Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2012, 3 (2):720-734

ISSN: 0976-8610
CODEN (USA): AASRFC

Effects of viscous dissipation of permeable fluid on laminar mixed convection in a vertical double passage channel

S. N. Gaikwad[1] and Rahuldev B. M.[2]

[1]Department of Mathematics, Jnana Ganga Campus, Gulbarga University, Gulbarga, Karnataka, India
[2]Department of Mathematics, Government Engineering College, Raichur, Karnataka, India

---

ABSTRACT

The fully developed flow and heat transfer in a vertical double passage channel containing permeable fluid is studied analytically using regular perturbation method. The Brinkman model is used for flow through porous media, viscous and Darcy dissipation terms are included in the energy equation. The channel is divided into two passages by means of a thin, perfectly conductive baffle and the walls are uniformly heated. The effect of porous parameter σ *and mixed convection parameter* λ *on the velocity and temperature profiles near the hot and cold wall are analyzed. The result shows that these effects mainly depend on the baffle position.*

Keywords: mixed convection, viscous dissipation, double-passage, Permeable fluid.

INTRODUCTION

Convective heat transfer is the study of heat transfer process between the layers of a fluid, when the fluid is in motion and/or between a fluid in motion and a boundary surface in contact with it when they are not at different temperatures. The convective mode of heat transfer is generally divided into two basic process. If the motion of the fluid arises from an external agent for example, fan, wind or the motion of heated object itself. Then the process is termed forced convection. If on the other hand no such externally induced force is provided and the flow arises naturally from the effect of a density difference, resulting from a temperature or concentration difference in a body force field such as the gravitation field, then the process is termed as natural convection. The density difference gives rise to buoyancy force to which flow is generated. Free convection heat transfer between a finite vertical parallel plates suspended in calm viscous fluid had been extensively investigated as one of the fundamental problem of heat transfer. Natural convection is an important heat transfer mechanism in the technology of building insulations. From the point of basic research in heat transfer, this phenomenon is being studied mainly in terms of simple model of free convection in rectangular enclosures, filled with viscous fluid. The subject of free convection in enclosures is extensive and has numerous applications in practical engineering situations. Fully developed heat transfer natural convection in vertical channel with symmetric constant wall temperatures has been studied by Bodoia and Osterle [1]. Aung [2] studied the case when the walls are heated asymmetrically. Aung and Worku [3] presented a theory for the fully developed heat transfer of combined convection in a vertical channel with asymmetric constant wall temperature. Nelson and Wood [4] presented an analytical solution for combined heat and mass transfer natural convection in vertical channel with asymmetric boundary conditions. The majority of existing studies on convective heat transfer in porous media are based on the Darcy flow model. Darcy's law, however, is found to be inadequate for the formulation of fluid flow and heat transfer problems in porous media when there is a solid boundary and the Reynolds number based on the pore size is greater than unity. Therefore, it is necessary to incorporate the boundary and inertia terms into the momentum equation. These effects have been studied for forced convection as well as for natural convection in porous media.

Natural convection heat transfer in porous media has received a world of careful attention because it frequently occurs in many physical problems and engineering applications for contemporary technology such as geothermal systems, grain storage, fiber and granular insulation, packed sphere beds, heat exchangers, chemical catalytic reactors, petroleum reservoirs, coal combustors, nuclear waste repositories, and filtration. A recent review by Tien and Vafai [6] gives the extent of the research information about natural convection in porous media and stress the importance of non-Darcy effects such as the inertia and boundary effects as the remedies of the Darcy's law in certain applications. Darcy law is an empirical formula relating the pressure gradient, the gravitational force and the bulk viscous resistance in a porous media. Thus, the mathematical formulation based on the Darcy's law will neglect the effects of solid boundary or the inertia forces on fluid flow and heat transfer through porous media. In general, the inertia and boundary effects become significant when the fluid velocity is high and the heat transfer is considered in the near-wall region, respectively. Theoretically a velocity square term and viscous term are incorporated in the momentum equations to model inertia and boundary effects, respectively. Cheng [7] *et al*. Studied numerically the non-Darcy effects on the transient natural convection boundary layer flow near an isothermal vertical flat surface embedded in a high-porosity medium. Certain porous materials, such as foam metals and fibrous media, usually have high porosities (with porosity about 0.9 □ 0.95). The analysis made by Cheng [8] *et al*. shows that the non-Darcy effects are much more consequential in high porosity media. It is also found that both the inertia and the boundary effects decrease the velocity of streaming fluid in the thermal boundary layer and reduce the heat transfer rate.

