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Entropy generation in the Poiseuille flow of a Temperature dependent viscosity fluid through a channel with a naturally permeable wall under thermal radiation

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ABSTRACT

In this study, we investigated the effects of thermal radiation on entropy generation in a temperature dependent viscosity fluid flow in a channel with a naturally permeable wall of very small permeability. Numerical solutions are presented for the isothermal and convective boundary conditions and the effects of various pertinent parameters are examined on the velocity field, temperature field, entropy generation number, and Bejan number through graphs and discussed.

Keywords: Entropy generation, Radiation, Poiseuille flow, naturally permeable, temperature dependent viscosity.

INTRODUCTION

The study of pressure-driven or shear driven flows through channels are important because of many applications in science and technology. There are numerous geophysical and industrial applications of the study of flow of a viscous fluid flows overlying a porous medium with associated heat transfer effects. Several such applications have been discussed by Nield and Bejan [1], and many others. When a viscous fluid in a channel bounded below by a porous medium, the no-slip condition at its surface is not valid, since there exists effectively a slip velocity. Instability of Poiseuille flow is examined by Chang et al.[2] in a fluid overlying fluid saturated porous medium layer Hill and Straughan [3], investigated the instability of Poiseuille flow numerically, in a viscous fluid overlying a same fluid-saturated porous material in a channel. The upper wall of the channel is impermeable while the lower one is composed of a Brinkman-type porous material layer and a layer of porous material of Darcy-type. Earlier, Beavers and Jospeh [4], Saffamn [5], Ochoa-Tapia and Whitaker [6, 7] and James and Davis [8] have investigated and discussed in detail matching conditions at the fluid-porous medium interface.

Numerous investigations have been conducted of viscous fluid flow and heat transfer in porous medium-wall bounded channels or coupled-flow in domains partially filled with a porous medium due to their recent technological implications in various configurations of engineering interest. Chauhan and Shekhawat [9] examined Couette compressible fluid flow in a channel bounded below by a porous medium of very small permeability. In a similar geometry Chauhan and Vyas [10] studied heat transfer effects in MHD Couette compressible fluid flow. Analytical investigation was presented by Kuznetsov [11] of fluid flow in the porous interface region in a channel partially filled with a porous medium. Kuznetsov [12,13] also investigated analytically Couette flow and heat transfer effects in a composite channel partially filled with a porous material . Alkam et al. [14] examined forced convection in channels partially filled with porous material. Al-Nimir and Khadrawi [15] studied Transient natural convection in domains partially filled with porous material. Umavathi et al. [16] examined heat transfer in a generalized Couette flow in a composite channel. Aguilar-Madera et al. [17] investigated convective heat transfer in a parallel-plate channel partially filled with a porous medium, where the forced convection is caused due to a pressure drop in the horizontal axis direction. They presented the modelling of flow and heat transfer in the channel using a one domain approach. Chauhan and Agrawal [18] examined MHD Couette flow with hall current effects in a partially porous material filled channel in a rotating system. Vyas and Srivastava [19] investigated generalized MHD Couette flow in a composite channel with entropy generation .Chauhan and Olkha [20] studied slip effects on wall driven flow of a non-Newtonian fluid in a channel bounded by a stretching sheet and a porous medium bed by employing homotopy perturbation method.

Most of the investigations dealing with heat transfer effects in various flow configurations considered heat analysis based on the First Law of thermodynamics, which expresses the energy conservation principle. In analysing the complete energy transfer process, this law is inadequate. Experimental studies indicated that when heat energy is transferred to a system, only a part of it is converted to work, which is useful. Thus useful energy associated to work must have low entropy. Bejan [21] studied entropy generation in thermal systems and investigated the importance of entropy minimization in improving their performance. The study of entropy generation, thus, is useful in the design of heat exchangers, and other devices. Therefore many researchers carried out studies in this field, e.g. Morosuk[22], Mahmud and Fraser[23] , Hooman and Gurgenci[24], Damesh et al.[25], Chauhan and Kumar[26], Makinde and Aziz[27].Radiation effects on fluid flows are important in context of thermal energy storage solar power technology, environmental , astrophysical and space technology processes involving high temperatures. The study of these effects are considered by many authors in their research works, e.g. Smith[28] , Whitaker[29],Lai and Kulacki[30] ,Chamkha[31], Raptis[32], Hossian et al.[33] , Bakier[34], Cortell[35], Shit and Haldar[36], Pantokratoras and Fang[37], Chauhan et al.[38]. In thermal-flow systems it is intended to utilize optimally the energy resources and avoid energy losses. If thermal radiation is appreciable, it also affects the entropy production in thermal flow system. Arpaci [39] investigated heat lost into entropy production under thermal radiation effects. Chen et al.[40], Makinde[41], Bull et al. [42] studied effects of thermal radiation on entropy generation due to flow along wavy or flat plate.

The aim of this research is to investigate the entropy generation in Poiseuille flow of a fluid overlying a porous medium under thermal radiation. In this study, a flow of temperature-dependent viscosity fluid in a channel bounded below by a naturally permeable bed of very small permeability is considered. Flow of fluid in the channel is driven by a constant pressure gradient applied at the mouth of the channel. Viscous dissipation and radiation effects are also considered, and Rosseland approximations for radiative heat transfer are assumed to be valid. Governing momentum and energy equations are solved numerically. The effects of various pertinent parameters are examined on the velocity field, thermal field, entropy generation number and Bejan number, and discussed graphically.

1. Formulation of the problem

We consider the flow of fluid with temperature dependent viscosity through a horizontal parallel channel of width $'d'$. The upper wall of the channel is impermeable, while the lower wall is a naturally permeable medium of very small permeability and saturated with fluid at constant temperature $'T_0'$.

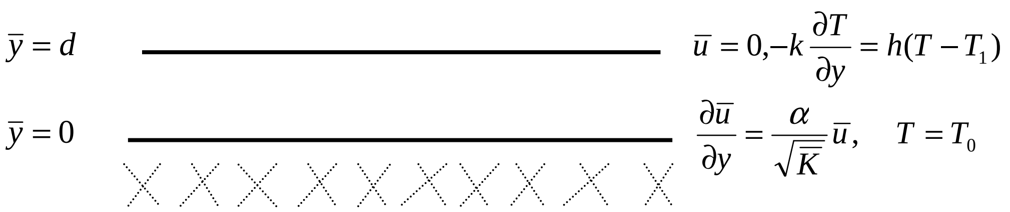

Figure1 Schematic diagram

The flow in the bounding porous medium wall is modelled by the Darcy's equation therefore in the absence of any external pressure gradient, the filter velocity in the porous matrix of very small permeability is assumed to be zero. The effect of the porous matrix is, thus to introduce a velocity slip at the lower bounding wall of the channel and its permeability affects flow in the channel through Saffman slip boundary condition [5].The flow in the parallel wall channel is driven by a constant pressure gradient applied at the mouth of the channel. The upper impermeable channel wall is assumed to have a negligible thickness and its upper face is in contact with another fluid at temperature $'T_1'$. The upper wall is thus heated by convection from external hot fluid which provides a convection heat transfer coefficient $'h'$. Further it is assumed that property variations of the viscous fluid in the channel because of temperature are limited to viscosity only, which is assumed to vary as an inverse linear function of temperature, following Lai and Kulacki [30], as follows:

