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Encircled energy factor in impulse response functions of optical systems with first-order parabolic filters

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ABSTRACT

In the present paper, we have studied the Encircled Energy Factor, $EEF(\delta), 0<\delta\leq 1$ within a circle of specified radius δ in the Impulse response function or the Green's function of an optical system with First –Order Parabolic Filters. As the most important conclusion from our present study, we find that the Encircled Energy Factor $EEF(\delta)$ is a monotonically increasing positive function. It vanishes at the origin, $\delta=0$, of the $EEF(\delta)$ versu δ curves and then increases monotonically approaching unity asymptotically as the 'bias' of the apodising function tends to increase.

Key-words: Mathematical Optics, Analogue Optical signal Processing, First Order Parabolic Filters….etc.

INTRODUCTION

The e**ncircled energy EEF (δ)** measures the fraction of the total energy contained within a circle of specified radius 'δ' in the image plane, centered on the diffraction head. This is one of the important parameters in the study of a diffraction image. LORD RAYLEIGH [1] first pointed out the importance of the studies on the encircled energy in the diffraction pattern as an image quality assessment parameter. Although the encircled energy is a good test for the quality of an optical system, it has not been used much in the early years, because, of the difficulties in its calculations. To overcome this difficulty, LANSRAUX and BOIVIN [2] introduced computing techniques for the numerical evaluation of the encircled energy in the diffraction pattern of an optical system having aberrations. The image structure of self- luminous or incoherently illuminated objects, formed by optical instruments like astronomical telescopes and cameras used in aerial photography is determined by the diffraction studies. The **Fractional Encircled Energy** gives an idea of the effective spread of the diffraction pattern. This spread will determine the resolution as in the astronomical imagery like imaging of distant point objects through a random stationary atmosphere. In what follows now, we will present a brief review of the previous works done on the EEF by various authors.

SOM and BISWAS [3] studied the fractional encircled energy distribution in the far-field diffraction pattern with circular apertures under partially coherent illumination. KINTNER [4] has calculated the encircled energy analytically by representing the point spread function in an orthogonal series form. It is now well-known that the use of an annular aperture makes the central maximum in the Airy pattern narrow and increases the depth of focus. WELFORD [5] has studied the focal depth with annual apertures and discussed about the encircled energy. TSCHUNKO [6] has shown that there will be increases in the resolution with the obscured apertures. STAMNES, HEIER and LJUNGGREN [7] have calculated the encircled energy for large no. of aberration free annular aperture systems using HOPKIN'S algorithm [8]. They have also assessed the focal shift tolerances. TSCHUNKO [6] derived the total energy function and determined the partial energy integrals for the system with an annular aperture apodised with different types of apodising functions. BISWAS and BOIVIN [9] derived a general formula to study the influence of wave aberrations on the performance of optimum apodisers particularly in the encircled energy values. In their study, they have used Straubel class and Lansraux-Boivin apodisers. DEVARAYALU, RAO and MONDAL [10] discussed about the possibility of identifying super- resolving and apodising properties of an optical system having shaded circular apertures from encircled energy considerations.

VISWANATHAM, RAO and MONDAL [11] have studied the fraction of the encircled energy with regard to the **dispersion factor** in the diffraction pattern of circular apertures. LUNEBERG [12] proposed four apodisations problems and suggested a method to solve them by using the calculus of variations. The third Lunenburg problem is to find the optimum pupil function which gives maximum energy in a given area in the image field at the receiving plane. LANSRAUX and BOIVIN [2], BARAKAT [13] also investigated this problem on the basis of the calculation of variations. UENO and ASAKURA [14] have solved this problem of maximum encircled energy for an aberration free, rotationally symmetric optical system apodised with a specified overall transmittance. CLEMENTS and WILLIKINS [15] investigated the problem of finding the diffraction pattern and corresponding pupil function having the maximum possible encircled energy ratio for an arbitrarily fixed radius and a fixed Raleigh limit of resolution. MONDAL [16] has derived an expression for the encircled energy within the circle of a specified radius in the Fraunhofer diffraction pattern due to an elliptical aperture.