The stratified situation occures for example, in cooling ponds, lakes for solar ponds and in the atmosphere. If the vertical surface is a part of an enclosure the ambient enhancement of the heat transfer in a vertical channel is a major aim because of its practical importance in many engineering systems, such as the solar energy collection and the cooling of electronics systems. The convective heat transfer may be enhanced in a vertical channel by using rough surface, inserts, swirl flow device, turbulent promoter, *etc*. Candra *et al*. [9] investigated the use of ribbed walls, Han *et al*. [10] used V-shaped turbulence promoters, Lin *et al*. and Beitelmal *et al*. [11] demonstrated the effect of jet impingement mechanism. Recently, Dutta and Hossain [12] investigated the heat transfer and the frictional loss in a rectangular channel with inclined solid and perforated baffles.

Unfortunately, most of these methods cause a considerable drop in the pressure. Guo *et al*. [13] suggested that the convective heat transfer could be enhanced by using special inserts, which can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector rather than to promote turbulence. So, the heat transfer is considerably enhanced with as little pressure drop as possible. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. To avoid a considerable increase in the transverse thermal resistance into the channel, a thin and perfectly conductive baffle is used. The effect of such baffle on the heat transfer in a vertical channel can be found elsewhere. In working dimensions are: length=1.2 m, width =0.2m and the volume flow rate $=1.10^{-5}\,m^{3}/s$. For double-passage channels, the length-to-width ratio becomes larger as the baffle becomes near the wall. So, viscous dissipation may become important.

Keeping in view the applications of mixed convective flows through porous medium as mentioned above and to analyze the heat and mass transfer by introducing the baffle in the channel has motivated us to choose this problem.

2 Mathematical formulation

The channel shown in Figure.1. is divided into two passages by means of perfectly conductive and thin baffle. Consideration is given to a laminar, two-dimensional, incompressible, steady flow of permeable fluid in a channel. The fluid enters the channel with a uniform upward vertical velocity and constant temperature. The channel walls are subjected to different constant temperatures, which are higher than that at the entrance. The fluid properties are assumed to be constant except for the buoyancy term of the momentum equation.

For fully developed flow, it is assumed that the transverse velocity and the temperature gradient in the axial direction are zero. Darcy–Lapwood Brinkman model is used in developing the basic equations. By taking into account the effect of viscous dissipation, the governing equations are

$$\frac{\mu_{eff}}{\rho_r}\frac{d^2u_i}{dy^2} = -g\beta\left(T_i - T_r\right) + \frac{1}{\rho_r}\frac{dp_i}{dx} + \frac{\mu u_i}{\rho_r s} \tag{2}$$

$$k_{eff}\frac{d^2T_i}{dy^2} + \mu_{eff}\left(\frac{du_i}{dy}\right)^2 + \frac{\mu u_i^{\ 2}}{s} = 0 \tag{3}$$

where, The subscript 'i' denotes stream 1 or stream 2. The boundary conditions are

$$\begin{aligned} &y=0: && u_1 = 0, && T_1 = T_c \\ &y=b^*: && u_1 = u_2 = 0, && T_1 = T_2, \quad \frac{dT_1}{dy} = \frac{dT_2}{dy} \\ &y=b: && u_2 = 0, && T_2 = T_h \end{aligned} \tag{4}$$

