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Advances in Applied Science Research, 2013, 4(3):27-32

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Advances in Applied Science Research

Pelagia Research Library

ISSN: 0976-8610
CODEN (USA): AASRFC

Energy gap in a two-qubit spin-1/2 XY model with added Dzyaloshinskii-Moriya interaction and longitudinal magnetic field

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ABSTRACT

We have examined the effect of a DM interaction and longitudinal external magnetic field on the energy gap of a two-qubit spin-1/2 XY mode. Also we have calculated the critical values of DM interaction and longitudinal external magnetic field. We have shown that DM interaction and external magnetic field have an effect on the energy gap behavior and critical values and also we have been able to get phase transition point.

Keywords: XY model, Dzyaloshinskii – Moriya Interaction, Energy gap.

INTRODUCTION

The quantum phase transition and Quantum magnetism are one of the most important and interesting issues in many body physics.[1-2] The quantum phase transition at the zero temperature is due to quantum fluctuations that create in the Hamiltonian by changing the parameters. A lot of methods as the DMRG method [3] , Lanczos method [4] and etc. are methods for examining quantum phase transitions. In recent decade, the quantum phase transition has been seen in many systems like optic nets.[5-6] We can point to the spin models for considering quantum phase transition. The Heisenberg model is a famous spin model that has got exact solution by Bethe Ansatz methods.[7] In one hand , XY model is exactly solvable. [8-10] In some of the experimental results, there are main differences between experimental measuring results and the result due to the extracted Heisenberg model.[11-13] The origin of this difference is DM interaction . [14,15], The Hamiltonian of DM interaction , by using phenomenological method is as $\vec{D}.(\vec{S}_j \times \vec{S}_k)$. For a better understanding of spin models, one can check two and three qubit models .[16-18]

The structure of the paper is following. In section two, we'll examine XY model in the presence of DM interaction and we'll examine D_c critical values and energy gap between the first excited state and ground state .In section three, we'll examine XY model in the presence of DM interaction and longitudinal external magnetic field , and consider and D_c critical h_c values and energy gap and in section four ,we will collect the information.

2- Two qubit XY spin model in the presence of DM interaction

The Hamiltonian of XY model in the presence of DM interaction is as :

$$H = \sum_j (J_x S_j^x S_{j+1}^x + J_x S_j^y S_{j+1}^y) + \vec{D}.\sum_j (\vec{S}_j \times \vec{S}_{j+1}) \tag{1}$$

and if $\vec{D} = D\hat{z}$, the Hamiltonian (1) will be as below for two qubit XY spin model :

$$H = J_x S_1^x S_2^x + J_x S_1^y S_2^y + D(S_1^x S_2^y - S_1^y S_2^x) \tag{2}$$

by calculating the eigenstates and eigenvalues of Hamiltonian energy (2) in legs $S_1^z \oplus S_2^z$ and if $J_x J_y \langle 0$, we have got transition point in $D_c = \sqrt{|J_x J_y|}$. D_c Behavior has come in Figure 1 according to J_x‹J_y changes. The energy gap also is as:

$$\Delta\varepsilon = -\frac{J_-}{4} + \frac{1}{4}\sqrt{4D^2 + J_+^2} \quad (3)$$

To study, we consider the following two conditions:

A) $J_x = J_y = J \rangle 0$

The energy gap (the difference between the energy of the first excited state and the energy of the ground state) is as : $\Delta\varepsilon = \frac{1}{2}\sqrt{D^2 + J^2}$ and the gap behavior has come in figure 2 according to D for $J = 1$.

B) $J_x = -J_y = J \rangle 0$

The transition point is in $D_c = J$ and the energy gap is as $\Delta\varepsilon = \frac{1}{2}(D - J)$ according to the D linear. The Figure of the energy gap has come in figure 3.

