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Effects of wall properties and heat transfer on the peristaltic transport of a jeffrey fluid in a channel

M. Arun Kumar[*], S. Sreenadh[*] and A. N. S. Srinivas[**]

**Department of Mathematics, Sri Venkateswara University, Tirupati(A.P), India*
***School of Advanced Sciences, VIT University, Vellore(T.N), India*

ABSTRACT

A mathematical model is constructed to study the effect of heat transfer and elasticity of flexible walls in swallowing of food bolus through the oesophagus. The food bolus is supposed to be Jeffrey fluid and the geometry of wall surface of oesophagus is considered as peristaltic wave. The expressions for temperature field, axial velocity, transverse velocity and stream function are obtained under the assumptions of low Reynolds number and long wavelength. The effects of thermal conductivity, Grashof number, rigidity, stiffness of the wall and viscous damping force parameters on velocity, temperature and stream function have been studied. It is noticed that increase in thermal conductivity β*, Grashof number* Gr *and the Jeffrey parameter* λ_1 *results in increase of velocity distribution. It is found that that the size of the trapped bolus increases with increase* λ_1*.*

Keywords: Peristaltic transport, Jeffrey fluid, Oesophagus, food bolus, channel.

INTRODUCTION

Peristaltic transport is a mechanism of pumping fluids in tubes when progressive wave of area contraction or expansion propagates along the length on the boundary of a distensible tube containing fluid. Peristalsis has quite important applications in many physiological systems and industry. It occurs in swallowing food through the oesophagus, chyme motion in the gastrointestinal tract, in the vasomotion of small blood vessels such as venules, capillaries and arterioles, urine transport from kidney to bladder. In view of these biological and industrial applications, the peristaltic flow has been studied with great interest. Many of the physiological fluids are observed to be non-Newtonian. Peristaltic flow of a single fluid through an infinite tube or channel in the form of sinusoidal wave motion of the tube wall is investigated by Burns and Parkes [1], Hanin [2],Shapiro et al.[3] etc,. In the literature some important analytical studies on peristaltic transport of non–Newtonian fluids are available Devi and Devanathan [4], Shukla and Gupta [5], Srivastava and Srivastava [6], Usha and Rao [7], Vajravelu et al. [8,9], Hayat et al. [10,11,12].

Further an interesting fact is that in oesophagus, the movement of food is due to peristalsis. The food moves from mouth to stomach even when upside down. Oesophagus is a long muscular tube commences at the neck opposite the long border of cricoids cartilage and extends from the lower end of the pharynx to the cardiac orifice of the stomach. The swallowing of the food bolus takes place due to the periodic contraction of the esophageal wall. Pressure due to reflexive contraction is exerted on the posterior part of the bolus and the anterior portion experiences relaxation so that the bolus moves ahead. The contraction is practically not symmetric, yet it contracts to zero lumen and squeezes it marvelously without letting any part of the food bolus slip back in the opposite direction. This shows the importance of peristalsis in human beings. Mitra and Prasad [13] studied the influence of wall properties on the Poiseuille flow under peristalsis. Mathematical model for the esophageal swallowing of a food bolus is analyzed by Mishra and Pandey [14]. Kavitha et al., [15] analysed the peristaltic flow of a micropolar fluid in a vertical channel with longwave length approximation. Reddy et al., [16] studied the effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining. Lakshminarayana et al., [17] studied the peristaltic pumping of a conducting fluid in a channel with a porous peripheral layer. Radhakrishnamacharya and Srinivasulu [18] studied the influence of wall properties on peristaltic transport with heat transfer. Rathod et al., [19] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. A new model for study the effect of wall properties on peristaltic transport of a viscous fluid has been investigated by Mokhtar and Haroun [20], Srinivas et al., [21] studied the effect of slip, wall properties and heat transfer on MHD peristaltic transport. Sreenadh et al., [22] studied the effects of wall properties and heat transfer on the peristaltic transport of food bolus through oesophagus. Afsar Khan et al., [23] analyzed the peristaltic transport of a Jeffrey fluid with variable viscosity through a porous medium in an asymmetric channel.

In view of the importance of non-Newtonian physiological fluid motion by peristalsis we consider a mathematical model to study the effects of wall properties and heat transfer in swallowing the food bolus through the oesophagus. The simplest non-Newtonian physiological fluid is taken as Jeffrey fluid. The results are analyzed for different values of physical parameters.

Mathematical Formulation

Consider the peristaltic flow of an incompressible Jeffrey fluid in a flexible channel with flexible induced by sinusoidal wave trains propagating with constant speed c along the channel walls. The wall deformation is given by

$$H(\bar{x},\bar{t}) = a - \bar{\phi}\, Cos^2 \frac{\pi}{\lambda}(\bar{x} - c\bar{t}) \qquad (1)$$

where $\bar{h}, \bar{x}, \bar{t}, a, \bar{\phi}, \lambda$ and c represent transverse vibration of the wall, axial coordinate, time, half width of the channel, amplitude of the wave, wavelength and wave velocity respectively.