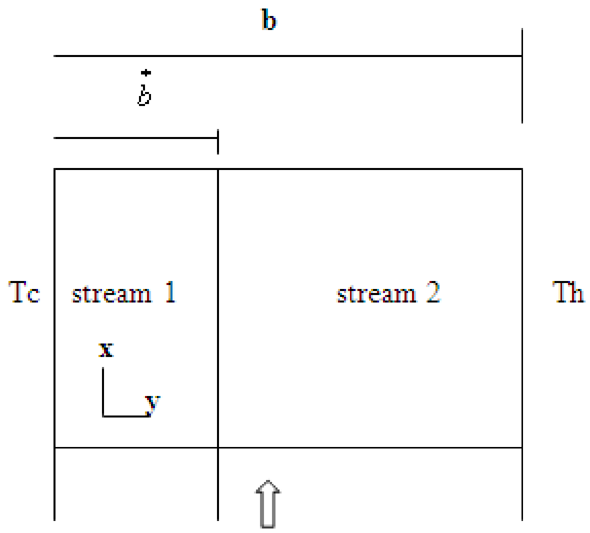

Figure. 1. Geometry and boundary conditions.

The momentum balance equation (2) has been written according to the Boussinesq approximation by invoking a linearization of the equation of state, $\rho(T)$ around the reference temperature T_r. Recently, Barletta and Zanchini (1998) have recommended the choice of the mean fluid temperature as the reference temperature in the fully developed region. In the present work, the mean temperature of the wall temperatures is chosen as the reference temperature, i.e.

$$T_r = \frac{T_c + T_h}{2}$$

The governing equations and boundary conditions can be expressed in the following dimensionless forms.

$$\frac{d^2U_i}{dY^2} = -\frac{Gr}{\mathrm{Re}}m^2\theta_i - m^2\frac{dP_i}{dX} - m^2\sigma^2U_i \tag{5}$$

$$\frac{d^2\theta_i}{dY^2}+\varepsilon\left(\frac{dU_i}{dY}\right)^2+\varepsilon\sigma^2 n^2 U_i^{\,2}=0 \qquad (6)$$

$$\begin{aligned}
&Y=0; \quad U_1=0, \quad \theta_1=-\frac{1}{2}\\
&Y=Y^*; \quad U_1=U_2=0, \quad \theta_1=\theta_{2,} \quad \frac{d\theta_1}{dy}=\frac{d\theta_2}{dy}\\
&Y=1; \quad U_2=0, \quad \theta_2=\frac{1}{2}
\end{aligned} \qquad (7)$$

where,

$$\begin{aligned}
&X=\frac{x}{b\,\mathrm{Re}}; \quad Y=\frac{y}{b}; \quad U=\frac{u}{u_r}; \quad \theta=\frac{T_h-T_r}{T_h-T_c}\\
&P=\frac{p}{\rho_r u^2{}_r}; \quad \mathrm{Re}=\frac{u_r b}{\upsilon}; \quad Gr=\frac{g\beta\left(T_h-T_c\right)b^3}{\upsilon^2}, \quad Br=\frac{\mu u_r^{\,2}}{k\left(T_h-T_c\right)}\\
&m^2=\frac{\mu}{\mu_{eff}}; \quad n^2=\frac{k}{k_{eff}}; \quad \sigma^2=\frac{b^2}{s};
\end{aligned} \qquad (8)$$

3. Analytical Solution

The basic equations governing the flow are defined in Equations (5) and (6) along with boundary conditions (7) are highly nonlinear and coupled. Hence finding exact solution is not possible. However, approximate analytical solutions can be found using the method of regular perturbation. We take flow field and the temperature field to be

Stream 1

$$U_1=U_{10}+\varepsilon U_{11} \qquad (9)$$

$$\theta_1=\theta_{10}+\varepsilon\theta_{11} \qquad (10)$$

Stream 2

$$U_2=U_{20}+\varepsilon U_{21} \qquad (11)$$

$$\theta_2=\theta_{20}+\varepsilon\theta_{21} \qquad (12)$$

Where $\varepsilon\ (=Br)$ is chosen as the perturbation parameter. Using equations (9) - (12) in the equations (5) - (7) becomes