3- Two qubit XY spin model in the presence of DM interaction and longitudinal magnetic field

If we add the longitudinal magnetic field to the Hamiltonian (1) , in this case, we can write the Hamiltonian of a two qubit XY spin model:

$$H = J_x S_1^x S_2^x + J_y S_1^y S_2^y + D(S_1^x S_2^y - S_1^y S_2^x) + h(S_1^z + S_2^z) \quad (4)$$

($\vec{D} = D\hat{z}$ has been selected) by calculating the eigenvalues and the eigenstate of the Hamiltonian energy(4) in the basis of $S_1^z \oplus S_2^z$, the critical value of D_c will be:

$$D_c = \sqrt{h^2 - J_x J_y} \quad (5)$$

is obtained and the behavior of D_c has come in Figure 4 according to h for $J_x = J_y = 1$.

$$h_c = \sqrt{D^2 + J_x J_y} \quad (6)$$

and also, the h_c changes has come in Figure 5 according to D for $J_x = J_y = 1$. We consider the following two conditions for more studying:

A) $J_x = J_y = J \rangle 0$

In this case, the energy gap is as:

$$\Delta\varepsilon = \frac{1}{2}(\sqrt{D^2 + J^2} - h) \quad (7)$$

The Figure of energy gap has come in figure 6 according to D, h for $J = 1$. And if $h = h_c = \sqrt{D^2 + J^2}$, the energy gap will be zero and h_c Figure will be as figure 5 according to D for $J = 1$.

B) $J_x = -J_y = J \rangle 0$

The energy gap is as $\Delta\varepsilon = 2D - 2\sqrt{4h^2 + J^2}$ and its changes has come in Figure 7 according to $J = 1$. The critical values h_c is as $h_c = \frac{1}{2}\sqrt{D^2 - J^2}$ that its Figure has come in figure 8 according to $J = 1$.

Figure 1: The energy gap Figure according to the J_x, J_y changes.

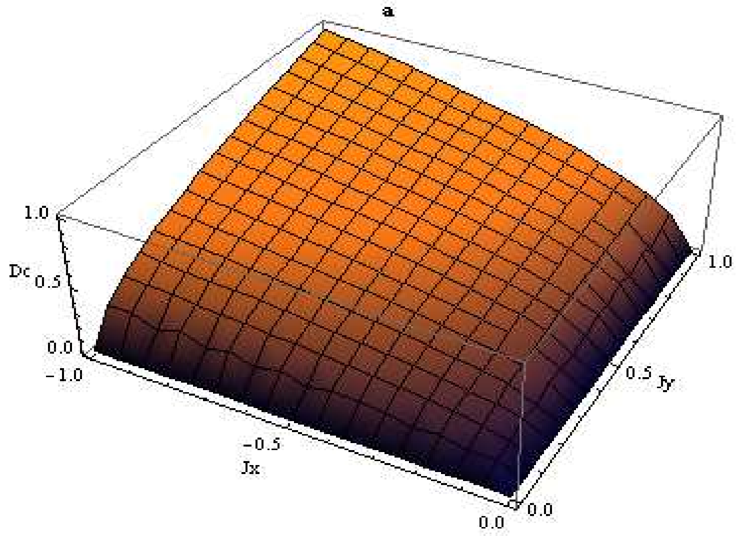


Figure 2: The energy gap Figure according to *D* changes for $J = 1.(J_x = J_y = J \rangle 0)$

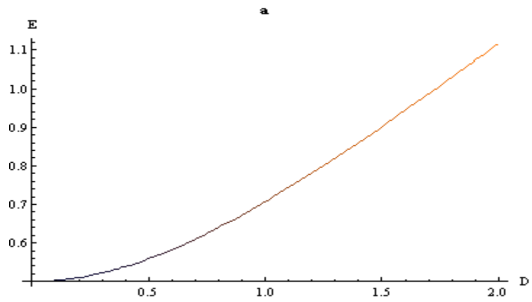


Figure 3: The energy gap Figure according to *D* changes for $J = 1.(J_x = -J_y = J \rangle 0)$