$$\mu(T) = \frac{\mu_0}{1 + \lambda(T - T_0)} \tag{1}$$

where μ_0, is the viscosity of the fluid at the temperature T_0 ; and λ is the viscosity variation parameter . The governing equations for the present problem are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}} = 0 \tag{2}$$

$$\frac{\partial}{\partial \bar{y}}\left(\mu(T)\frac{\partial \bar{u}}{\partial \bar{y}}\right) - \frac{\partial \bar{p}}{\partial \bar{x}} = 0 \tag{3}$$

$$\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial \bar{y}^2}\right) + \frac{\mu(T)}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} = 0 \tag{4}$$

The corresponding boundary conditions are given by

$$at \;\; \bar{y} = 0, \quad \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\alpha}{\sqrt{\bar{K}}}\bar{u} \;, \; T = T_0$$

$$at \;\; \bar{y} = d, \;\; \bar{u} = 0, \; -k\frac{\partial T}{\partial \bar{y}} = h(T - T_1) \tag{5}$$

Where,

$\bar{u}$ is the velocity in the x -direction ; T is the temperature; ρ , the density; C_p , the specific heat at the constant pressure; k , the thermal conductivity ; $\frac{-\partial \bar{p}}{\partial \bar{x}}$, the pressure gradient; and $\mu(T)$, the temperature dependent viscosity of the fluid; $\bar{K}$, the permeability of the porous medium ; and α , the dimensionless constant depending on the local geometry of interstices of the porous matrix.

In this study ,the Rosseland diffusion flux model is taken to simulate radiative heat transfer which is suitable for an optical thick fluid and gray , absorbing-emitting , but non scattering medium. Following Siegel and Howell [43] , it takes the form :

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T^4}{\partial \bar{y}} \tag{6}$$

Where, σ , the Stefan –Boltzmann constant; and k_1 , the mean absorption coefficient.

The term T^4 can be expanded for small temperature differences in a Taylor series about T_0 as follows $T^4 \approx T_0^4 + (T - T_{0)}4T_0^3 = 4T_0^3 T - 3T_0^4$, and neglecting higher terms:

Thus, we have

$$q_r = -\frac{16\sigma T_0^3}{3k_1}\frac{\partial T}{\partial \bar{y}} \tag{7}$$

Let us introduce the following non-dimensional quantities:

$$u = \frac{\bar{u}}{U}, x = \frac{\bar{x}}{d}, y = \frac{\bar{y}}{d}, \; \theta = \frac{T - T_0}{T_1 - T_0}, P = \frac{d^2}{\mu_0 U}\left(-\frac{\partial \bar{p}}{\partial \bar{x}}\right), \; K = \frac{\bar{K}}{d^2}, \; a = \lambda(T - T_0) \tag{8}$$

Substituting (1),(7) and above non-dimensional quantities (8) in equations (2)-(5), we obtain

$$\frac{du}{dx} = 0 \tag{9}$$

$$\frac{d}{dy}\left(\frac{1}{(1+a\theta)}\frac{du}{dy}\right)=-P \tag{10}$$

$$\frac{d^2\theta}{dy^2}+\left(\frac{3N_R}{4+3N_R}\right)\frac{Br}{(1+a\theta)}\left(\frac{du}{dy}\right)^2=0 \tag{11}$$

And the boundary conditions are given by

$$at \quad y=0, \frac{du}{dy}=\frac{\alpha}{\sqrt{K}}u, \qquad \theta=0$$

$$at \quad y=1, u=0, \frac{d\theta}{dy}=-Bi(\theta-1) \tag{12}$$

Where, P is the non dimensional axial pressure gradient; U, the flow characteristic velocity; and a, the variable viscosity parameter. For constant viscosity case we have, $a=0$; and for $a>0$, the viscosity of the fluid decreases with rise in temperature .

Here,

$N_R=\dfrac{kk_1}{4\sigma T_0^2}$, the Stark number;

$Br=\dfrac{\mu_0 U^2}{k(T_1-T_0)}$, the Brinkman number;

$Bi=\dfrac{hd}{k}$, the Biot number;

$K=\dfrac{\overline{K}}{d^2}$, the Permeability parameter.

2. Numerical method of Solution

Let us introduce the following new variables

$$u=x_1, \frac{du}{dy}=x_2, \; \theta=x_3, \; \frac{d\theta}{dy}=x_4 \tag{13}$$

Using the above new variables (13) the BVP consisting of the set of non-linear differential equations (10) and (11) with the boundary conditions (12), is reduced to a system of first order differential equations as follows:

$$x_1'=x_2,$$

$$x_2'=-P(1+a\theta)+\left(\frac{1}{1+a\theta}\right)x_2x_4,$$

$$x_3'=x_4,$$

$$x_4'=-\left(\frac{3N_r}{3N_r+4}\right)Br\left(\frac{1}{1+a\theta}\right)x_2^2 \tag{14}$$

and the corresponding boundary conditions are given by

$$at \; y=0, \qquad x_2=\left(\frac{\alpha}{\sqrt{K}}\right)x_1, \qquad x_3=0$$

$$and \; at \quad y=0, \qquad x_1=0, \quad x_4=-Bi(x_3-1) \tag{15}$$

The set of equations (14) are solved by using MATLAB solver bvp4c with the boundary conditions (15). The solution is obtained up to the desired accuracy 10^{-6} .

3. Entropy Generation

In a thermal-flow system, entropy generation is finite positive provided in the medium temperature/ velocity gradients are present. Following Bejan [43, 21] and Arpaci [39], the local volumetric rate of entropy generation in the presence of appreciable radiation, is given by

$$S''' = \frac{k}{T'^2}\left[\left(\frac{\partial T}{\partial \bar{y}}\right)^2 + \frac{16\sigma T_0^3}{3kk_1}\left(\frac{\partial T}{\partial \bar{y}}\right)^2\right] + \frac{\mu}{T'}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{16}$$

where T', is a reference temperature.