When a converging monochromatic spherical wave is diffracted at a circular aperture, the classical theory predicts that the intensity distribution in the focal region will be symmetrical about the focal plane. But subsequent studies show that the principal maximum of the diffraction pattern may not be at the geometrical focuses and moves towards the aperture depending upon the Fresnel number (N) of the system. This effect is referred to as **'focal shift'**. BARAKAT [13] has studied experimentally the variation of encircled energy with Fresnel number N of the system. MAHAJAN [17] has discussed about the encircled energy of systems with non- centrally obscured apertures and has shown that non-central obscuration yields an equal or higher encircled energy value than that with central obscuration. VENKAT REDDY, PRASAD and MONDAL [18] have computed the encircled energy of optical systems with non- centrally apodised pupils.

ENCIRCLED ENERGY FACTOR (EEF)

The principal corollary of the PSF is the **Encircled Energy Factor (EEF).** As already stated, it is the ratio of the flux inside a circle of radius 'δ' centered on the diffraction head, to the total flux in the diffraction pattern. It represents the amount of energy contained within a circle of radius 'δ' in the image plane normalized to a total energy value of 1.0 as '$\delta \to \infty$'.Thus, by definition,

$$EEF(\delta) = \frac{\int_0^{2\pi}\int_0^{\delta} \left|G_p(0,z)\right|^2 z\,dz\,d\phi}{\int_0^{2\pi}\int_0^{\infty} \left|G_p(0,z)\right|^2 z\,dz\,d\phi} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Where $\phi = \tan^{-1}\left(\frac{v'}{u'}\right)$ and $z = \left(u'^2 + v'^2\right)^{\frac{1}{2}}$ are the polar coordinates of a point in the diffraction pattern on the image plane; ϕ is the azimuthally angle and $G_p(0,z)$ is the amplitude in the image plane at a point z units away from the diffraction head; z is expressed in dimensionless diffraction units. Since the integration under ϕ introduces just a constant in both numerator and denominator, the above equation reduces to:

$$EEF(\delta) = \frac{\int_0^{\delta} |G_p(0,z)|^2 zdz}{\int_0^{\infty} |G_p(0,z)|^2 zdz} \quad \text{..............(2)}$$

The denominator in the equation (2) represents the total flux and is equal to the twice the **pass flux ratio** τ . The amplitude of the light diffracted in the far field region associated with rotationally symmetric pupil function can be expressed by the following equation:

$$G(0,z) = 2\int_0^1 f(r) J_0(zr) rdr \quad \text{..............(3)}$$

When $f(r)$ is the pupil function, for our apodization filter. The pupil function $f(r)$ is here represented as:

$$f(r) = (\alpha + \beta r^2) \qquad \text{..............(4)}$$

Substituting equations (4) and (3) in equation (2) we obtain for the,

$$EEF(\delta) = \frac{\int_0^{\delta} \left[2\int_0^1 (\alpha + \beta r^2) J_0(zr) rdr \right]^2 zdz}{\int_0^{\infty} \left[2\int_0^1 (\alpha + \beta r^2) J_0(zr) rdr \right]^2 zdz} \quad \text{.............. (5)}$$

It is a monotonically increasing positive function. It vanishes at the origin $\delta = 0$ and increases monotonically approaching unity asymptotically as α tends to ∞ .

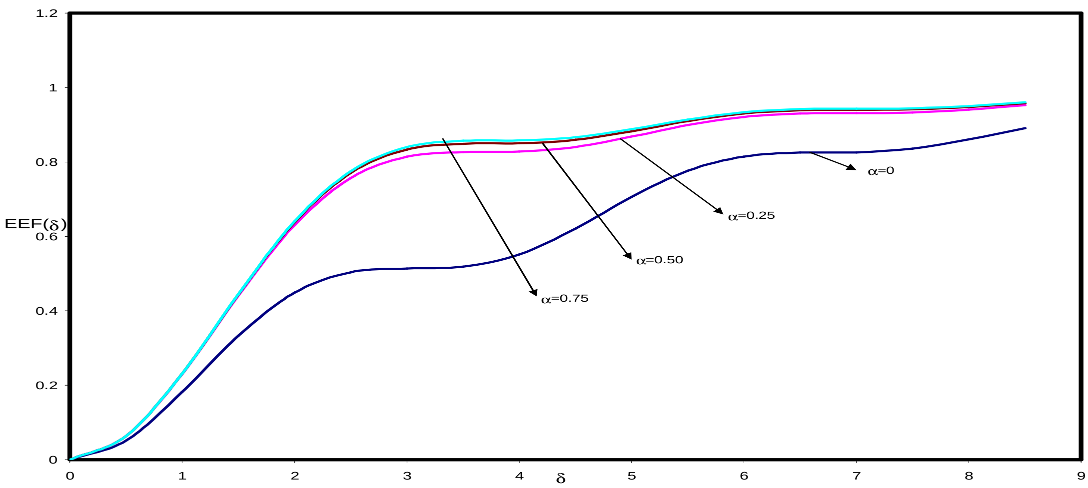

Fig.1(a) : Variation of EEF(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.1