Stream 1

Zeroth order equations

$$\frac{d^2U_{10}}{dy^2}+\frac{Gr}{\mathrm{Re}}m^2\theta_{10}-m^2\gamma_1-m^2\sigma^2U_{10}=0 \qquad (13)$$

$$\frac{d^2\theta_{10}}{dy^2}=0 \qquad (14)$$

First order equations

$$\frac{d^2U_{11}}{dy^2}+\frac{Gr}{\mathrm{Re}}m^2\theta_{11}-m^2\sigma^2U_{11}=0 \tag{15}$$

$$\frac{d^2\theta_{11}}{dy^2}+\frac{n^2}{m^2}\left(\frac{du_{10}}{dy}\right)^2+\sigma^2n^2u^2{}_{10}=0 \tag{16}$$

Stream 2
Zeroth order equations

$$\frac{d^2U_{20}}{dy^2}+\frac{Gr}{\mathrm{Re}}m^2\theta_{20}-m^2\gamma_2-m^2\sigma^2U_{20}=0 \tag{17}$$

$$\frac{d^2\theta_{20}}{dy^2}=0 \tag{18}$$

First order equations

$$\frac{d^2U_{21}}{dy^2}+\frac{Gr}{\mathrm{Re}}m^2\theta_{21}-m^2\sigma^2U_{21}=0 \tag{19}$$

$$\frac{d^2\theta_{21}}{dy^2}+\frac{n^2}{m^2}\left(\frac{du_{20}}{dy}\right)^2+\sigma^2n^2u^2{}_{20}=0 \tag{20}$$

Zeroth order boundary conditions

$$\begin{aligned}
&U_{10}=0; && at && Y=0\\
&U_{10}=0; && at && Y=Y^*\\
&U_{20}=0; && at && Y=1\\
&U_{20}=0; && at && Y=Y^*\\
&\theta_{10}=-1/2 && at && Y=0\\
&\theta_{20}=1/2 && at && Y=1\\
&\theta_{10}=\theta_{20} && at && Y=Y^*\\
&\frac{d\theta_{10}}{dy}=\frac{d\theta_{20}}{dy} && at && Y=Y^*
\end{aligned} \tag{21}$$

First order boundary conditions

$$\begin{aligned}
&U_{11}=0 && at && Y=0\\
&U_{11}=0 && at && Y=Y^*\\
&U_{21}=0 && at && Y=1\\
&U_{21}=0 && at && Y=Y^*\\
&\theta_{11}=0 && at && Y=0\\
&\theta_{21}=0 && at && Y=1\\
&\theta_{11}=\theta_{21} && at && Y=Y^*
\end{aligned}$$

$$\frac{d\theta_{11}}{dy} = \frac{d\theta_{21}}{dy} \quad at \;\; Y = Y* \tag{22}$$

Zeroth order solutions

The solutions of zeroth order differential equations (13), (14), (17), (18) along with boundary and interface conditions (21) are

Stream 1

$$U_{10} = d_1 c\,\text{osh}(\alpha y) + d_2 s\,\text{inh}(\alpha y) + l_1 y - l_2 y \tag{23}$$

$$\theta_{10} = c_1 y + c_2 \tag{24}$$

Stream 2

$$U_{20} = d_3 c\,\text{osh}(\alpha y) + d_4 s\,\text{inh}(\alpha y) + l_1 y - l_3 y \tag{25}$$

$$\theta_{20} = c_3 y + c_4 \tag{26}$$

First order solutions

The solutions of first order differential equations (15) - (16) using boundary and interface conditions (22) are,

Stream 1

$$\begin{aligned} U_{11} =& d_5 cosh(\alpha y) + d_6 sinh(\alpha y) + f_{10} cosh(2\alpha y) + f_{11} s\,\text{inh}(2\alpha y) \\ &+ f_{12} y^2 cosh(\alpha y) + f_{13} y^2 sinh(\alpha y) + f_{14} y cosh(\alpha y) \\ &+ f_{15} y sinh(\alpha y) + f_{16} y^4 + f_{17} y^3 + f_{18} y^2 \\ &+ f_{19} y + f_{20} \end{aligned} \tag{27}$$

$$\begin{aligned} \theta_{21} =& f_1 cosh(2\alpha y) + f_2 sinh(2\alpha y) + f_3 y cosh(\alpha y) + f_4 y sinh(\alpha y) \\ &+ f_5 cosh(\alpha y) + f_6 sinh(\alpha y) + f_7 y^4 + f_8 y^3 + f_9 y^2 + c_5 y + C_6 \end{aligned} \tag{28}$$

