charts play the dominant role to ensure a state of statistical control of the process. Acceptance Inspection is a necessary part of a manufacturing system and may be applied to incoming materials, final products and to the semi items in a production line. The word sampling inspection is used when the quality of the product is evaluated by sampling rather than 100% inspection (Guenther W.C [1977]).

Sampling plans uses a random sample as the basis for assessing the quality of a finite population of units called a lot(H.F.Dodge[1943]). The supplier of the lot is generally called the producer and the buyer is called the consumer. Acceptance Sampling is a statistical procedure that specifies a rule to accept or reject a lot, based on the quality observed in the sample drawn from that lot. That is why it is called lot sentencing procedure. A sampling plan is thus a set of rules to execute acceptance inspection. Several basics ideas of Acceptance sampling can be found in Montgomery (1997).

PRELIMINERIES

If the decision about accepting or rejecting a lot is taken on the basis of only one sample drawn from the lot, it is called a single sampling plan. This plan is based on the following technical terms.

a. Acceptable Quality Level (AQL) : This is the proportion of defectives with which a lot can be accepted. It is based on the observation that in spite of all the efforts made to avoid non – conformities, certain defectives occur in the lots and the consumer also agrees to accept such lots. It is usually expressed as a percent like 1 % or 0.5% defectives being admitted. It is conventionally denoted by $p_{1.}$

b. Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD) :
This is the worst – case fraction defective at which the consumer can accept the lot. If the observed fraction defective toucher LTPD the lot is rejected. It is denoted by p_2 and takes values higher than AQL.

C. Producer's Risk :
Since the decision on the lot is based on random sample, there is every possibility that one sample may show a higher number defectives than another sample drawn from the same lot. The Producer, after inspection may reject a lot even though the lot really does not warrant rejection ! This is called Type-1 error and the probability of committing such an error is known as producer's risk. This is denoted by α and given by the **conditional probability** $P(X \leq c / P \leq AQL)$.

d. Consumer's Risk :
It is the probability of accepting a lot, based on sample, given that the lot truly contains LTPD. This error, known as Type-II error, occurs becausethe sample might some times fail to reflect the real quality of the lot. The risk of committing this error is known as consumer's risk, denoted by β and given by the conditional probability $P(X \leq c / P \geq LTPD)$.

NEW APPROACH

While defining the apparent fraction defective, it is assumed that the type-I and type-II inspection risks are known and fixed(SK.Khadar Babu 2007). In practice, variation in the values of φ and ε occur due to several uncontrollable factors. When the inspector changes the gage or inspector is changed from the testing station or the operating environment gets disturbed, it is possible that the inspection risks are dragged to one of the extremes say 0 or 1. In other words φ and ε my come closer 0 or 1. It is therefore, reasonable to describe the inspection risk as a continuous random variable Y, $0 \leq Y \leq 1$.We can use either Uniform distribution in [0,1] or Beta distribution of type – 1 to describe that behavior of Y.

In the following section we use type 1 Beta distribution and examine its properties to describe the uncertainty in φ and ε.

3.1 Beta Distribution and its properties

A continuous random variable Y is said to have a Beta distribution of type 1 with parameter (m,n) if its probability density function (pdf) is given by

$$f(y) = \begin{cases} \frac{1}{\beta(m,n)} y^{m-1}(1-y)^{n-1}, 0 \le y \le 1 \end{cases}$$

Where $\beta(m,n) = \int_{0+}^{1-} y^{m-1}(1-y)^{n-1} dx.$ The value of $\beta(m,n)$, for m,n positive integers is given by

$$\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$$

Since $\Gamma(n) = (n-1)!, we$ get the relationship

$$\beta(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$$

The distribution function of Y is given by G(y) = p(Y< y)

$$G(y) = \begin{cases} \frac{1}{\beta(m,n)} \longrightarrow \int_{0+}^{y} y^{m-1} (1-y)^{n-1}, dx, 0 \le y \le 1 \end{cases}$$

The r^{th} moment of Y about origin "o" can be shown to be equal to

$$E(y^r) = \frac{\Gamma(r+m).\Gamma(m+n)}{\Gamma(r+m+n).\Gamma(m)}$$

Hence $E(y) = \dfrac{m}{m+n}$ and

$$V(y) = \frac{mn}{(m+n)^2(m+n+1)}$$

If Y Follows $\beta(m,n)$*then*$(1-y)$ *follows* $\beta(n,m)$,*When* $m = n = 1$,*thedistribution*
Y will be uniform on$[0,1]$.