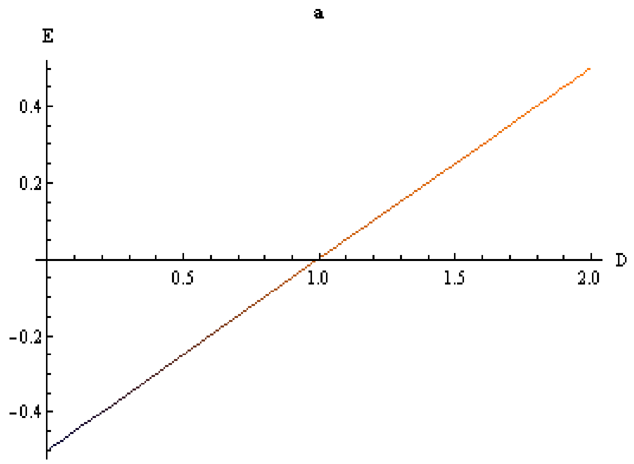


Figure 4: D_c **Figure according to** h **changes for** $J_x = J_y = 1$.

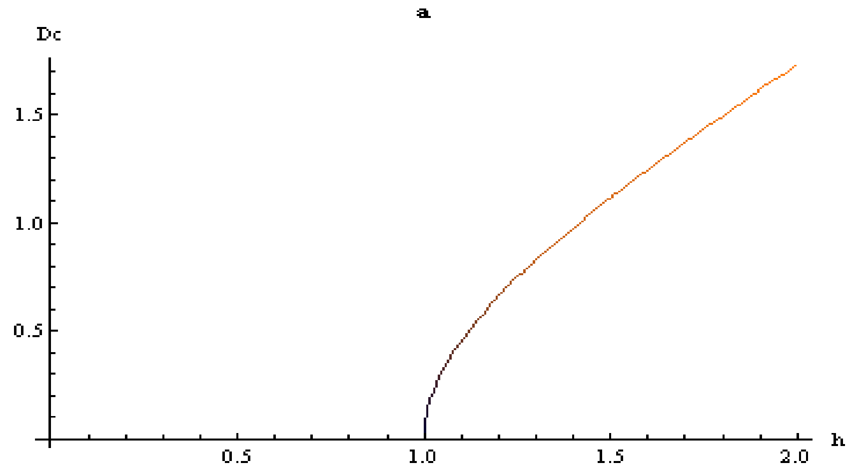


Figure 5: h_c **Figure according to *D* changes for** $J_x = J_y = 1$.

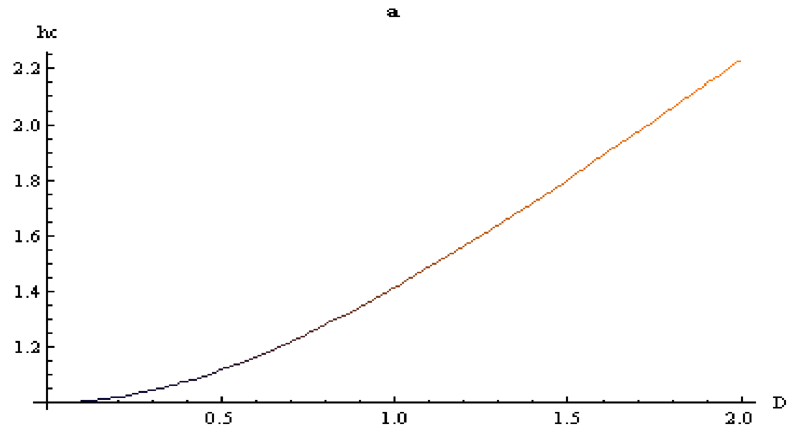


Figure 6: The energy gap Figure according to h, D **changes for** $J_x = J_y = 1$.