Stream 2

$$\begin{aligned} U_{21} =& d_7 cosh(\alpha y) + d_8 sinh(\alpha y) + f_{30} cosh(2\alpha y) + f_{31} s\,\text{inh}(2\alpha y) \\ &+ f_{32} y^2 cosh(\alpha y) + f_{33} y^2 sinh(\alpha y) + f_{34} y cosh(\alpha y) \\ &+ f_{35} y sinh(\alpha y) + f_{36} y^4 + f_{37} y^3 + f_{38} y^2 \\ &+ f_{39} y + f_{40} \end{aligned} \tag{29}$$

$$\begin{aligned} \theta_{21} =& f_{21} cosh(2\alpha y) + f_{22} sinh(2\alpha y) + f_{23} y cosh(\alpha y) + f_{24} y sinh(\alpha y) \\ &+ f_{25} cosh(\alpha y) + f_{26} sinh(\alpha y) + f_{27} y^4 + f_{28} y^3 + f_{29} y^2 + c_7 y + C_8 \end{aligned} \tag{30}$$

RESULTS AND DISCUSSION

The fully developed flow and heat transfer in a vertical double passage channel containing permeable fluid is studied analytically using regular perturbation method. The Brinkman model is used for flow through porous media, viscous and Darcy dissipation terms are included in the energy equation. Figures 2-5 represents that the effect of porous parameter σ and mixed convection parameter λ on the velocity and the temperature are shown in when the baffle is placed near the cold wall, Figures 6-8 are the graphs when the baffle is placed in the middle of the channel

The variation of velocity for different values of porous parameter σ and mixed convection parameter λ which is the ratio of Grashof number to Reynolds number is shown in Figure 2 and 3 respectively. We observe that the velocity decreases with increase in the value of the porous parameter σ. For large porous parameter, the frictional drag resistance against the flow in the porous region is very large and as a result, the velocity decreases as the porous parameter σ increases. Since the baffle is placed near the cold wall, the effect of σ is insignificant in stream1 compared to stream2.

The effect of mixed convection parameter λ on the velocity is shown in Figure 3. As λ increases velocity increases in both the streams but its effect is more influential in stream 2 compared to stream 1. Physically increase of mixed convection parameter λ implies increase in Grashof number, where Grashof number is the ratio of buoyancy force to viscous force. Hence increase in Grashof number increases the buoyancy force which in turn promotes the flow.

Figures 4 and 5 shows that as the porous parameter σ and mixed convection parameter λ increases, temperature increases. From Figure 6 it is seen that as the porous parameter σ increases velocity decreases in both the streams when the baffle is placed in the middle of the channel. It is observed that the magnitude of suppression is large when the baffle is placed near the cold wall compared to the baffle placed in the middle of the channel. As the mixed convection parameter λ increases velocity increases in both the streams. As the porous parameter σ and mixed convection parameter λ increases, temperature increases in both the streams. From figure 9 it is seen that the porous parameter σ increases, the velocity decreases in both the streams.

CONCLUSION

In the present study, fully developed flow and heat transfer in a vertical double passage channel containing permeable fluid is studied analytically using regular perturbation method. The Brinkman model is used for flow through porous media, the velocity and temperature profiles have been presented. Hence one can conclude that porous parameter σ is to suppress the velocity, promotes the temperature where as mixed convection parameter λ promotes both the velocity and temperature when the baffle is placed at cold, middle and near the hot walls of the channel.

Acknowledgment
This work is supported by University Grants Commission, New Delhi under major
Research Project with F.No. 37-174/2009 (SR) dated 12-01-2010.