The dimensionless entropy generation number (NS) is defined as

$$NS = \frac{S'''}{S_0'''}, \tag{17}$$

where $S_0''' = \dfrac{k\Delta T^2}{T'^2 d^2}$ is the characteristic entropy generation rate; and $\Delta T = T_1 - T_0$

Using equations (18),(19) and non dimensionless quantities (8), to obtain dimensionless entropy generation number as follows:

$$NS = \left(\frac{4+3N_R}{3N_R}\right)\left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega(1+a\theta)}\left(\frac{du}{dy}\right)^2, \tag{18}$$

$$NS = NS_1 + NS_2 \tag{19}$$

Where $\Omega = \dfrac{\Delta T}{T'}$, the dimensionless temperature difference; NS_1 is the dimensionless entropy generation due to heat transfer in the presence of radiation; and NS_2, the dimensionless entropy generation due to fluid friction.

Another important irreversibility parameter is the Bejan number (Be) which is defined as

$$Be = \frac{NS_1}{NS} = \frac{Entropy\ \ generation\ \ due\ to\ heat\ transfer}{Total\ Entropy\ generation} \tag{20}$$

DISCUSSION

In this paper, a second law analysis of a temperature dependent viscosity fluid Poiseuille flow is investigated in the presence of thermal radiation. A pressure –driven flow through a channel with a naturally permeable wall of very small permeability is considered, and the effects of various parameters, such as, the Permeability parameter K, the variable viscosity parameter a, the stark number N_R, the Brinkman number Br, the Biot number Bi, and the pressure gradient parameter P, are examined and discussed.

The results on fluid flow are depicted graphically in figures 2-4 for various values of the parameters. It is found that the effect of the permeability of the lower porous medium wall is to enhance the flow in the channel. There is a velocity slip at the lower wall which increases by the Permeability parameter K of the porous medium. In the limit when $K \to 0$, the porous medium wall becomes impermeable, and the corresponding boundary conditions at it will be of no-slip. We see that for this case $(K = 0)$ the velocity profile is parabolic and the maximum velocity occurs at the centre line of the channel. For $K \neq 0$, there is a slip at the lower porous medium wall, the maximum velocity is not at the centre line of the channel, but shifted a little towards lower wall. The location of peak velocity in the channel shifts towards lower wall as permeability K increases. Further it is seen that as the viscosity variation parameter a increases, which in turn cause a decrease in the fluid viscosity, consequently the flow in the channel increases. It is also observed that an increase in the value of the Brinkman number Br, enhances the velocity of the fluid in the channel due to viscous heating effect.

Figures 5-8 illustrate the temperature profiles against y for various values of the parameters. It is found that with the increase in the permeability K of the lower porous medium wall the temperature in the channel rises since permeability K increases flow in the channel causing more viscous dissipation. The similar effect is observed by increasing the value of the viscosity variation parameter a. Further it is observed that the temperature profile in the absence of viscous dissipation $(Br = 0)$ is linear, which indicates that the process of heat transfer in this case is purely by conduction , and with the increase in the value of Br the temperature rises in the channel. The effect of the Stark number N_R or the Biot number Bi is also to increase the temperature in the channel.

The effects of various parameters on the entropy generation rate NS are displayed in figures 9-15. The effect of the permeability K of the porous medium lower wall is to reduce NS at the lower wall, while NS enhances at the upper wall of the channel by increasing K. Further it is seen that with the increase in the value of the viscosity variation parameter a, the viscosity of the fluid decreases which affects significantly the velocity gradients in the channel causing an increase in the viscous dissipation, consequently increasing the entropy generation rate NS near the channel walls. It is further seen that NS increases as the Brinkman number Br increases. The effects are more pronounced at the channel walls, as expected. However, with the increase in the value of Stark number N_R, the entropy generation rate reduces in the channel at all values of y. Similar effect is observed with the increase in the value of the difference temperature parameter Ω. The effect of the Biot number Bi is to increase NS at all values of y in the channel. Similar results are observed with the increase in the value of the pressure gradient parameter P. The profiles for Bejan number Be are illustrated in the figures 16-22. It is observed that the effect of the permeability K is to increase Be at the lower wall, while Be decreases at the upper wall by K. There is a point in the channel where $Be = 1$ for some value of y. This point where Be attains its maximum value shifts towards lower wall from mid channel with the increase in K. The effect of the viscosity variation parameter a is to reduce the Bejan number Be in the channel. Similarly it is seen that with the increase in Br value, the Bejan number Be decreases in the channel. Be, attains its maximum value one for $Br = 0$ for all values of y in the channel. There is also a point in the middle part of the channel , where Be also attains one for $Br \neq 0$, which shows that at this point in the channel the fluid friction irreversibility is zero for the values of the parameters taken in this graph . Similar results are observed by the pressure gradient parameter P. However, the effect of the Stark number N_R, or the Biot number Bi, or the temperature difference parameter Ω, is to increase the Bejan number Be.