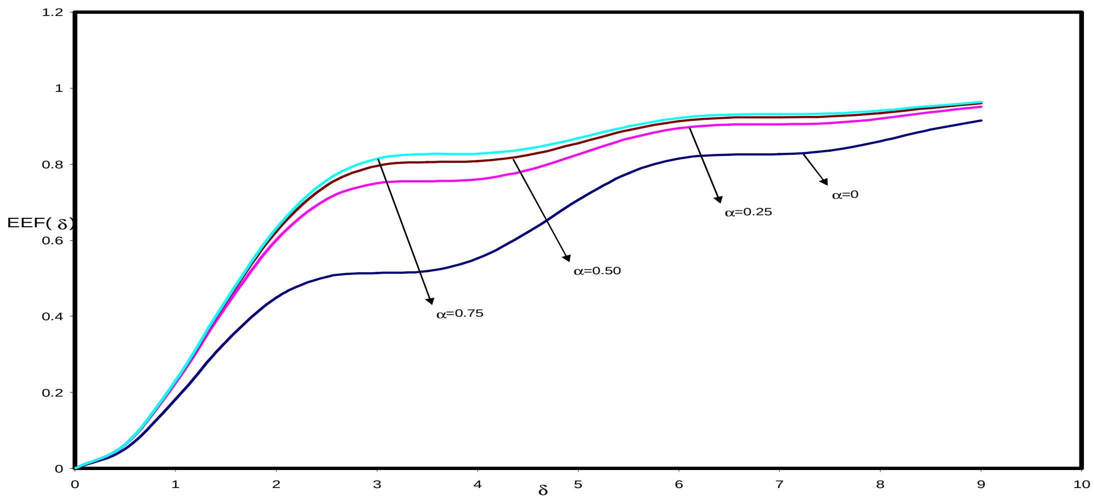

Fig.1(b) : Variation of EEF(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.3

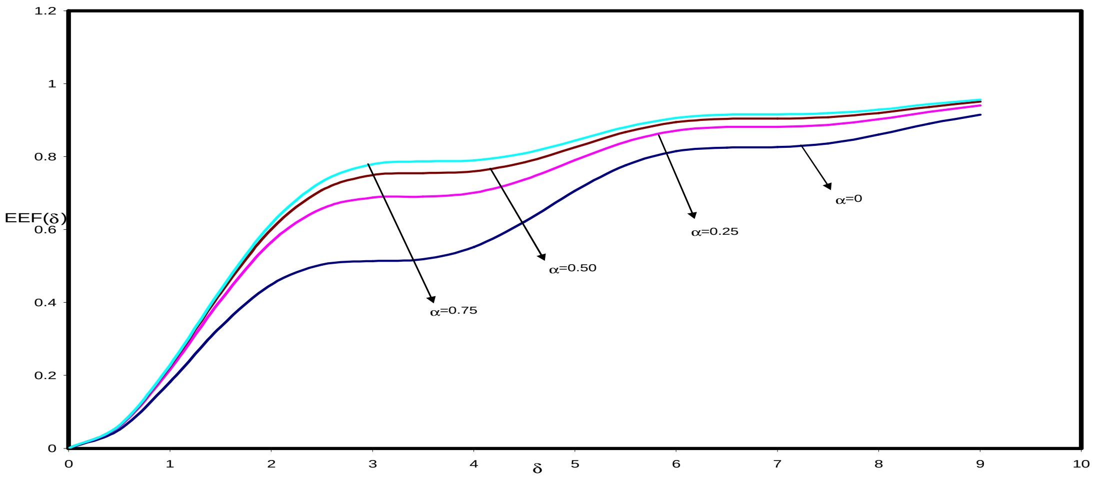

Fig.1(c) : Variation of EEF(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.6