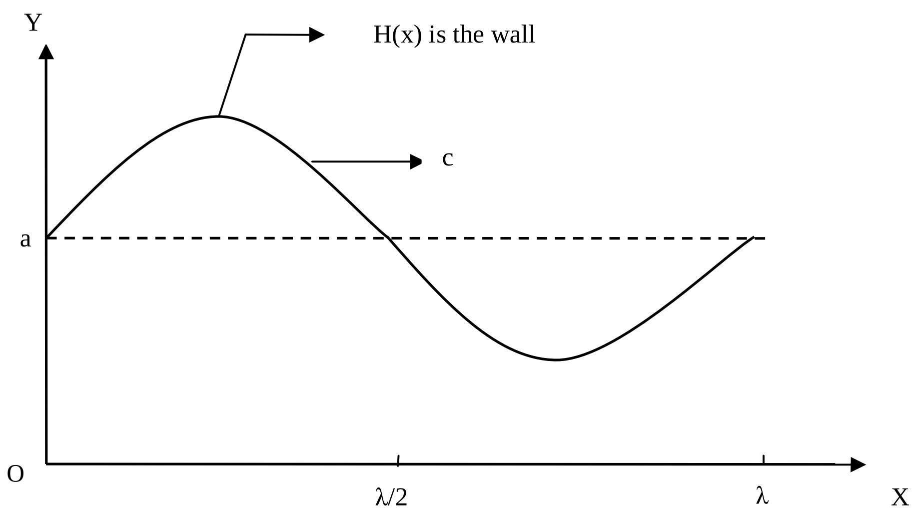


Figure1. Physical Model

The governing equations of motion of incompressible Jeffrey fluid are given as

$$\rho\left(\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{u}=-\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\mu}{1+\lambda_1}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\rho g\alpha(T-T_0) \tag{2}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{v}=-\frac{\partial \bar{p}}{\partial \bar{y}}+\frac{\mu}{1+\lambda_1}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{3}$$

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{4}$$

$$\rho c_p\left(\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)T=K\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right)+\Phi \tag{5}$$

where ρ is the fluid density, $\bar{u}$ axial velocity , $\bar{v}$ Transverse velocity, $\bar{y}$ transverse coordinate, $\bar{p}$ pressure, μ fluid viscosity, g acceleration due to gravity, α coefficient of linear thermal expansion of fluid, T temperature, c_p specific heat at constant pressure, K thermal conductivity and Φ constant heat addition/absorption.

The velocity and temperatures at the central line and the wall of the peristaltic channel are given as

$$T=T_0 \ at \ \bar{y}=0$$
$$T=T_1 \ at \ \bar{y}=\bar{h}$$

where T_0 is the temperature at centre is line and T_1 is the temperature on the wall of peristaltic channel.

The governing equation of motion of the flexible wall may be expressed as $L^* = \bar{p}-\bar{p}_0$ (6)

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$L^* = -\tau\frac{\partial^2}{\partial x^2}+m_1\frac{\partial^2}{\partial t^2}+c\frac{\partial}{\partial t}$$

Continuity of stress at $y=\bar{h}$ and using momentum equation, yield

$$\frac{\partial}{\partial \bar{x}}L^*\left(\bar{h}\right)=\frac{\partial \bar{p}}{\partial \bar{x}}=\frac{\mu}{1+\lambda_1}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\rho g\alpha(T-T_0)-\rho\left(\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{u} \tag{7}$$

Here τ is the elastic tension in the membrane, m_1 is the mass per unit area, C is the coefficient of viscous damping forces. Introducing the following non-dimensional quantities,

$$x=\frac{\bar{x}}{\lambda},\ y=\frac{\bar{y}}{a},\ u=\frac{\bar{u}}{c},\ v=\frac{\bar{v}}{c\delta},\ \delta=\frac{a}{\lambda},\ p=\frac{a^2\bar{p}}{\mu c\lambda},\ t=\frac{c\bar{t}}{\lambda},\ h=\frac{\bar{h}}{a},\psi=\frac{\bar{\psi}}{ac},Q=\frac{\bar{Q}}{ac}$$
$$\phi=\frac{\bar{\phi}}{a},\mathrm{Re}=\frac{\rho ca\delta}{\mu},Gr=\frac{gpa^2\alpha(T_1-T_0)}{c\mu^2},\theta=\frac{T-T_0}{T_1-T_0},\beta=\frac{a^2\Phi}{k(T_1-T_0)},\ pr=\frac{\mu c_p}{k} \tag{8}$$

where δ is the length of the channel, $\bar{\psi}$ is the Stream function, $\bar{Q}$ is the Volume flow rate, Re is Reynolds number, Gr is the Grashof number, θ dimensionless temperature, β is the dimensionless heat source/sink parameter and $\Pr$ is Prandtl number, we obtain the dimensionless governing equations and boundary conditions as follows

$$h(x,t)=1-\phi\,Cos^2\pi(x-t) \tag{9}$$

$$Re\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+Gr\theta \tag{10}$$

$$Re\,\delta^3\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\frac{1}{1+\lambda_1}\left(\delta^4\frac{\partial^2 v}{\partial x^2}+\delta^2\frac{\partial^2 v}{\partial y^2}\right) \tag{11}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{12}$$

$$\frac{(Re)\,(Pr)}{(T_1-T_0)}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\left(\theta\left(T_1-T_0\right)+T_0\right)=\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\beta \tag{13}$$

$$\frac{\delta^2}{1+\lambda_1}\frac{\partial^2 u}{\partial x^2}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+Gr\,\theta-Re\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right)(h) \tag{14}$$

$$\begin{aligned}&\frac{\partial u}{\partial y}=0 \ \text{ at } y=0\\ &u=0 \ \text{ at } y=h\\ &v=0 \ \text{ at } y=0\\ &\theta=0 \ \text{ at } y=0,\ \theta=1 \ \text{ at } y=h\end{aligned} \tag{15}$$

3 Solution of the problem
Under the assumptions of long wavelength $\delta \ll 1$ and low Reynolds number, equations (9)-(15) reduce to

$$h(x,t)=1-\phi Cos^2\pi(x-t) \tag{16}$$

$$\frac{\partial p}{\partial x}=\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+Gr\,\theta \tag{17}$$

$$\frac{\partial p}{\partial y}=0 \tag{18}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{19}$$

$$\frac{\partial^2\theta}{\partial y^2}+\beta=0 \tag{20}$$

$$\theta=0 \text{ at } y=0,\ \theta=1 \text{ at } y=h \tag{21}$$

$$\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+Gr\,\theta=\left(E_1\frac{\partial^3 h}{\partial x^3}+E_2\frac{\partial^3 h}{\partial x\partial t^2}+E_3\frac{\partial^2 h}{\partial x\partial t}\right) \tag{22}$$