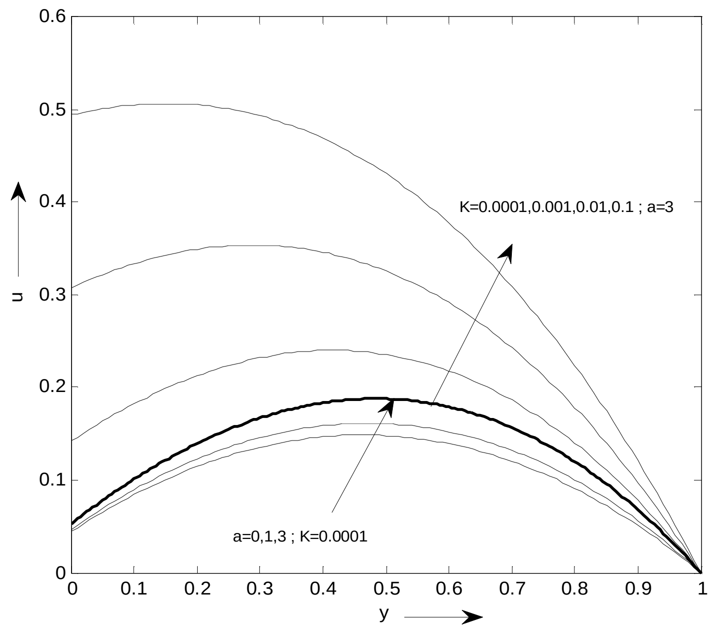

Figure2. Velocity profiles for $Br = 1, N_R = 1, P = 1, \alpha = 0.1, Bi = 0.2$

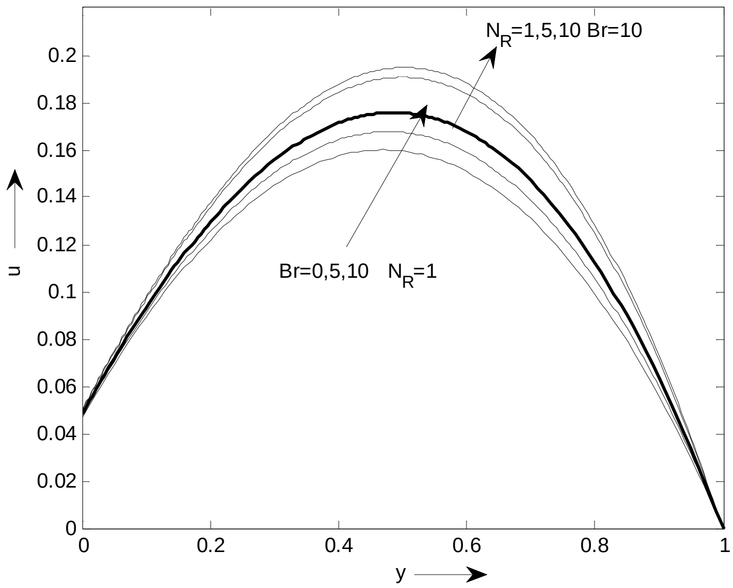

Figure3.Velocity profiles for $Br = 1, a = 1, \alpha = 0.1, K = 0.0001, Bi = 0.2$

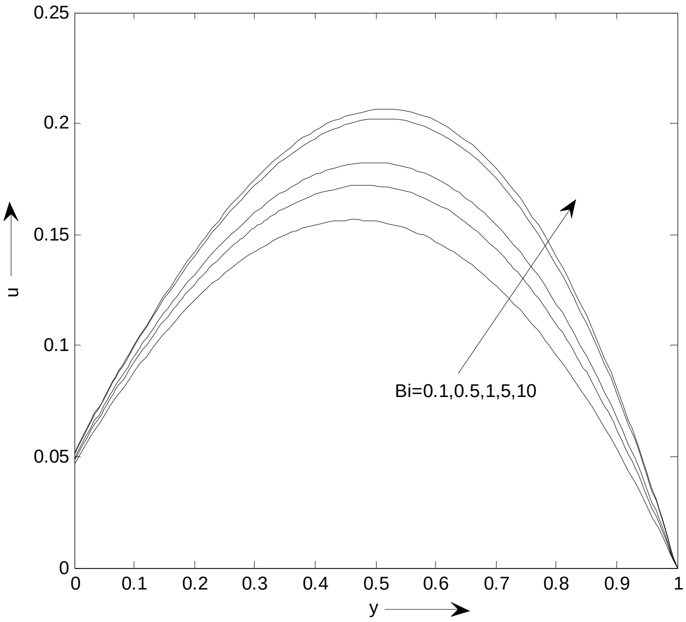

Figure4. Velocity profiles for $Br = 1, N_R = 1, P = 1, a = 1, \alpha = 0.1, K = 0.0001$

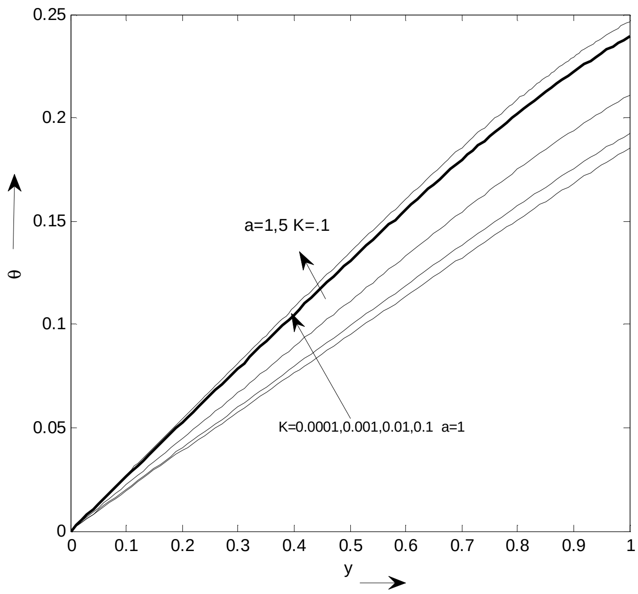

Figure5. Temperature profiles for $P=1, N_R=1, Br=1, \alpha=0.1, Bi=0.2$

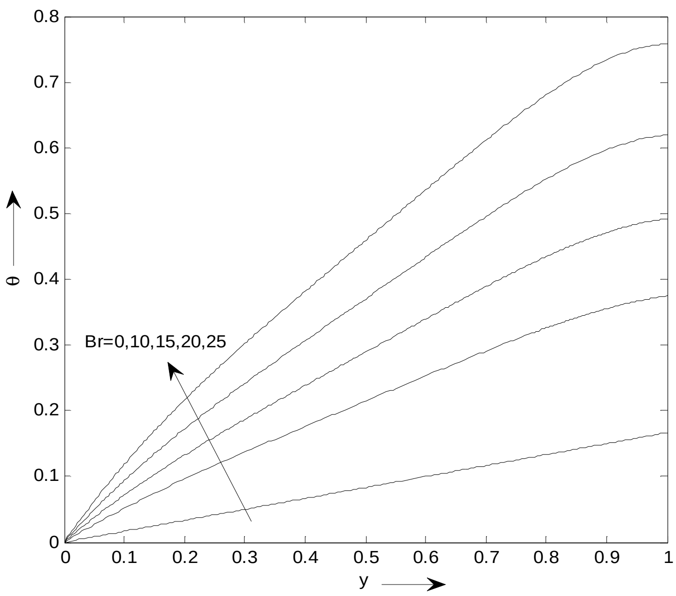

Figure6. Temperature profiles for $N_R=1, P=1, a=1, K=0.0001, \alpha=0.1, Bi=0.2$

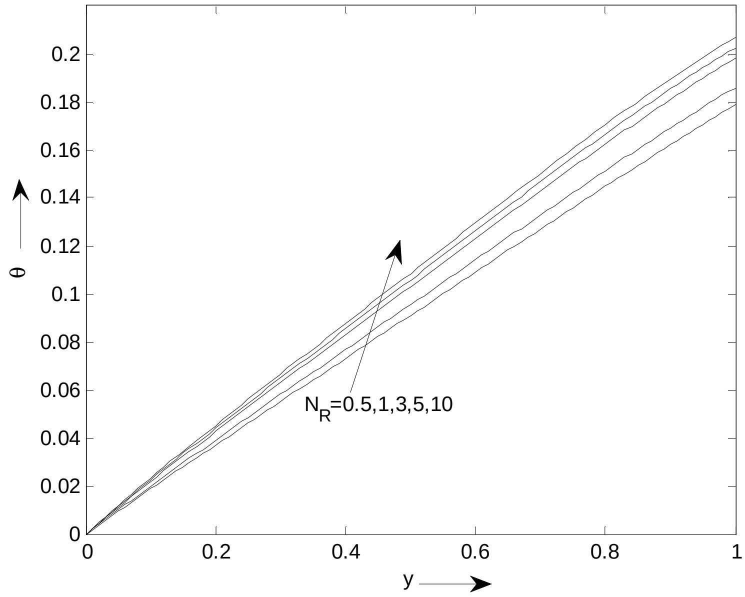

Figure7. Temperature profiles for $Br = 1, P = 1, a = 1, K = 0.0001, \alpha = 0.1, Bi = 0.2$