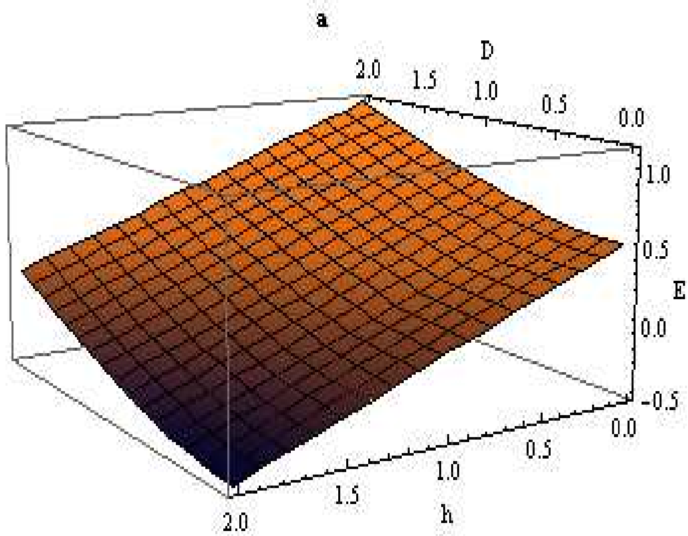


Figure 7: The energy gap Figure according to h, D changes for $J_x = -J_y = 1$.

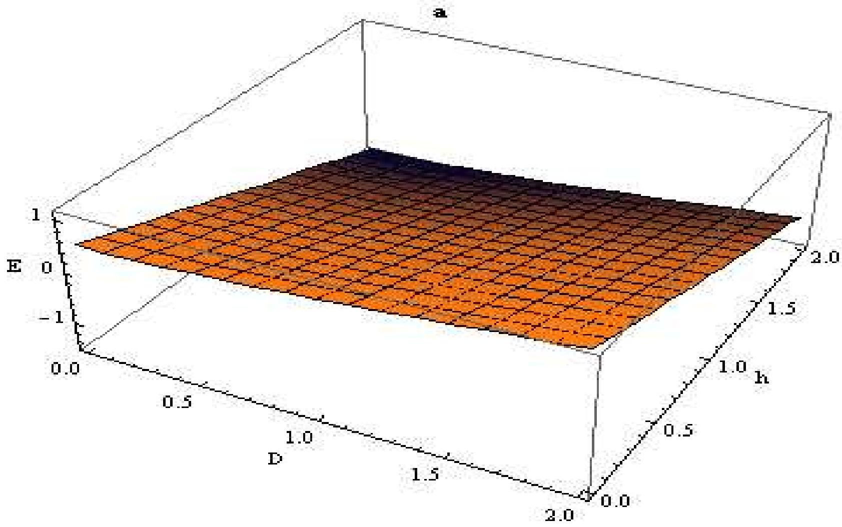
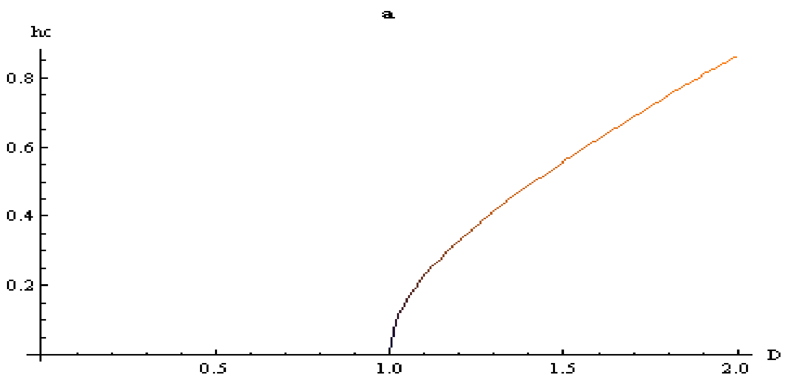


Figure 8: h_c Figure according to *D* changes for $J_x = -J_y = 1$.



CONCLUSION

We have studied the two qubit spin-1/2 XY model in the presence of DM and longitudinal magnetic field. We have shown that the energy gap between the first excited state and the ground state depends on the both the DM interaction and the external magnetic field. It has been shown that the energy gap increases by increasing DM interaction and will be zero in critical values of the DM interaction and field, D_c, h_c, respectively. The effect of the DM interaction on the critical magnetic field and also effect of the magnetic filled on the critical DM interaction has been studied.

Acknowledgment
The authors gratefully acknowledge the financial and other support of this research, provided by the Islamic Azad University, South- Tehran Branch, Tehran, Iran.

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