The following boundary conditions are imposed on the governing equations to model the problem under consideration:

$$\frac{\partial u}{\partial y}=0 \text{ at } y=0 \tag{23}$$

$$u=0 \text{ at } y=h \tag{24}$$

$$v=0 \text{ at } y=0 \tag{25}$$

Equation (18) shows that P is not a function of y. Now on differentiating equation (17) with respect to y, the compatibility equation as follows

$$\frac{1}{1+\lambda_1}\frac{\partial^3 u}{\partial y^3}+Gr\frac{\partial\theta}{\partial y}=0 \tag{26}$$

$$\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+Gr\,\theta=\left(E_1\frac{\partial^3 h}{\partial x^3}+E_2\frac{\partial^3 h}{\partial x\partial t^2}+E_3\frac{\partial^2 h}{\partial x\partial t}\right) \tag{27}$$

The closed form solution for equations (17) and (20) with the boundary conditions (21), (23) and (24) is given by

$$\theta=\frac{y}{h}+\frac{\beta}{2}\left(hy-y^2\right) \tag{28}$$

$$u=(1+\lambda_1)\left\{\begin{array}{l}\pi^2\phi(y^2-h^2)\left\{E_3\left(Sin^2\pi(x-t)-Cos^2\pi(x-t)\right)-(E_1+E_2)\left(4\pi Cos\pi(x-t)\,Sin\pi(x-t)\right)\right\}\\ \quad+\frac{Gr}{6}\left(\frac{\beta}{4}\left(h^4-2y^3h+y^4\right)-\frac{1}{h}\left(y^3-h^3\right)\right)\end{array}\right\} \tag{29}$$

Integrating the continuity equation with respect to y, using the above equation and the boundary condition (25), we obtain transverse velocity as

$$v=-(1+\lambda_1)\left\{\begin{array}{l}\pi^2\phi y\left(\frac{y^3}{3}-h^2\right)\left\{4\pi E_3\left(Sin4\pi(x-t)-(E_1+E_2)4\pi^2Cos2\pi(x-t)\right)\right\}\\-2\pi^2\phi hy\frac{\partial h}{\partial x}\left\{E_3Cos4\pi(x-t)+(E_1+E_2)\left(2\pi\, Sin2\pi(x-t)\right)\right\}\\-\frac{Gr}{6}\left(\frac{\beta}{4}\left(4h^4y-2\frac{y^4}{4}\right)-\left(\frac{y^3}{3h^2}+2hy\right)\right)\frac{\partial h}{\partial x}\end{array}\right\} \tag{30}$$

Stream function can be obtained by integrating equation and using the condition $\psi=0$ at $y=0$. It is given by

$$\psi=(1+\lambda_1)\left\{\begin{array}{l}\pi^2\phi y\left(\frac{y^3}{3}-h^2\right)\left\{E_3\left(Sin^2\pi(x-t)-Cos^2\pi(x-t)\right)-(E_1+E_2)\left(4\pi Cos\pi(x-t)\,Sin\pi(x-t)\right)\right\}\\+\frac{Gr}{6}\left(\frac{\beta}{4}\left(h^4y-\frac{y^4h}{2}+\frac{y^5}{5}\right)-\frac{1}{h}\left(\frac{y^4}{4}-h^3y\right)\right)\end{array}\right\} \tag{31}$$

RESULTS AND DISCUSSION

In order to observe the quantitative effects of various parameters involved in the analysis, the velocity, temperature and stream functions are calculated for various values of these physical parameters. The numerical evaluations of the analytical results and some significant results are displayed graphically from Figures (2) - (14). From Figures (2), (3) and (4), it is observed that increase in thermal conductivity β, Grashof number Gr and the Jeffrey parameter λ_1 results in increase of velocity distribution. Figure (5) displays the effect of rigidity parameter in the presence of stiffness $(E_2 \neq 0)$ and viscous damping force $(E_3 \neq 0)$. It is noticed that the velocity increases with increase in rigidity parameter. A similar observation is made for different values of E_2 in the presence of other parameters i.e., rigidity and viscous damping force which is shown in Figure (6). From figure (7), we can see the influence of viscous damping force on velocity distribution in the presence of rigidity and stiffness. One can observe that the velocity decreases with the increase in E_3. The variation in temperature for various values of thermal conductivity is shown in Figure (8). The temperature increases with the increase in β.

An interesting phenomenon of peristalsis is trapping in which streamlines split to trap a bolus in the wave frame. The effect of thermal conductivity on trapping is analyzed in Figure (9). It can be concluded that the size of the trapped bolus in the left side of the channel decreases when β increases where as it has opposite behavior in the right hand side of the channel. The influence of Grashof number on trapping is analyzed in Figure (10). It shows that the size of the left trapped bolus decreases with increase in Gr where as the size of the right trapped bolus increases with increase in Gr. The effect of λ_1 on trapping can be seen in Figure (11). We notice that the size of the bolus increases with increase λ_1. The effect of E_1 on trapping can be seen in figure (12). We notice that the size of the bolus increases with increase in E_1. Figure (13) shows the influence of E_2 on trapping. We observe that the size of the trapped bolus decreases with increase in E_2. The effect of E_3 on trapping is shown in figure (14). It is shown that the size of the left bolus decreases where as the right bolus increases with increase in E_3.