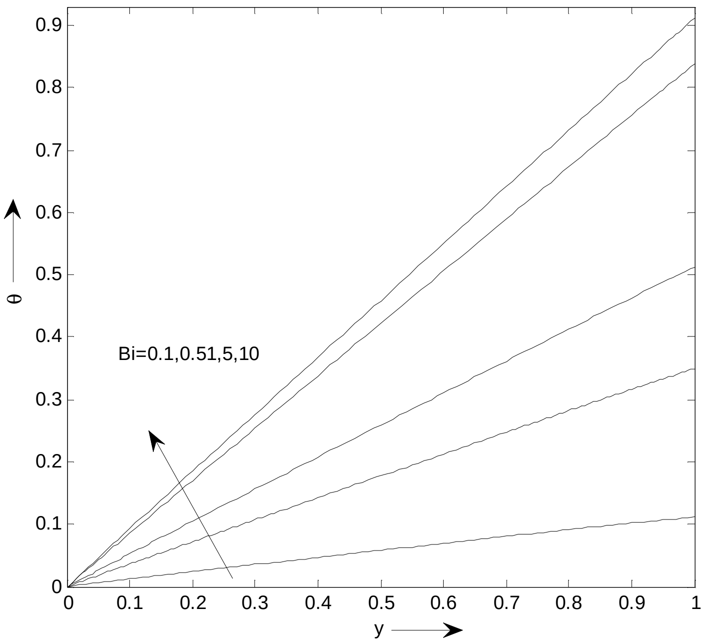

Figure8. Temperature profiles for $Br = 1, N_R = 1, P = 1, a = 1, K = 0.0001, \alpha = 0.1$

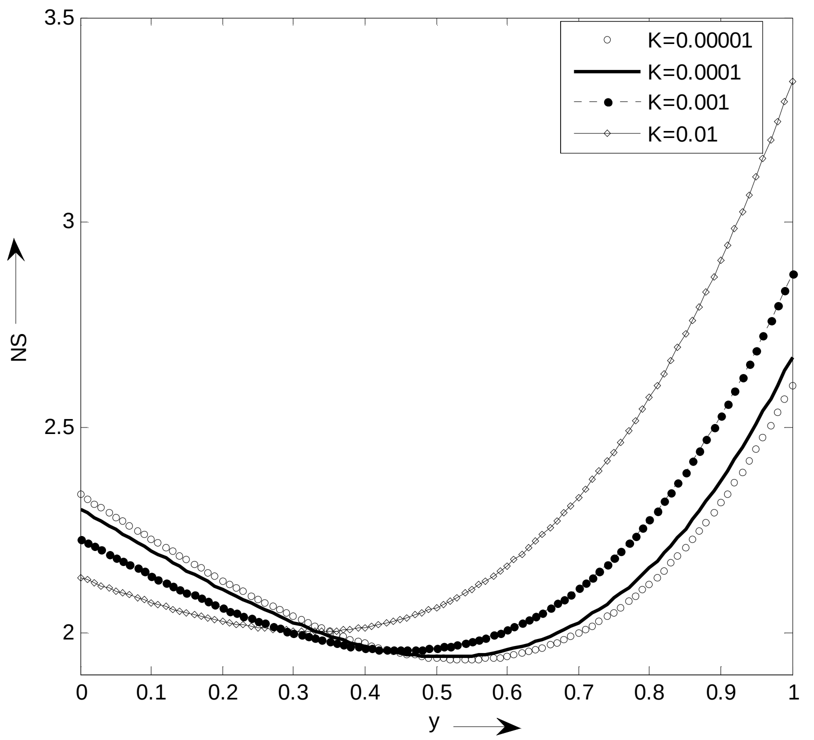

Figure9. Entropy profiles for $Br = 1, N_R = 1, P = 1, a = 1, \alpha = 0.1, Bi = 10, \Omega = 1$

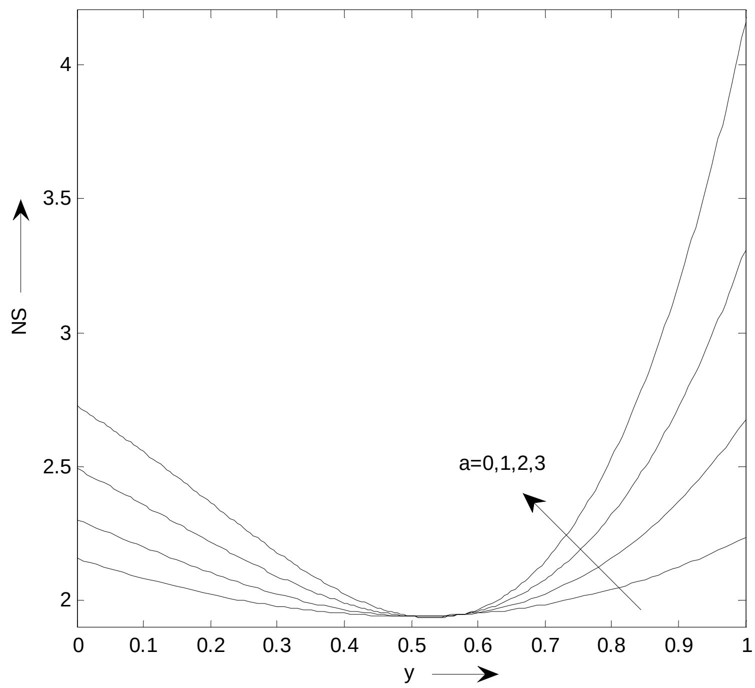

Figure10. Entropy profiles for $Br = 1, N_R = 1, P = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