Appendix

$$c_1 = c_3 = -1, \qquad c_2 = c_4 = -1/2 \qquad l_1 = \frac{m^2}{\alpha^2}\frac{Gr}{Re}$$

$$l_2 = \frac{m^2}{\alpha^2}\frac{Gr}{2\,\mathrm{Re}} + \frac{m^2}{\alpha^2}\gamma_1, \qquad l_3 = \frac{m^2}{\alpha^2}\frac{Gr}{2\,\mathrm{Re}} + \frac{m^2}{\alpha^2}\gamma_2, \qquad d_1 = l_2$$

$$d_{21} = \left(\frac{m^2}{\alpha^2}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right) co\sec h\left(\alpha y^*\right) + \left(\frac{m^2}{\alpha^2}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right) coth\left(\alpha y^*\right) - co\sec h\left(\alpha y^*\right) + l_1 y*$$

$$d_{22} = \left(\frac{m^2}{\alpha^2}\right) coth\left(\alpha y^*\right) + \left(\frac{m^2}{\alpha^2}\right) co\sec h\left(\alpha y^*\right)$$

$$d_{31} = \left(\frac{m^2}{\alpha^2}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right) s\,ec\,h(\alpha) - l_1 sech(\alpha) - d_{41} tanh(\alpha)$$

$$d_{32} = \left(\frac{m^2}{\alpha^2}\right) s\,ec\,h(\alpha) - d_{42} tanh(\alpha)$$

$$d_{411} = \frac{\left(l_1\left(y^*cosh(\alpha) - cosh(\alpha y^*)\right)\right)}{sinh(\alpha)cosh(\alpha y^*) - sinh(\alpha y^*)cosh(\alpha)}$$

$$d_{412} = \frac{-\left(\frac{m^2}{\alpha^2}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right)cosh(\alpha)}{sinh(\alpha)cosh(\alpha y^*) - sinh(\alpha y^*)cosh(\alpha)}$$

$$d_{41} = d_{411} - d_{412}$$

$$d_{42} = \frac{\left(\frac{m^2}{\alpha^2}\right)cosh(\alpha y^*) - \left(\frac{m^2}{\alpha^2}\right)cosh(\alpha)}{sinh[\alpha]cosh(\alpha y^*) - sinh(\alpha y^*)cosh(\alpha)}$$

$$\gamma_1 = \frac{\left(\frac{-d_{21}cosh(\alpha y^*)}{\alpha} + \left(\frac{m^2}{\alpha^2}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right)y^* - \left(\frac{m^2}{\alpha^3}\right)\left(\frac{Gr}{2\,\mathrm{Re}}\right)sinh(\alpha y^*)\right) - \frac{l_1 y^{*2}}{2} + \frac{d_{21}}{\alpha} + y^*}{\left(\frac{m^2}{\alpha^3}\right)sinh(\alpha y^*) - \left(\frac{m^2}{\alpha^2}\right)y^* + \frac{d_{22}cosh(\alpha y^*)}{\alpha} - \frac{d_{22}}{\alpha}}$$

$$Gm2n = \frac{d_{31}\left(sinh(\alpha y^*) - sinh(\alpha)\right)}{\alpha} + \frac{d_{41}\left(cosh(\alpha y^*) - cosh(\alpha)\right)}{\alpha} + \frac{l_1\left(y^{*2} - 1\right)}{2} + \left(\frac{m^2}{\alpha^2}\frac{Gr}{2\,\mathrm{Re}} + 1\right)\left(1 - y^*\right)$$

$$Gm2d = \frac{d_{32}sinh(\alpha)}{\alpha} + \frac{d_{42}cosh(\alpha)}{\alpha} - \left(\frac{m^2}{\alpha^2}\right) - \frac{d_{32}sinh(\alpha y^*)}{\alpha} - \frac{d_{42}cosh(\alpha y^*)}{\alpha} + \left(\frac{m^2}{\alpha^2}\right)y^*$$

$$\gamma_2 = \frac{Gm2n}{Gm2d}\;; \quad d_2 = d_{21} + d_{22}\gamma_2\,; \quad d_3 = d_{31} + d_{32}\gamma_2\,; \quad d_4 = d_{41} + d_{42}\gamma_2\,;$$

$$g_1 = -\left(\frac{n^2}{m^2}\right)\alpha^2 d_1^{\,2} - \sigma^2 n^2 d_2^{\,2}\;;\; g_2 = \left(\frac{n^2}{m^2}\right)\alpha^2 d_2^{\,2} + \sigma^2 n^2 d_1^{\,2}$$