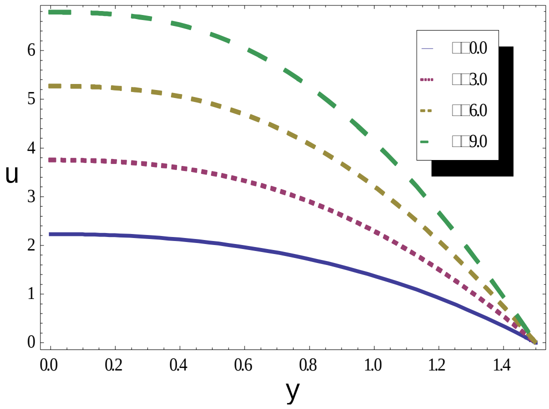


Fig 2. Velocity distribution for different values of β with
$E_1 = 0.7, E_2 = 0.5, E_3 = 0.1, t = 0.5, \beta = 2, y = 0.5, \lambda_1 = 0.2$

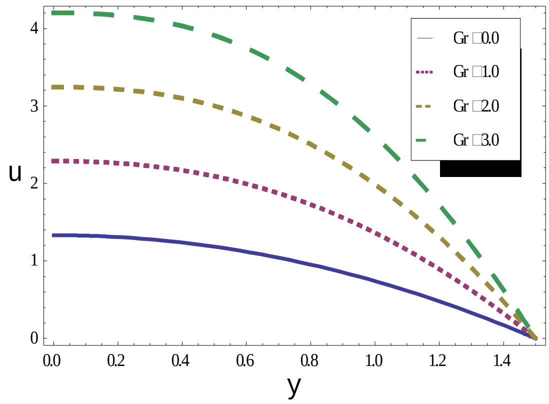


Fig3. Velocity distribution for different values of Gr with
$E_1 = 0.7, E_2 = 0.5, E_3 = 0.1, t = 0.5, \beta = 2, y = 0.5, \lambda_1 = 0.2.$

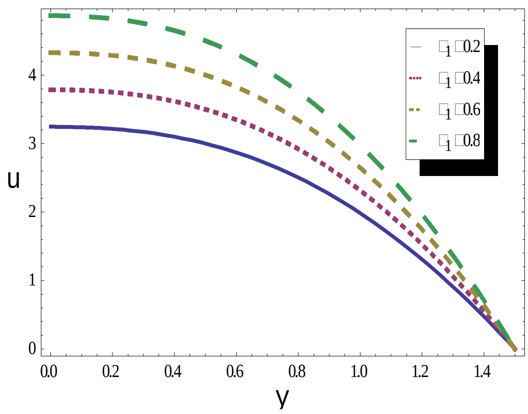


Fig 4. Velocity distribution for different values of λ_1 with
$E_1 = 0.7, E_2 = 0.5, E_3 = 0.1, t = 0.5, \beta = 2, y = 0.5, Gr = 2.$

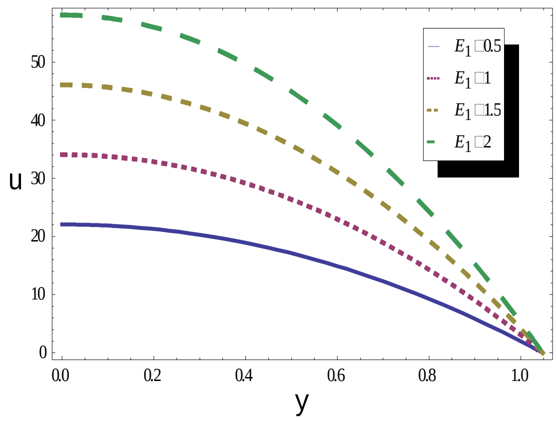


Fig 5. Velocity distribution for different values of E_1 with
$E_2 = 0.5, E_3 = 0.1, \lambda_1 = 0.2, t = 0.5, \beta = 2, y = 0.5, Gr = 2.$

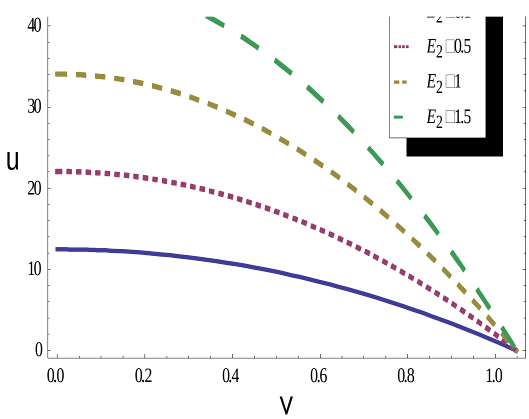


Fig 6. Velocity distribution for different values of E_2 with
$E_1 = 0.7, E_3 = 0.1, \lambda_1 = 0.2, t = 0.5, \beta = 2, y = 0.5, Gr = 2.$

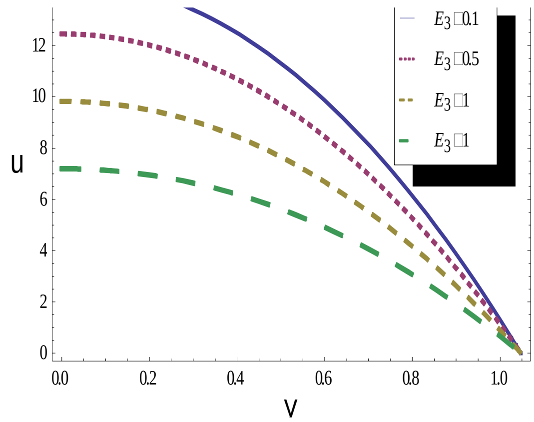


Fig 7. Velocity distribution for different values of E_3 with
$E_1 = 0.7, E_2 = 0.5, \lambda_1 = 0.2, t = 0.5, \beta = 2, y = 0.5, Gr = 2.$

