Original Article

Particle Swarm Optimization for the Design of High Diffraction Efficient Holographic Grating

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ABSTRACT

We present a Particle Swarm Optimization (PSO) algorithm for the design of holographic gratings using coupled mode theory. The algorithm is based on a new objective function that has been proposed for achieving desired diffraction efficiency. The performance of the algorithm is determined by designing a grating with specific application to solar concentrator. The algorithm could predict the different parameters like thickness of grating, modulation index and angular deviation for achieving maximum diffraction efficiency.

Keywords: Diffraction efficiency, Holograms, Holographic grating, Particle swarm optimization.

INTRODUCTION

In recent years, different stochastic global optimization algorithms like Genetic Algorithm, Simulated Annealing, have been applied to solve different physical problems. The drawback of Genetic Algorithm is its expensive computational cost[1]. In simulated annealing, repeatedly annealing with a $1/\log k$ schedule is very slow, especially if the cost function is expensive to compute[2]. Recently PSO has been proposed as an alternative tool for solving optimization problems. Due to its simplicity, this algorithm is used to optimize almost every physical problem.

We present a Particle Swarm Optimization (PSO) algorithm[3] for the design of holographic gratings using coupled mode theory[4]. The algorithm is based on a new objective function that has been proposed for achieving desired diffraction efficiency. The performance of the algorithm is determined by designing a grating with specific application to solar concentrator. The algorithm could predict the different parameters like thickness of grating, modulation index and angular deviation for achieving maximum diffraction efficiency.

Holographic grating

The hologram grating, which we have considered for this investigation is a transmission hologram as shown in fig (1) and is characterized by Λ -Grating period , $\overline{K}$ -Grating vector (perpendicular to the fringe planes), θ- the angular deviation from the Bragg's angle, φ - slant angle, $\overline{\rho}$ -Propagation vectors of diffracted waves, $\overline{\sigma}$ -propagation vectors of reference wave, n -Spatial modulation amplitudes of the refractive index, d -Grating thickness.

The diffraction efficiency (η) for such a holographic grating using Kogelnik coupling theory [5] can be expressed as:

$$\eta = \frac{c_s}{c_R} S S^* \qquad (1)$$

Where,

$$S = -j\left(\frac{c_R}{c_s}\right)^{\frac{1}{2}} e^{i\xi} \sin\left(\frac{\left(v^2+\xi^2\right)^{\frac{1}{2}}}{\left(1+\frac{\xi^2}{v^2}\right)^{\frac{1}{2}}}\right) \qquad (2)$$

$$c_R = \cos\theta \qquad (3)$$

$$c_s = \cos\theta - \frac{K\lambda\cos\varphi}{2\pi n} \qquad (4)$$

Using fig (1) the diffraction efficiency of holographic grating can be shown to be:

$$\eta = \sin^2\left(\frac{\left(v^2+\xi^2\right)^{\frac{1}{2}}}{1+\frac{\xi^2}{v^2}}\right) \qquad (5)$$

Where,

$$\xi = \frac{\vartheta d}{2c_s} \qquad (6)$$

$$\vartheta = K\cos(\varphi-\theta) - \frac{K^2}{4\pi n} \qquad (7)$$

$$k = \frac{2\pi}{\Lambda} \qquad (8)$$

$$v = \frac{\pi n_1 d}{\lambda\left(c_R c_s\right)^{\frac{1}{2}}} \qquad (9)$$

Equation (5) is the basic equation which is used for optimization of diffraction efficiency.

OBJECTIVE FUNCTION

First we set 9 different points on the desired flat curve and then minimize the distance between desired curve and objective plot as shown in fig (2) by using the fitness function in equation (10).

$$fitness = \sum_{k_i=1}^{n}\left(\eta_{(k)} - \eta(k_i)\right)^2 \qquad (10)$$

Where,

$\eta(k_i) =$ The desired value of efficiency at k_i , where $i = 1, 2 \ldots 9$

$\eta_k =$ It is the diffraction efficiency of holographic grating as given in equation (5).

Particle swarm optimization (PSO)

Particle Swarm Optimization is a population based optimization technique. It emulates some aspects of social behavior of a flock of birds and a school of fish. The swarm initially has a population of random solutions. Each potential solution, called a particle, is given a random velocity and is flown through the problem space. The particles have memory and each particle keeps track of its previous best position (pbest) and the corresponding fitness value. The swarm has another value called gbest, which is the best value of the entire particle's pbests in the swarm. The PSO algorithm[6,7] used in our paper is as summarized below:

Step-1: Initialize a population of particles (30 in our case) with random positions (s_i^k) and velocities (v_i^k).where s_i^k and v_i^k represent the position of particle i at iteration k and velocity of particle i at iteration k with a number of dimension respectively within the lower and upper bounds of index modulation (n_1), thickness of hologram (d) and angle of deflection from Bragg's angle (θ). Evaluate the fitness of each particle using equation (10) and assign the particle position as pbest. The best among the pbests is global best (gbest).