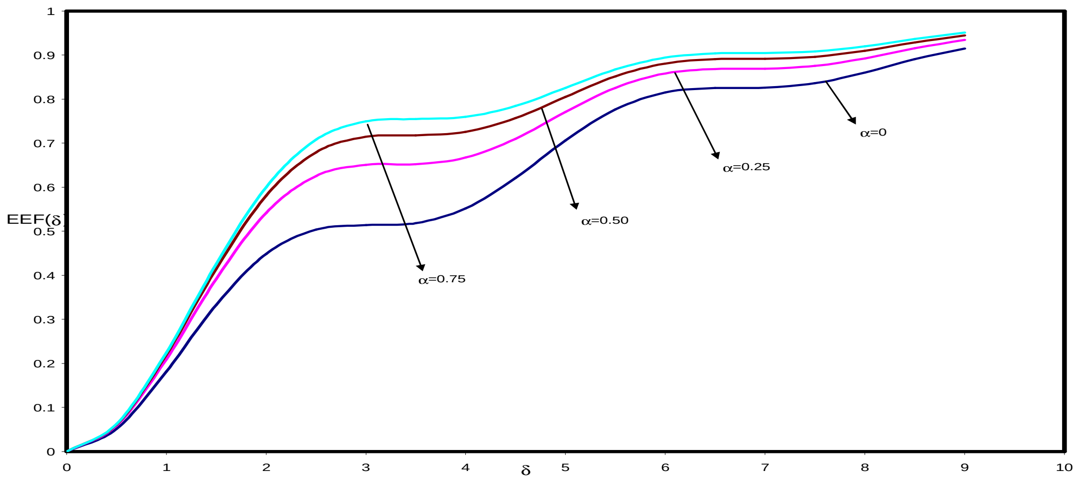

Fig.1(d) : Variation of EEF(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.9

We have shown the variation of EEF (δ) with δ for α =0, 0.25, 0.50 & 0.75 in the figures 1 (a) to 1 (d). Each curve in the figures is for a particular value of α as indicated therein. It is observed from the figures that a particular value of δ and for smaller value of α, EEF (δ) has lower values. Further, the EEF (δ) curves are not well dispersed for higher values of α. Also, this happens when the value of β is very small. As the value of β is increased , the EEF (δ) curves start getting well-separated from one another to show the desirable effects of higher values of β on the amount of Encircled Energy within a pre-specified radius of δ.

3 RELATIVE ENCIRCLED ENERGY $\left[EEF(\delta)\right]_R$

It is defined as the light flux enclosed by a circle of radius ‘ δ ’ in the image plane due to the apodised pupil as a function of the total flux in the image plane due to the Airy pupil. Thus,

$$\left[EEF(\delta)\right]_R = \frac{\int_0^{\delta} \left|G_p(0,z)\right|^2 zdz}{\int_0^{\infty} \left|G_A(0,z)\right|^2 zdz} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

G_A and G_p are the amplitudes for the free aperture and the given case of apodisation in our study respectively.