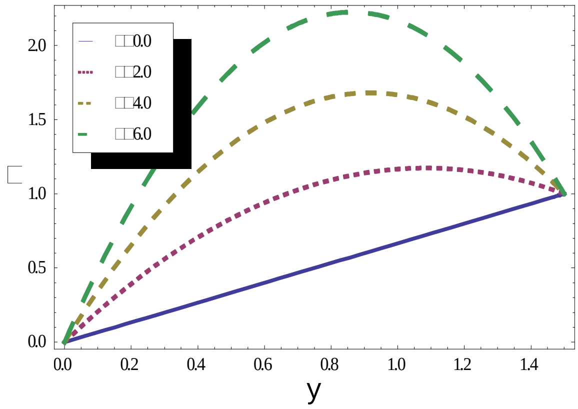
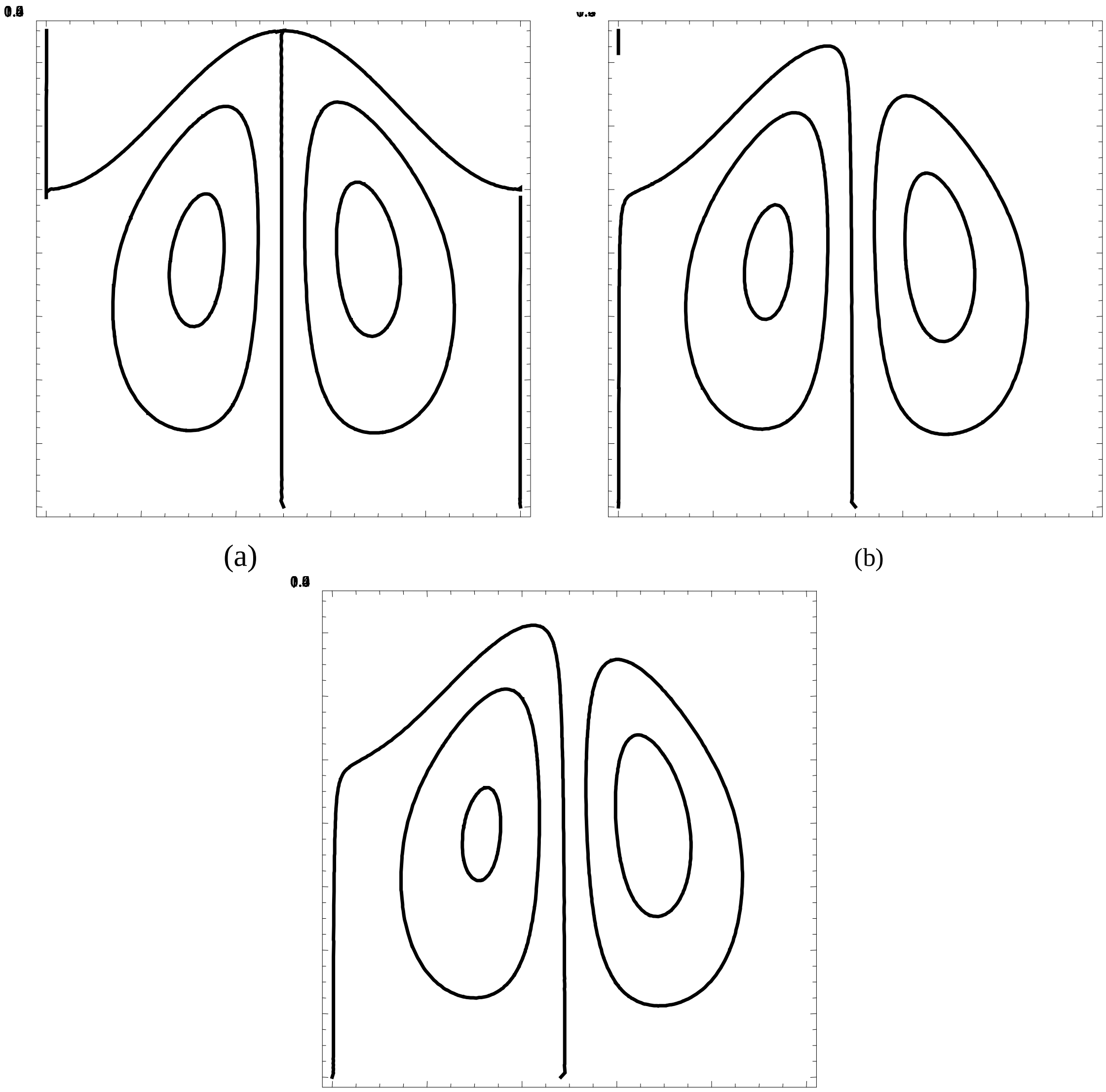