Step-2: Plot graph between wave length λ versus diffraction efficiency η for different optimum values of modulation index (n_1), thickness of hologram (d) and angle of deflection from Bragg's angle (θ) and angle of deviation from Bragg's angle (θ) versus diffraction efficiency η for different optimum values of wavelength λ, modulation index (n_1), thickness of hologram (d).

Step-3:

$$V_i^{k+1} = wV_i^k + c_1 * rand_1 * (\text{pbest}_i - s_i^k) + c_2 * rand_2 * (\text{gbest} - s_i^k) \quad (11)$$

$$w = \text{w}Max - [(\text{wMax} - \text{wMin}) \times \text{iter}] / \text{maxIter} \quad (12)$$

$$s_i^{k+1} = s_i^k + V_i^{k+1} \quad (13)$$

Change the velocity and position of the particle according to equations (11) and (13), respectively. V_i^k & s_i^k represents the velocity of particle i at iteration k and current position of particle i at iteration k with a no. of dimensions respectively. $rand_1$ & $rand_2$ are two uniform random functions between 0 and 1, w is the inertia weight calculated according to equation (12) and c_1 & c_2 are the acceleration constants.

Step-4: For each particle, evaluate the fitness using equation (10) if all variables are within the search range[8].

Step-5: For each particle, if the current fitness value is better than pbest, then update pbest. If the current best fitness is better than the gbest, update gbest to the current best position and fitness value.

Step-6: Repeat steps 4 & 5 until the maximum number of function evaluation is completed.

SIMULATION RESULT AND ANALYSIS

For an effective holographic grating to be used in photovoltaic applications, its diffraction efficiency should not vary much with respect to wavelength and deviation from Bragg's angle of incidence. In practical situation, both wavelength and deviation from Bragg's angle of incidence are not constant, but vary in a given range. For example the position of sun with respect to concentrator changes with time. As the diffraction efficiency of the holographic grating depends on other parameters like thickness and modulation index, it is possible to optimize the diffraction efficiency such that it does not vary much with the change of wavelength(λ) or deviation from Bragg's angle of incidence (θ). To achieve this, one has to search for the values of thickness of hologram(d), angle of deviation from Bragg's angle(θ), modulation index (n_1) for which diffraction efficiency becomes maximum and does not change much with respect to change of wavelength. In other words, the variation of diffraction efficiency with respect to wavelength and Bragg angle variation should be almost flat and maximum. We use the algorithm presented in the section IV, to find these values of d, n_1, θ. Simulation are made with the following values of $\eta_{t_1} = 0.996, \eta_{t_2} = 0.997, \eta_{t_3} = 0.998, \eta_{t_4} = 0.999, \eta_{t_5} = 0.1, \eta_{t_6} = 0.999, \eta_{t_7} = 0.998, \eta_{t_8} = 0.997, \eta_{t_9} = 0.996$

The wavelength range of operation is 500-700nm. For example the proposed algorithm is implemented for photovoltaic application and the different parameters for maximum and flat diffraction efficiency are found to be $\theta = 6.6721, n_1 = 0.0526, d = 5.3839 \times 10^{-6}\, m$ as shown in fig (3) and $\lambda = 5.7670 \times 10^{-6}\,m, n_1 = 0.4784, d = 9.0835 \times 10^{-6}\,m$ as shown in fig (4).

CONCLUSION

In this paper we proposed an algorithm based on Particle Swarm Optimization to optimize diffraction efficiency of a holographic grating. A new fitness function has been proposed and the algorithm is applied for a holographic grating which can be used as a solar concentrator to achieve maximum and flat diffraction efficiency.

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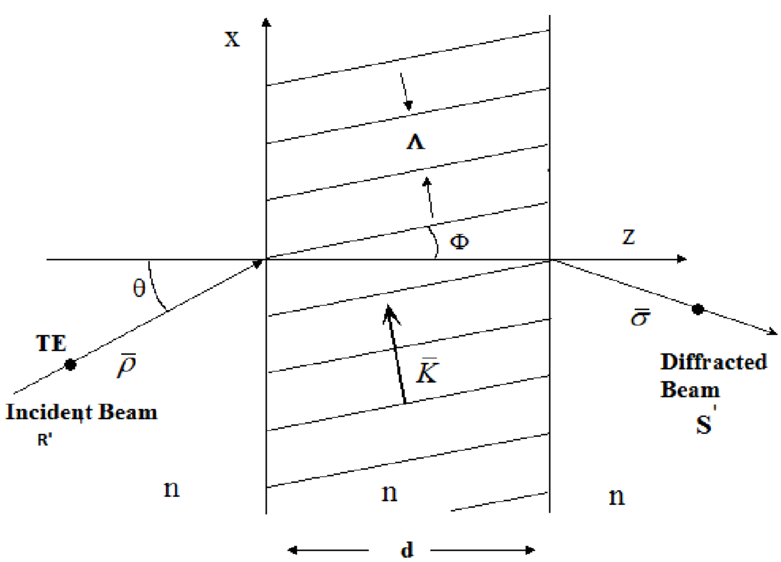