$$g_3 = \left(\frac{n^2}{m^2}\right)\alpha^2 d_1 d_2 + \sigma^2 n^2 d_1 d_2\,; \;\; g_4 = \left(\frac{2n^2}{m^2}\right)\alpha d_1 l_1 - 2\sigma^2 n^2 d_2 l_2\,; \;\; g_5 = 2\sigma^2 n^2 d_2 l_1$$

$$g_6 = \left(\frac{2n^2}{m^2}\right)\alpha d_2 l_1 - 2\sigma^2 n^2 d_1 l_2\,; \;\; g_7 = 2\sigma^2 n^2 d_1 l_1\,; \; g_8 = \sigma^2 n^2 l_1^{\,2}\,; \;\; g_9 = 2\sigma^2 n^2 l_1 l_2\,;$$

$$g_{10} = \left(\frac{n^2}{m^2}\right)l_1^{\,2} + \sigma^2 n^2 l_2^{\,2}\,; \;\; g_{11} = \frac{g_1}{2} + \frac{g_2}{2} + g_{10}\,; \;\; g_{12} = -\left(\frac{n^2}{m^2}\right)\alpha^2 d_3^{\,2} - \sigma^2 n^2 d_4^{\,2}$$

$$g_{13} = \left(\frac{n^2}{m^2}\right)\alpha^2 d_4{}^2 + \sigma^2 n^2 d_3{}^2; \quad g_{14} = \left(\frac{n^2}{m^2}\right)\alpha^2 d_3 d_4 + \sigma^2 n^2 d_3 d_4$$

$$g_{15} = \left(\frac{2n^2}{m^2}\right)\alpha d_3 l_1 - 2\sigma^2 n^2 d_4 l_3; \quad g_{16} = 2\sigma^2 n^2 d_4 l_1; \; g_{17} = \left(\frac{2n^2}{m^2}\right)\alpha d_4 l_1 - 2\sigma^2 n^2 d_3 l_3$$

$$g_{18} = 2\sigma^2 n^2 d_3 l_1$$

$$g_{19} = 2\sigma^2 n^2 l_1{}^2; \qquad g_{21} = \left(\frac{n^2}{m^2}\right) l_1{}^2 + \sigma^2 n^2 l_3{}^2$$

$$g_{22} = \frac{g_{12}}{2} + \frac{g_{13}}{2} + g_{21};$$

$$f_1 = \frac{g_1 - g_2}{8\alpha^2}; \qquad f_2 = \frac{-g_3}{4\alpha^2}; \qquad f_3 = \frac{-g_7}{\alpha^2}$$

$$f_4 = \frac{-g_5}{\alpha^2}; \qquad f_5 = \frac{2g_5}{\alpha^3} - \frac{g_6}{\alpha^2}; \qquad f_6 = \frac{2g_7}{\alpha^3} - \frac{g_4}{\alpha^2}$$

$$f_7 = \frac{-g_8}{12}; \qquad f_8 = \frac{g_9}{6}; \qquad f_9 = \frac{-g_{11}}{2}$$

$$f_{10} = \frac{-l_4 f_1}{3\alpha^2}; \qquad f_{11} = \frac{l_4 f_2}{3\alpha^2}; \qquad f_{12} = \frac{l_4 f_4}{4\alpha}$$

$$f_{13} = \frac{l_4 f_3}{4\alpha}; \qquad f_{14} = \frac{l_4 f_6}{2\alpha} - \frac{l_4 f_3}{4\alpha^2}; \qquad f_{15} = \frac{l_4 f_5}{2\alpha} - \frac{l_4 f_4}{4\alpha^2}$$

$$f_{16} = \frac{-l_4 f_7}{\alpha^2}, \qquad f_{17} = \frac{-l_4 f_8}{\alpha^2}, \qquad f_{18} = \frac{12 l_4 f_7}{\alpha^4} - \frac{l_4 f_9}{\alpha^2}$$

$$f_{19} = -\frac{6 l_4 f_8}{\alpha^4} - \frac{l_4 c_5}{\alpha^2}; \qquad f_{20} = \frac{24 l_4 f_7}{\alpha^6} - \frac{2 l_4 f_9}{\alpha^4} - \frac{l_4 c_6}{\alpha^2}$$