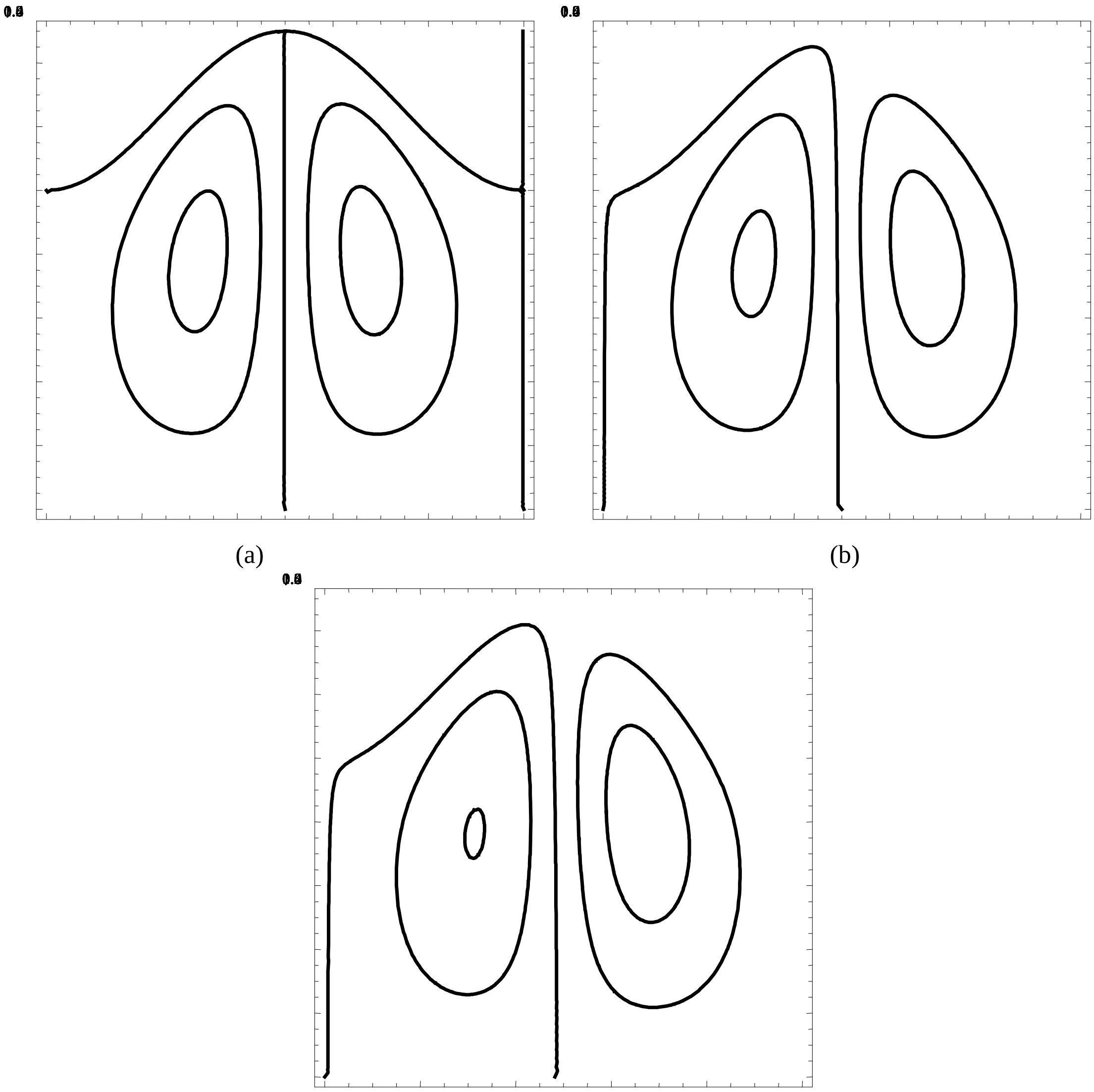
Fig 8: The temperature distribution for different values of θ **with** $E_1 = 0.7, E_2 = 0.5, \lambda_1 = 0.2, t = 0.5, y = 0.5, Gr = 2.$



(a) (b)

(c)

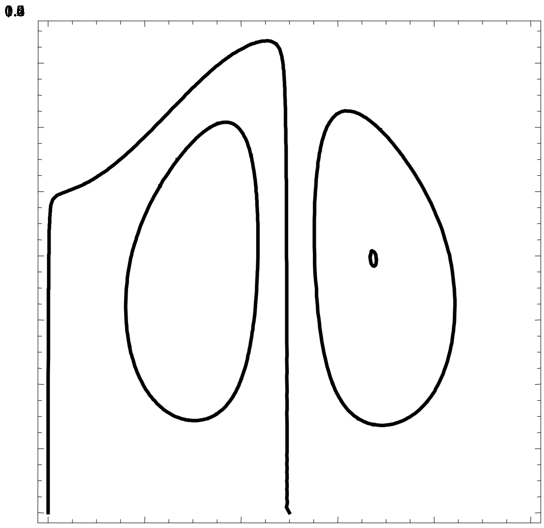
Fig 9: Effect of β **on Trapping (a)** $\beta = 0$ **(b)** $\beta = 4$ **(c)** $\beta = 8$ **for** $E_1 = 0.7, E_2 = 0.5, E_3 = 1, \lambda_1 = 0.2, t = 0.1, Gr = 2.$



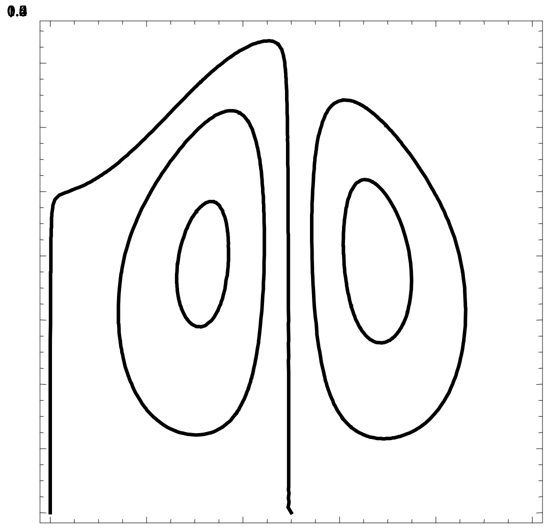
(a) (b) (c)

Fig 10: Effect of Gr **on Trapping (a)** $Gr=0$ **(b)** $Gr=2$ **(c)** $Gr=4$ **for**

$E_1=0.7, E_2=0.5, E_3=1, \lambda_1=0.2, t=0.1, \beta=2.$



(a)



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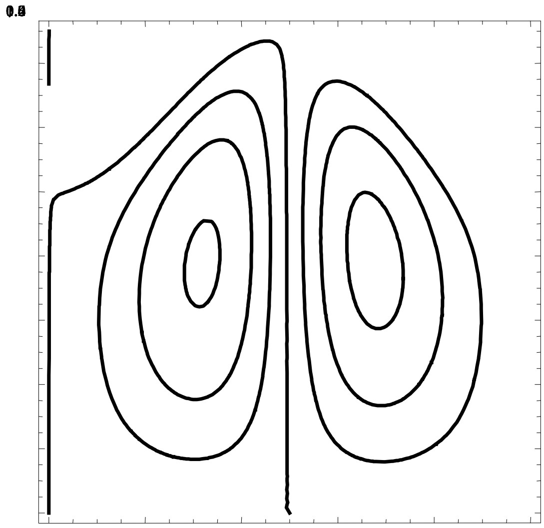