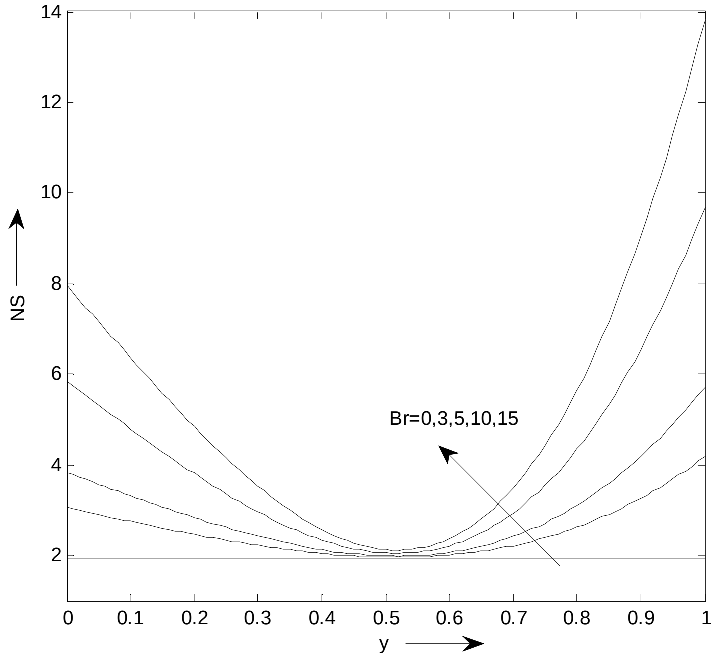

Figure11. Entropy profiles for $N_R = 1, P = 1, a = 1, K = 0.001, \alpha = 0.1, Bi = 10, \Omega = 1$

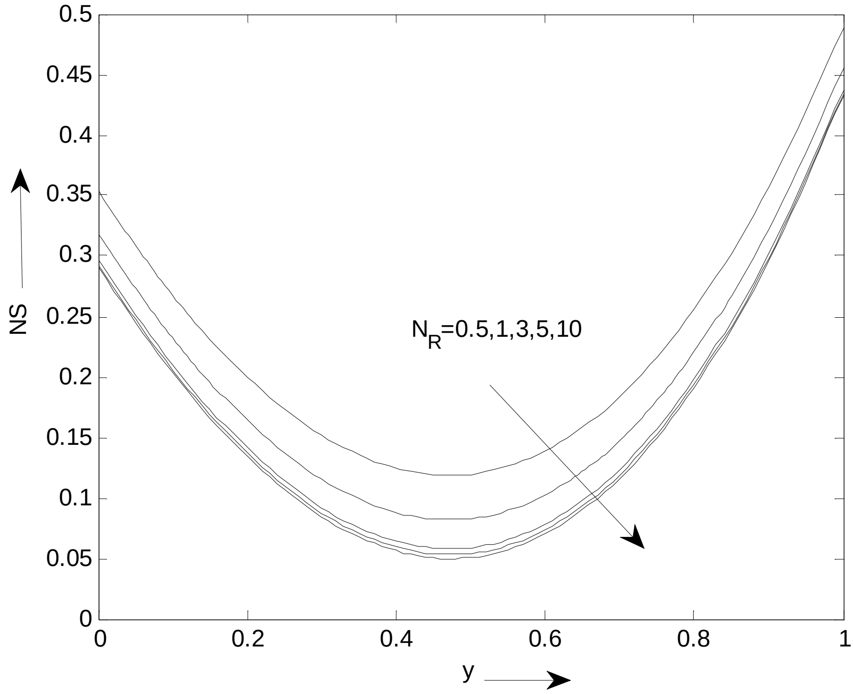

Figure12. Entropy profile for $Br = 1, P = 1 a = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

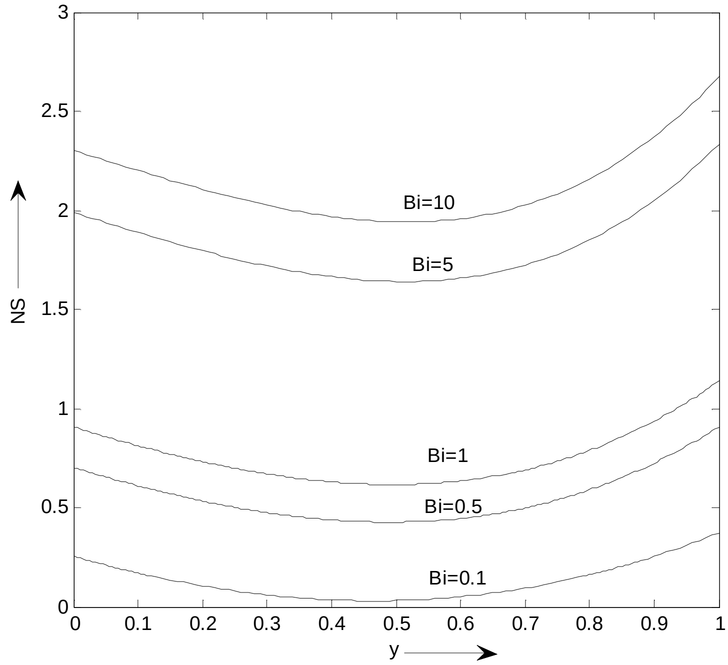

Figure13. Entropy profiles for $Br = 1, P = 1, N_R = 1, a = 1, K = 0.001, \alpha = 0.1, \Omega = 1$

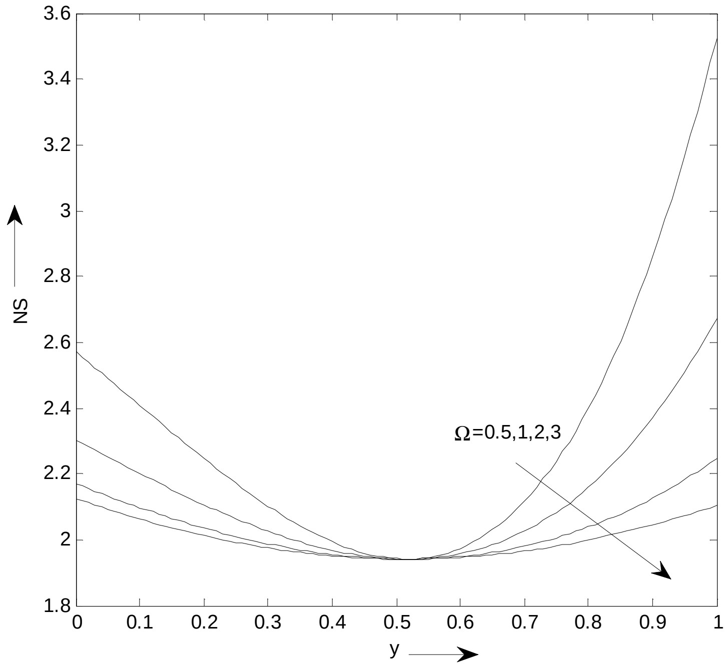

Figure14. Entropy profiles for $Br = 1, N_R = 1, P = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