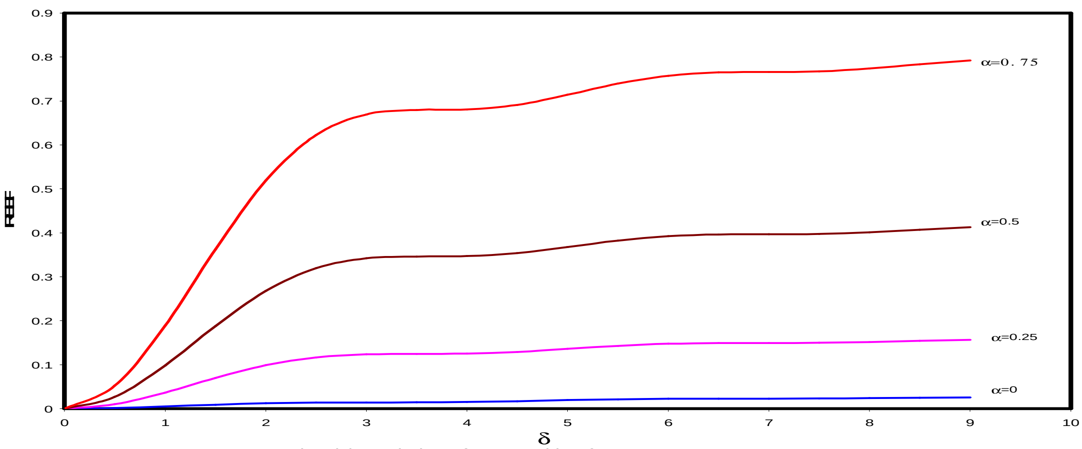

Fig.2(a): Variation of REEF with δ for α =0,0.25,0.50&0.75; β=0.3

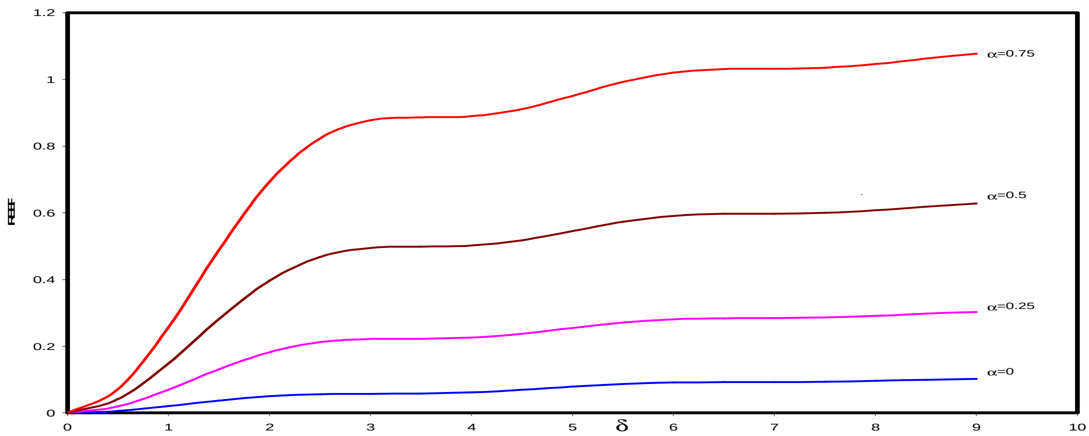

Fig.2(b): Variation of REEF with δ for α =0,0.25,0.50&0.75; β=0.6

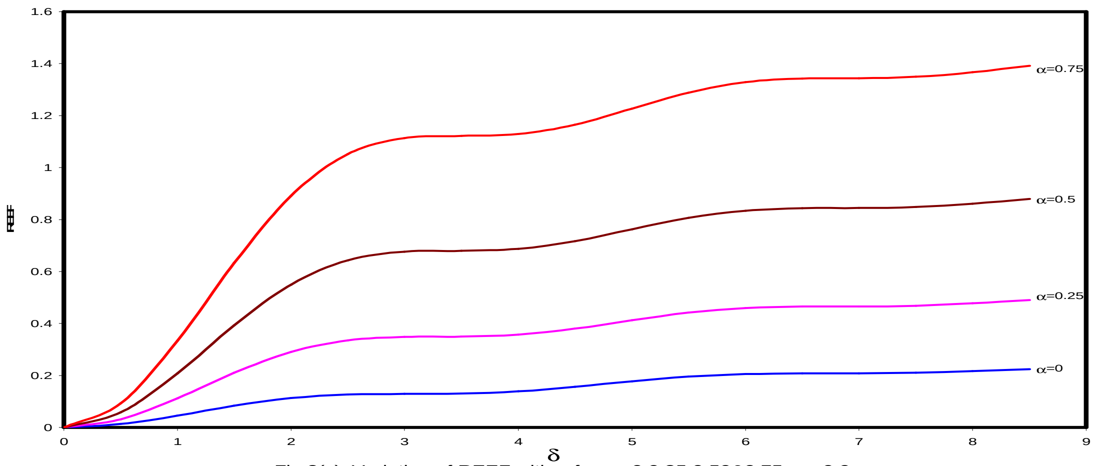

Fig.2(c): Variation of REEF with δ for α =0,0.25,0.50&0.75; β=0.9

We have shown the variation of Relative Encircled Energy Factor with specified values of radii δ for a smaller value of $\beta = 0.3$ an intermediate value of $\beta = 0.6$ and a higher value of $\beta = 0.9$ for $\alpha = 0, 0.25, 0.50 \,\&\, 0.75$ in the figures 2 (a) to 2 (c). It is observed from the figures that for a particular value of β the effects of increasing the α values are to increase the **bias or the average value** of the REEF curves, as it should be expected. So far as the effects of β, for a particular value of α are concerned, an increasing the value of β increases the individual values of REEF. Thus, for the relative EEF the effects of both α and β are similar.