(c)

Fig 11: Effect of λ_1 **on Trapping (a)** $\lambda_1 = 0$ **(b)** $\lambda_1 = 0.2$ **(c)** $\lambda_1 = 0.4$ **for** $E_1 = 0.7, E_2 = 0.5, E_3 = 1, Gr = 2, t = 0.1, \beta = 2.$

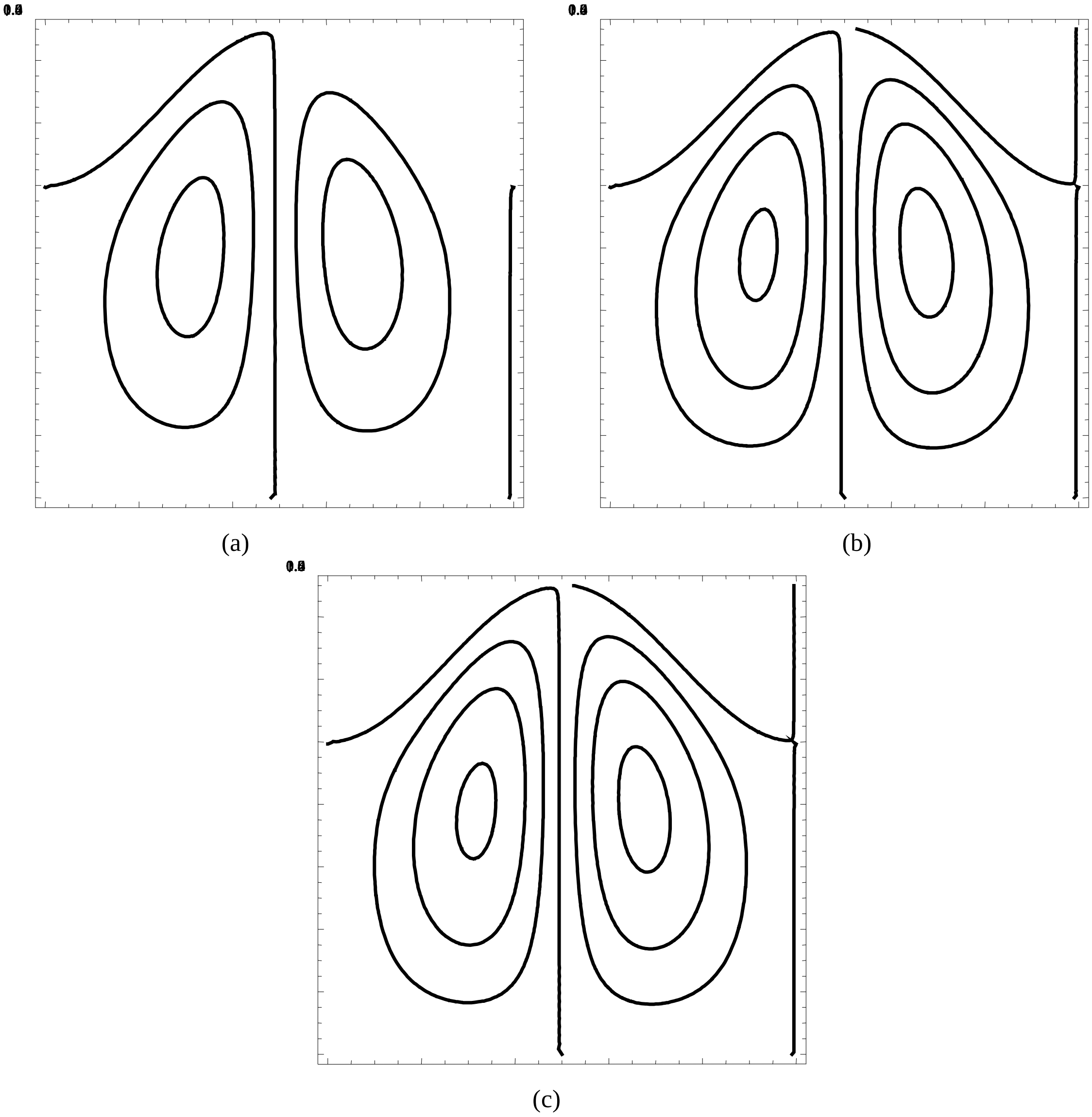
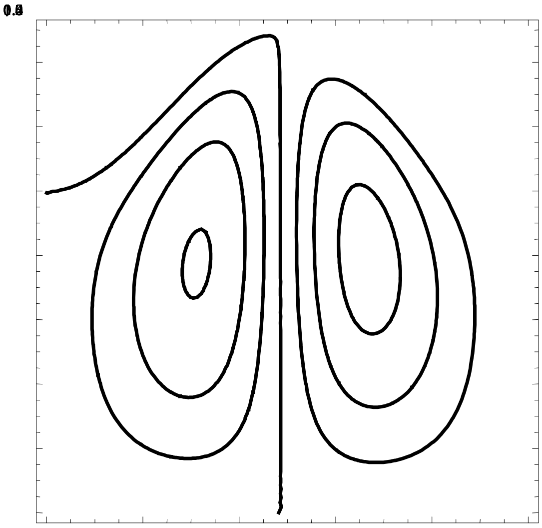


Fig 12: Effect of E_1 **on Trapping (a)** $E_1 = 1$ **(b)** $E_1 = 1.5$ **(c)** $E_1 = 2$ **for** $E_2 = 0.5, E_3 = 1, \lambda_1 = 0.2, Gr = 2, t = 0.1, \beta = 2.$