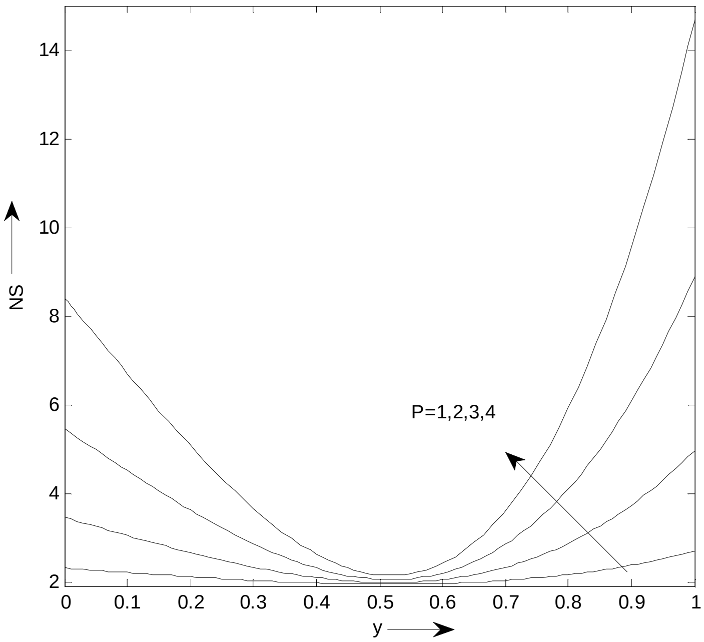

Figure15. Entropy profiles for $Br = 1, N_R = 1, a = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

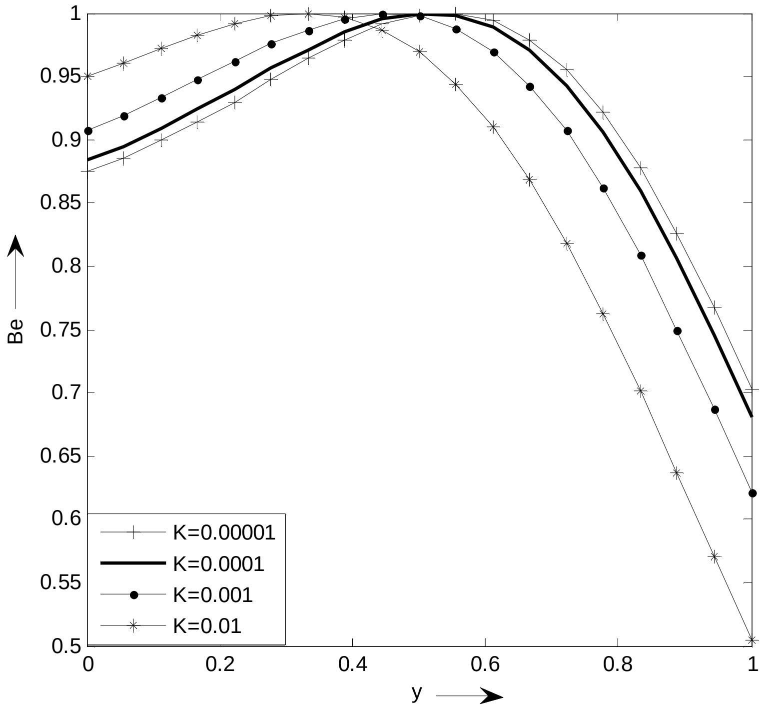

Figure16. Bejan number profiles for $Br = 1, N_R = 1, P = 1, a = 1, \alpha = 0.1, Bi = 10, \Omega = 1$

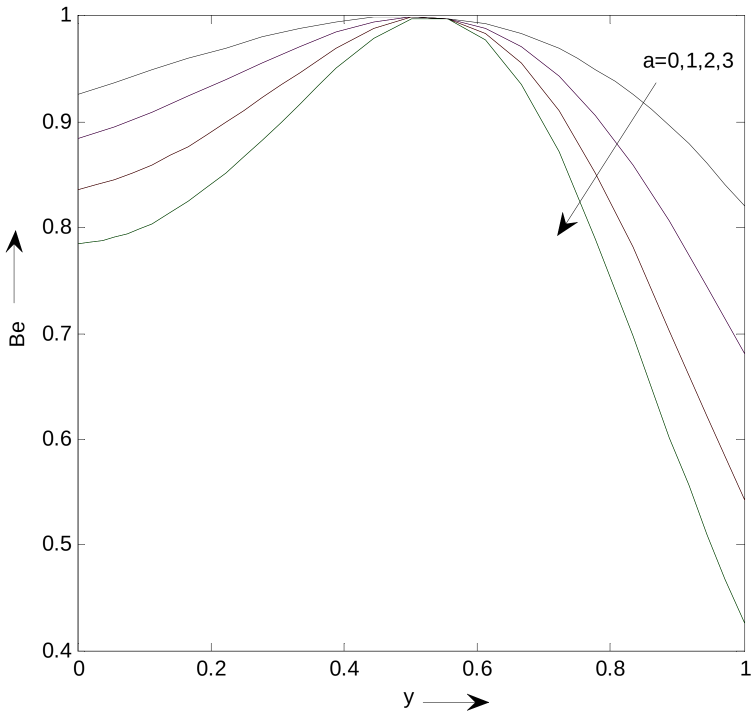

Figure17. Bejan number profiles for $Br = 1, N_R = 1, P = 1, K = 0.001, \alpha = 0.2, Bi = 10, \Omega = 1$

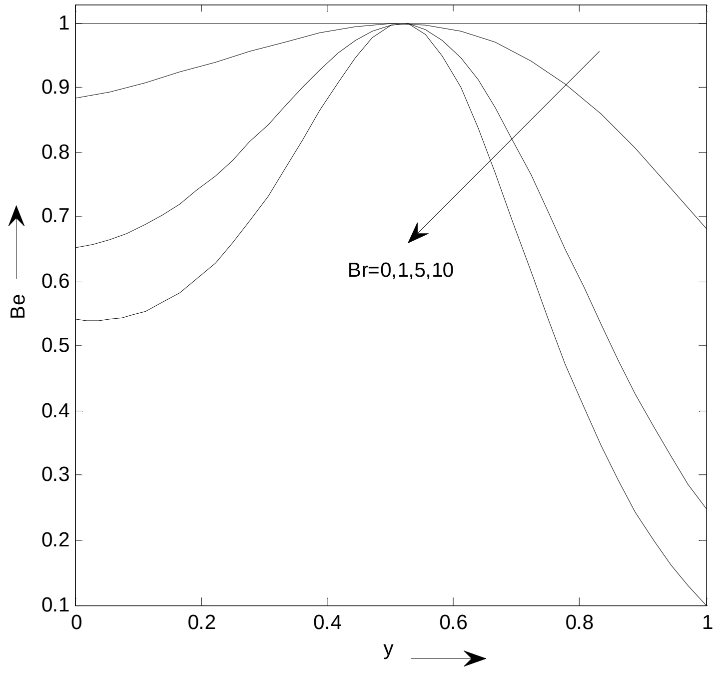

Figure18. Bejan number profiles for $a = 1, N_R = 1, P = 1, K = 0.001, \alpha = 0.2, Bi = 10, \Omega = 1$