4 EXCLUDED ENERGY $EE(\delta)$

The distribution of energy in the outer ring structure is called **Excluded Energy** $EE(\delta)$ which is defined as

$EE(\delta)$ = 1- $EEF(\delta)$ ....(7)

where $EEF(\delta)$ is the **encircled energy factor** within the circle of radius 'δ'. This parameter $EE(\delta)$ is also known as the **"dispersion factor"**. Mathematically, the excluded energy can be defined as:

$$EE(\delta) = \frac{\int_{\delta}^{\infty} \left|G_p(0,z)\right|^2 zdz}{\int_{0}^{\infty} \left|G_p(0,z)\right|^2 zdz} \text{..............(8)}$$

This is used in evaluating apodisation techniques for suppressing the ring structure. It is evident from the definition of Excluded Energy $EE(\delta)$ as given in the expression (8) that $EEF(\delta)$ and $EE(\delta)$ are complementary in nature. Therefore, the effects of α and β for a particular value of δ on the $EEF(\delta)$ curves will be reversed for the $EE(\delta)$ curves for the same value of δ.

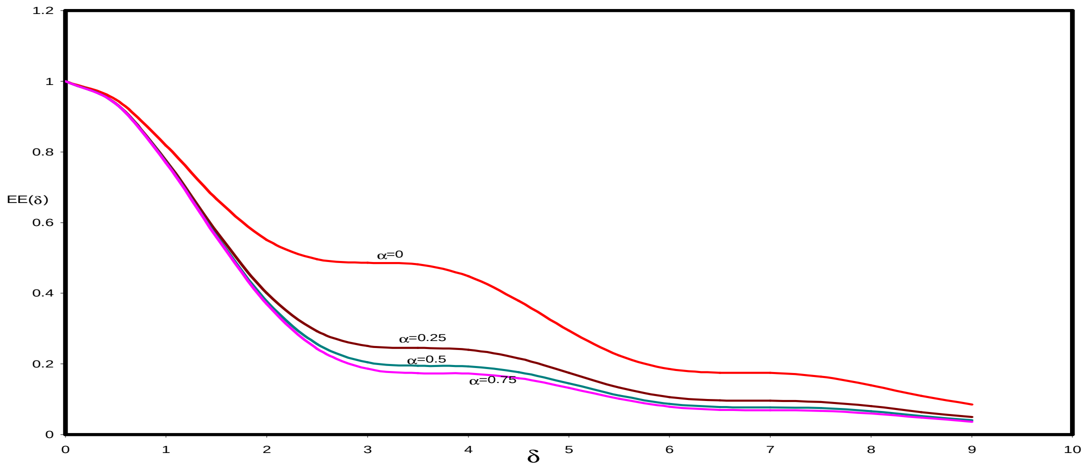

Fig.3(a): Variation of EE(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.3

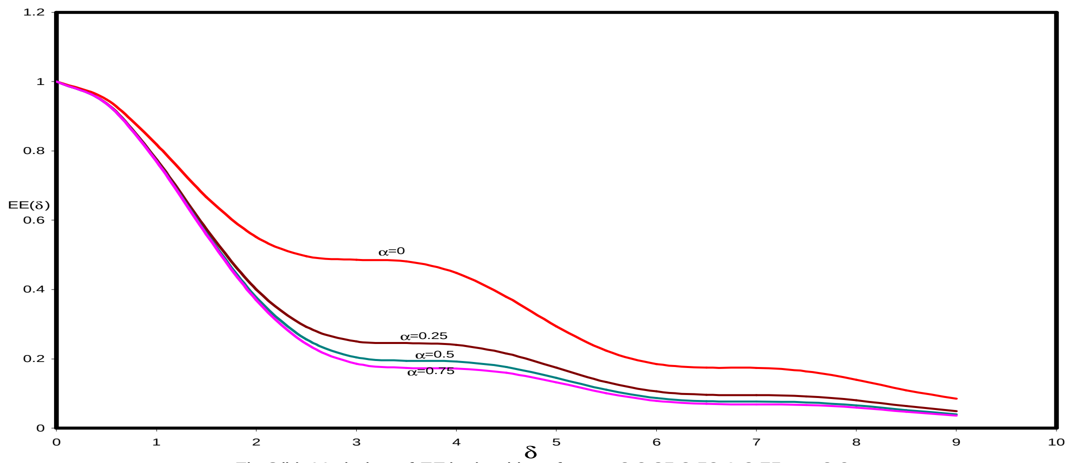

Fig.3(b): Variation of EE(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.6