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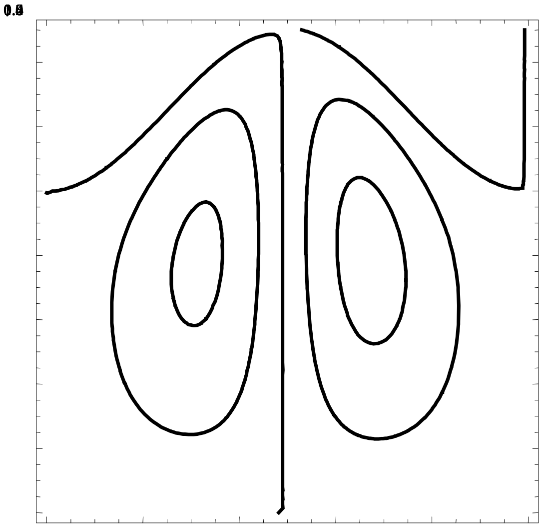

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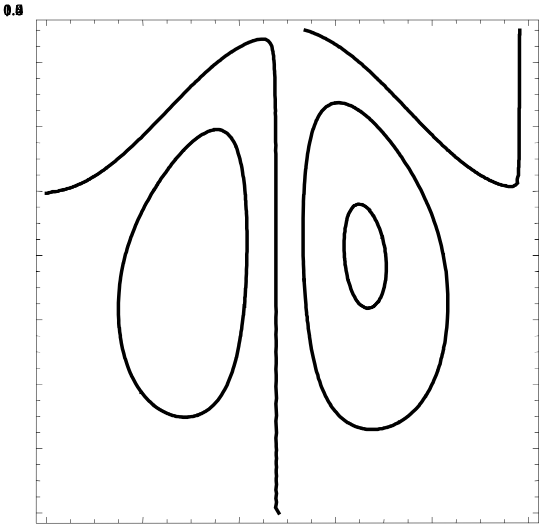
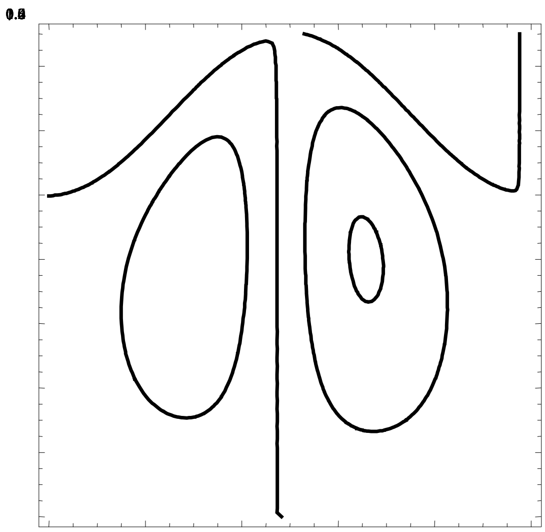
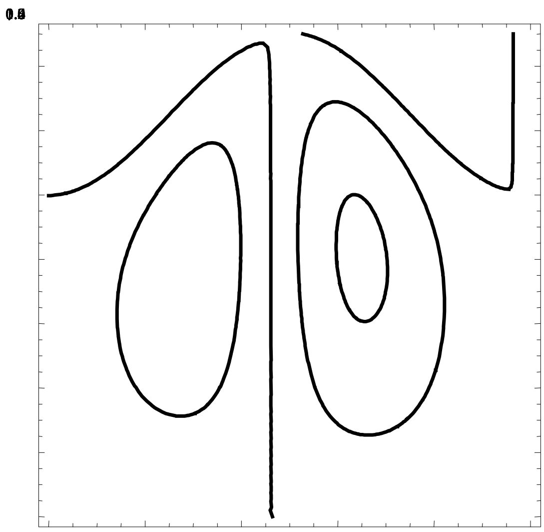


(c)

Fig 13: Effect of E_2 **on Trapping (a)** $E_2 = 0.1$ **(b)** $E_2 = 0.5$ **(c)** $E_2 = 0.9$ **for**
$E_1 = 0.7, E_3 = 1, \lambda_1 = 0.2, Gr = 2, t = 0.1, \beta = 2.$



(a)



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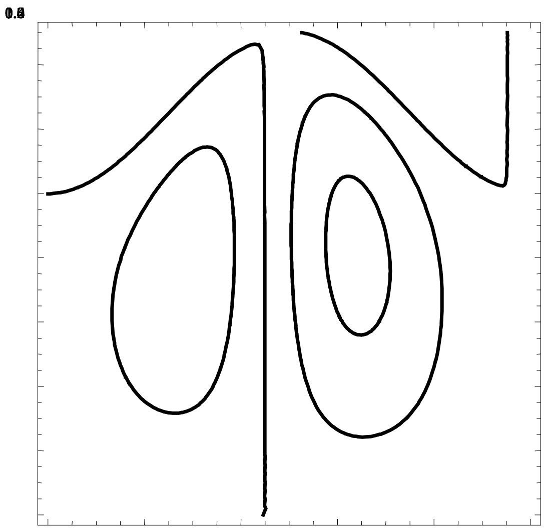

(c)

Fig 14: Effect of E_3 **on Trapping (a)** $E_3 = 1$ **(b)** $E_3 = 1.5$ **(c)** $E_3 = 2$ **for**
$E_1 = 0.7, E_2 = 0.1, \lambda_1 = 0.2, Gr = 2, t = 0.1, \beta = 2.$

CONCLUSION

The present study deals with the combined effect of wall properties and heat transfer on the peristaltic transport of a Jeffrey fluid in a two dimensional channel. We obtained the analytical solution of the problem under long wavelength and low Reynolds number assumptions. Some of the interesting findings are

1. The velocity increases with increase in thermal conductivity β , Grashof number Gr and the Jeffrey parameter λ_1 .
2. It is found that that the size of the trapped bolus increases with increase λ_1 .
3. The coefficient of temperature increases with increasing values of thermal conductivity.

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