3.2 The distribution of β (2,2) to describe φ and ε

One of the particulars cases of Beta distribution of first kind β (2,2) in general is given by
$f(y) = 6y(1-Y), o \le y \le 1$

For this distribution E(y) = ½ and V(Y) = 7/20. Clearly E(Y) > V(y)

This distribution can be used as a model to explain φ and ε.

If we assume that each one of the inspection risks follow β (2,2) distribution then it follows that

$$E(\varphi) = \frac{1}{2} \, and \, V(\varphi) = \frac{7}{20}$$

and $E(\varepsilon) = \frac{1}{2} \, and \, V(\varepsilon) = \frac{7}{20}$

Using these values the expected apparent fraction defective denoted by Π_e is given by

$$\pi_e = E(\pi) = p(1 - E(\varphi)) + (1 - p)\, E(\varepsilon)$$

$$= p\left(1 - \frac{1}{2}\right) + (1 - p)\, 1/2 = \frac{1}{2}$$

This is an expected result in which the apparent fractoon defective is found to be independent of the incoming lot quality. E $(\varepsilon) = \frac{1}{2}$ implies that inspector is indifferent in classifying the item as good or bad. Similar is the case with E (φ) and these two expected risks create the highest uncertainty in decision making.

3.3. Estimating φ using β (2,2) distribution truncated at b

With regard to the inspection risks, it is reasonable to assume that either of the risks of misclassification is not more than 0.1 or 0.2. These values correspond to 1% and 2% risks of misclassification. It is also possible that due to fatigue or monotony of inspection of the misclassification risks, some times happen to be on the higher side, starting with minimum of 0.5 or 0.6. This is only a theoretical possibility but a good system expects both the risk to be very small.

a. Upper truncated β (2,2) distribution :

We denote this distribution by β (2,2) and the pdf is given by

$$f_u(y) = \{12y(1-y),\ 0 < y < 0.5$$

For this distribution $E_u(y) = \frac{5}{16}$ and $V_u(y) = \frac{67}{1280}$

b. Lower truncated β (2,2) distribution

For the lower truncated β (2,2) distribution we get

$$f_L(y) = \{12y(1-y),\ 0.5 < y < 1$$

For this distribution $V_L(y) = \dfrac{19}{1280}$

In the following discussion we examine this Beta(2,2) distribution and study their effect on the apparent fraction defective as well as on the properties of the single sampling paln.

When both φ and ε are truncated on the upper side at b<1, the apparent fraction defective can be worked out in a closed form. Consider the following.

$$\pi_e = \frac{p(6-12b+6b^2)+(4b-3b^2)}{(6-4b)}$$

3.4 Apparent fraction defective when φ and ε are at their expected values.

In this section we determine $E(\pi)$ under three conditions.

1. φ Follows Upper truncated β (2,2,0.5) & ε Follows lower truncated β (2,2,0.5)

2.. φ Follows lower truncated β (2,2,0.5) & ε Follows upper truncated β (2,2,0.5).
3. Both are truncated in one direction (Lower)
4. Both are truncated in one direction (Upper)

Substituting the values of E(φ) and E(ε)

$\pi_e = p(1-E(\varphi))+(1-p)E(\varepsilon)$ *and simplifying we get the Following possible values.*

Table: Types of inspection errors

Case	**Type of truncation for**		**Value of π_e**
	φ	ε	
I	Lower	Lower	$\frac{5+6p}{16}$
II	Lower	Upper	$\frac{11}{16}$
III	Upper	Lower	$\frac{5}{16}$
IV	Upper	Upper	$\frac{11-6p}{16}$

CONCLUSION

We observe the following results from the value of for π_e

a. When φ and ε are based on the different types of truncation the expected fraction defective π_e becomes independent of the incoming lot fraction defective. (Case – II and Case – III).

b. when the type of truncation is changed from lower to upper the resulting becomes complementary to the previous combination. This is true between Case – II and Case – III and also between Case – I and Case- IV.

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