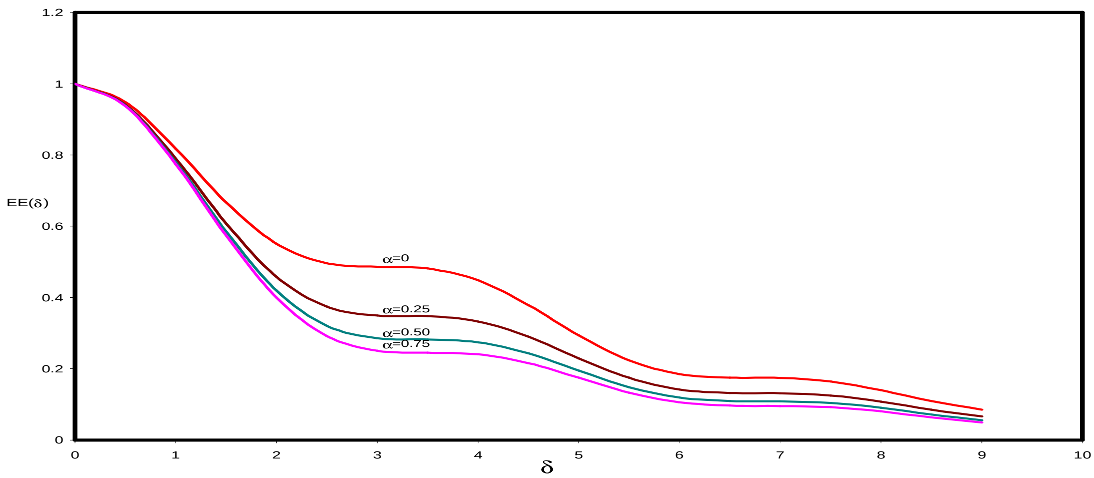

Fig.3(c): Variation of EE(δ) with δ for α =0,0.25,0.50 & 0.75; β=0.9

5 DISPLACED ENERGY $DE(\delta)$

The two parameters encircled energy factor and excluded energy represents distribution of energy within a circle of specified radius and outside it in the point spread function of a particular real optical system respectively. The displaced energy $DE(\delta)$ is the difference of the encircled energy factor of the perfect lens system and that of the real one.

$$DE(\delta) = \left[EEF(\delta)\right]_A - \left[EEF(\delta)\right]_p \ldots\ldots\ldots\ldots\ldots(9)$$

where $\left[EEF(\delta)\right]_A$ *and* $\left[EEF(\delta)\right]_p$ refer respectively to the perfect and the real system. In the above expression $\left[EEF(\delta)\right]_A$ gives encircled energy for the perfect ideal system $(\beta = 0)$ and $\left[EEF(\delta)\right]_p$ is the encircled energy for the actual apodised case for the same radius 'δ'.