Figure 1. Schematic diagram showing the basic model for diffraction from a volume hologram

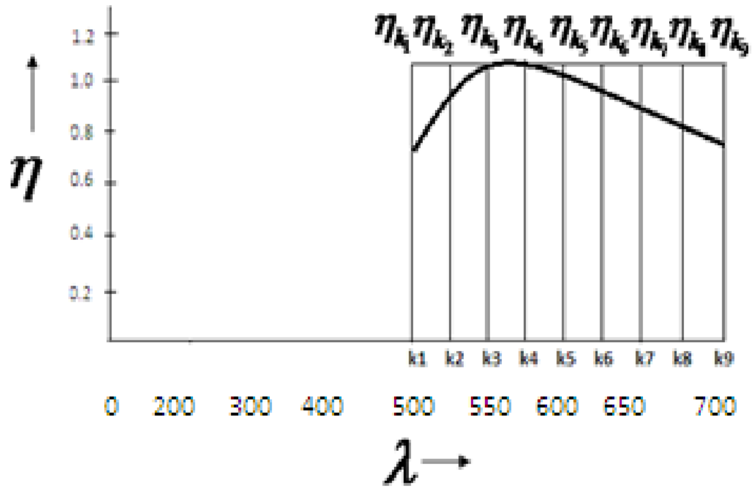

Figure 2. Setting of the desired function

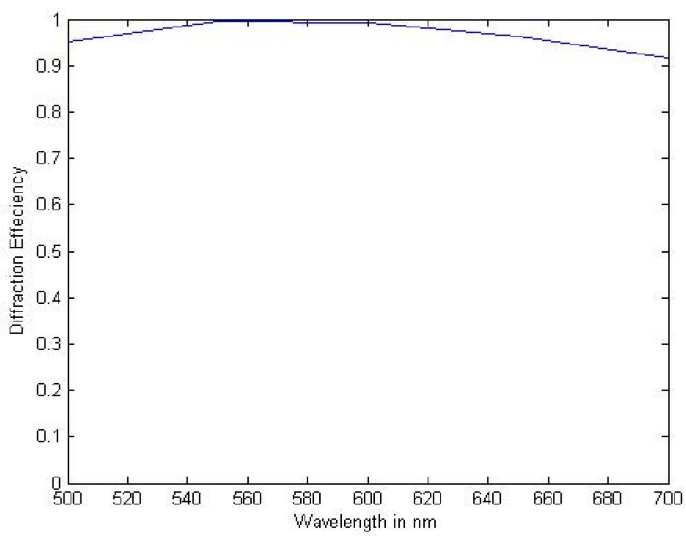

Figure 3. Variation of diffraction efficiency with wavelength at Bragg's angle illumination at fixed fringe spacing $\Lambda = 1.9231\ \mu m$, the angular deviation from the Bragg's angle $\theta = 6.6721^o$, modulation index $n_1 = 0.0526$, and thickness of the grating $d = 5.3839 \times 10^{-6}\ m$

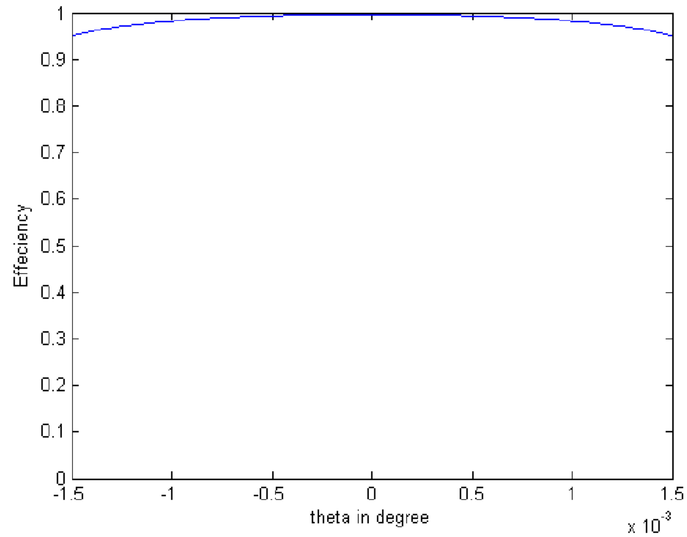

Figure 4. Variation of diffraction efficiency with angular deviation from the Bragg's angle at fixed fringe spacing $\Lambda = 1.9231\ \mu m$, wavelength $\lambda = 5.7670 \times 10^{-7}\ m$, modulation index $n_1 = 0.4784$, and thickness of the grating $d = 9.0835 \times 10^{-6}$ m