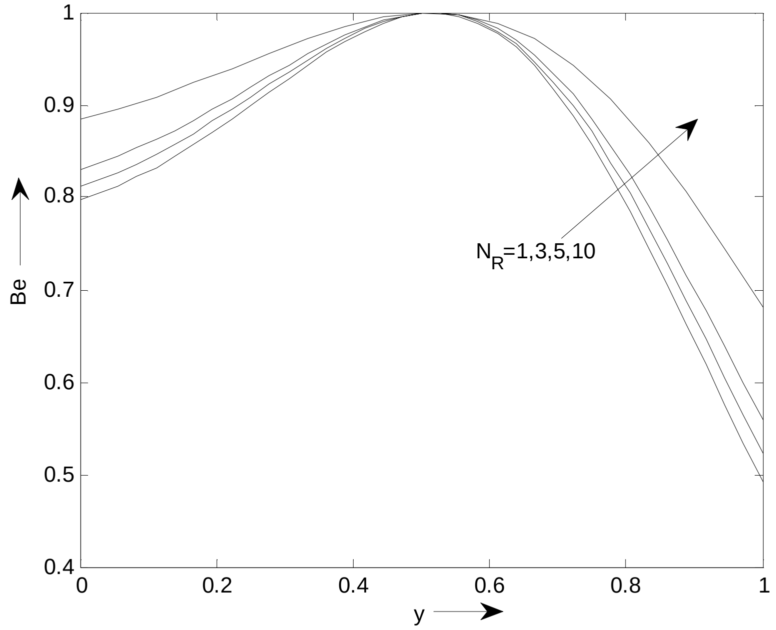

Figure19. Bejan number profiles for $Br = 1, P = 1, a = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

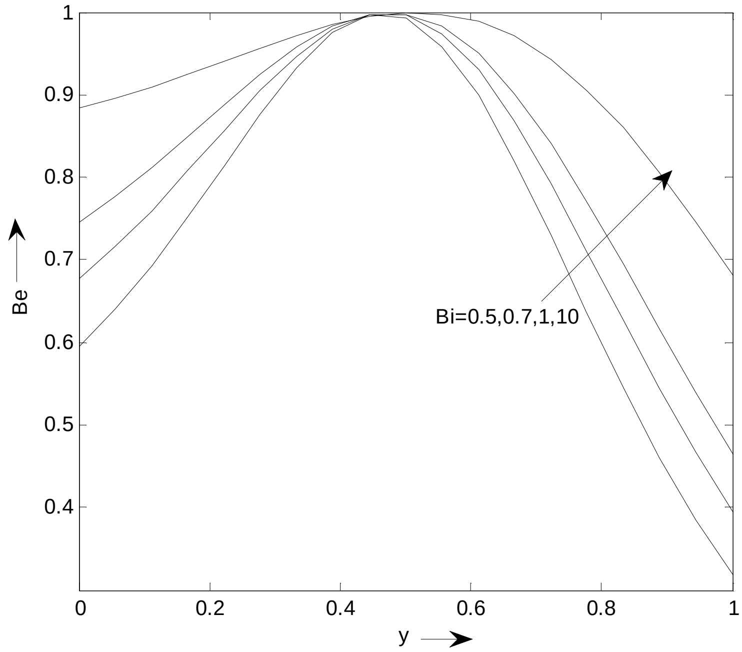

Figure20. Bejan number profiles for $Br = 1, P = 1, N_R = 1, a = 1, K = 0.0001, \alpha = 0.1, \Omega = 1$

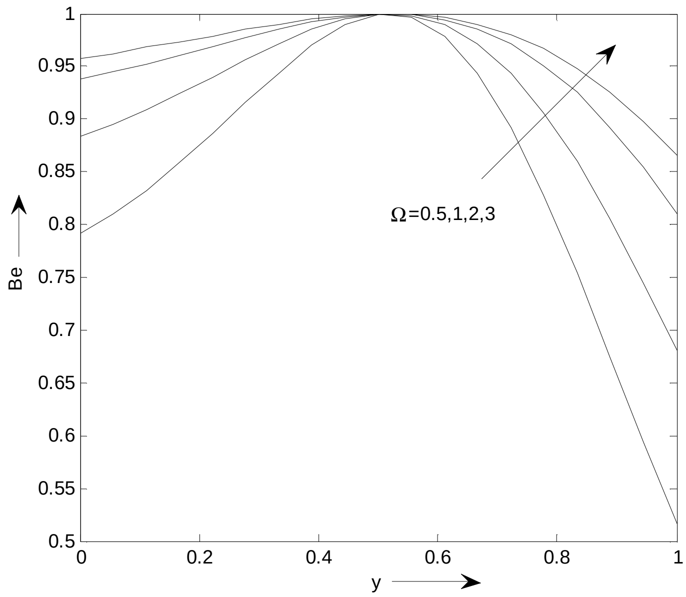

Figure21. Bejan number Profiles for $Br = 1, N_R = 1, a = 1, P = 1, K = 0.0001, \alpha = 0.1, Bi = 10$

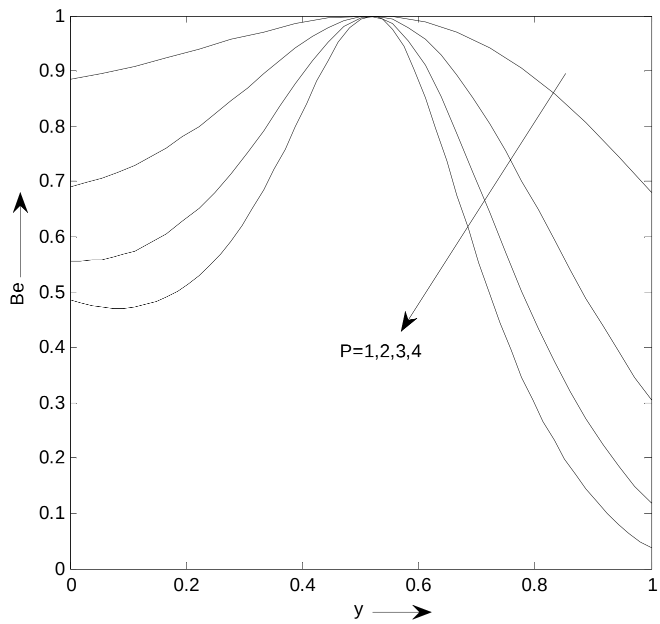

Figure22. Bejan number profiles for $Br = 1, N_R = 1, a = 1, K = 0.0001, \alpha = 0.1, Bi = 10, \Omega = 1$

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