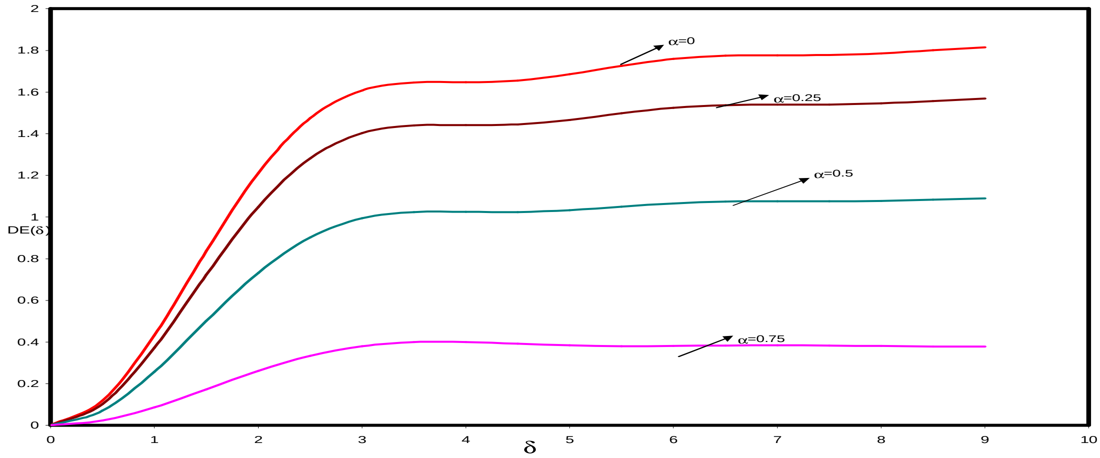

Fig.4(a): Variation of DE(δ) with δ for α =0,0.25,0.50 & 0.75 ; β=0.3

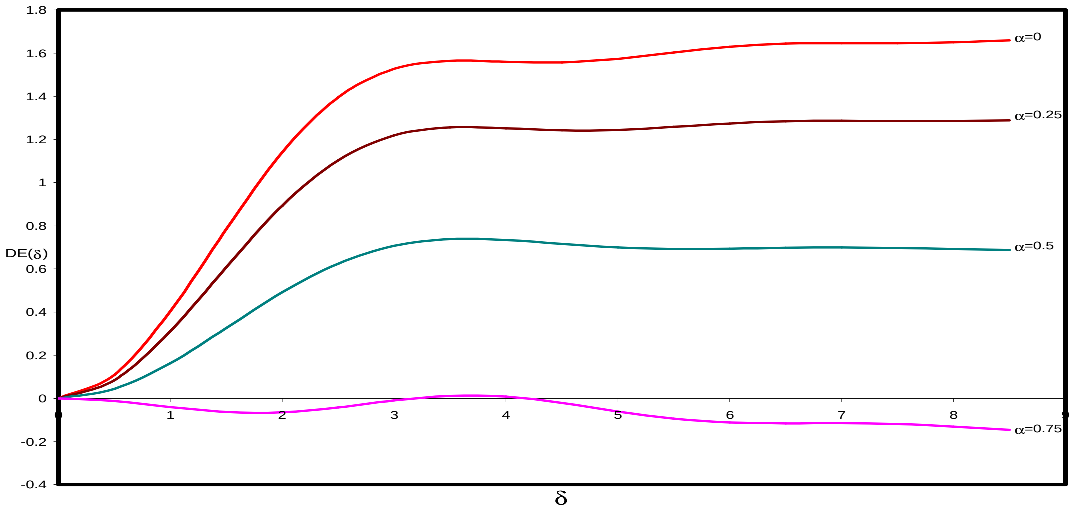

Fig.4(b): Variation of DE(δ) with δ for α =0,0.25,0.50 & 0.75 ; β=0.6

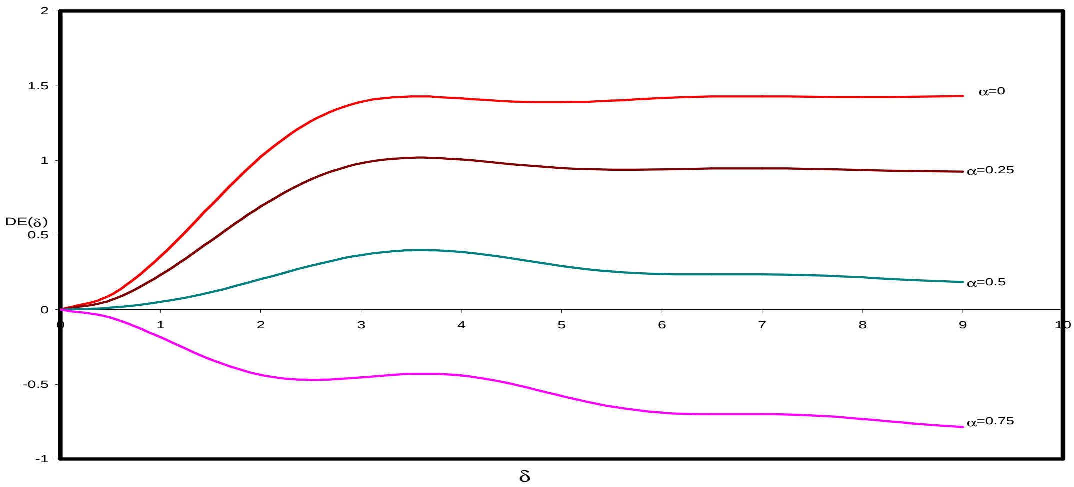

Fig.4(c): Variation of DE(δ) with δ for α =0,0.25,0.50 & 0.75 ; β =0.9

We have shown the variation of DE (δ) with δ for various values of α =0, 0.25, 0.50 & 0.75 for particular values of β =0.3, 0.6 & 0.9 in figures 4.4(a) to 4.4(c). The curves are self evident and need not be discussed any further.

We have shown the variation of $DE(\delta)$ with δ for various of $\alpha = 0, 0.25, 0.50 \,\&\, 0.75$ for particular values of $\beta = 0.3, 0.6 \,\&\, 0.9$ in Fig 4(a) to 4(c). The curves are self- Evident and need not be discussed any further.

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