$$f_{21} = \frac{g_{12} - g_{13}}{8\alpha^2}; \qquad f_{22} = \frac{g_{14}}{4\alpha^2}; \qquad f_{23} = -\frac{g_{18}}{\alpha^2}$$

$$f_{24} = -\frac{g_{16}}{\alpha^2}; \qquad f_{25} = \frac{2g_{16}}{\alpha^3} - \frac{g_{17}}{\alpha^2}; \qquad f_{26} = \frac{2g_{18}}{\alpha^3} - \frac{g_{15}}{\alpha^2}$$

$$f_{27} = -\frac{g_{19}}{12}; \qquad f_{28} = -\frac{g_{20}}{6}; \qquad f_{29} = -\frac{g_{22}}{2}$$

$$f_{30} = \frac{l_4 f_{21}}{3\alpha^2}; \qquad f_{31} = \frac{l_4 f_{22}}{3\alpha^2}; \qquad f_{32} = \frac{l_4 f_{24}}{4\alpha}$$

$$f_{33} = \frac{l_4 f_{23}}{4\alpha}; \qquad f_{34} = \frac{l_4 f_{26}}{2\alpha} - \frac{l_4 f_{23}}{4\alpha^2}; \quad f_{35} = \frac{l_4 f_{25}}{2\alpha} - \frac{l_4 f_{24}}{4\alpha^2}$$

$$f_{36}=\frac{-l_4 f_{27}}{\alpha^2}; \qquad f_{37}=\frac{-l_4 f_{28}}{\alpha^2}; \qquad f_{38}=\frac{12 l_4 f_{27}}{\alpha^4}-\frac{l_4 f_{29}}{\alpha^2}$$

$$f_{39}=-\frac{6 l_4 f_{28}}{\alpha^4}-\frac{l_4 c_7}{\alpha^2}; \qquad f_{40}=\frac{24 l_4 f_{27}}{\alpha^6}-\frac{2 l_4 f_{29}}{\alpha^4}-\frac{l_4 c_8}{\alpha^2}$$

$$z_1 = f_{10} cosh(2\alpha) + f_{11} sinh(2\alpha) + f_{12} y^{*2} cosh(\alpha y^*) + f_{13} y^{*2} sinh(\alpha y^*) + f_{14} y^* Cosh(\alpha y^*) +$$
$$f_{15} y^* sinh(\alpha y^*) + f_{16} y^{*4} + f_{17} y^{*3} + f_{18} y^{*2} + f_{19} y^* + f_{20}$$

$$z_2 = f_1 cosh(2\alpha y^*) + f_2 sinh(2\alpha y^*) + f_3 y^* cosh(\alpha y^*) + f_4 y^* sinh(\alpha y^*) +$$
$$f_5 cosh(\alpha y^*) + f_6 sinh(\alpha y^*) + f_7 y^{*4} + f_8 y^{*3} + f_9 y^{*2}$$

$$z_3 = f_{21} cosh(2\alpha y^*) + f_{22} sinh(2\alpha y^*) + f_{23} y^* cosh(\alpha y^*) + f_{24} y^* sinh(\alpha y^*) +$$
$$f_{25} cosh(\alpha y^*) + f_{26} sinh(\alpha y^*) + f_{27} y^{*4} + f_{28} y^{*3} + f_{29} y^{*2}$$

$$z_4 = 2\alpha f_1 sinh(2\alpha y^*) + 2\alpha f_2 cosh(2\alpha y^*) + \alpha f_3 y^* sinh(\alpha y^*) + f_3 cosh(\alpha y^*)$$
$$+\alpha f_4 y^* cosh(\alpha y^*) + f_4 sinh(\alpha y^*) + \alpha f_5 sinh(\alpha y^*) + \alpha f_6 cosh(\alpha y^*) +$$
$$4 f_7 y^{*3} + 3 f_8 y^{*2} + 2 f_9 y^*$$

$$z_5 = 2\alpha f_{21} sinh(2\alpha y^*) + 2\alpha f_{22} cosh(2\alpha y^*) + \alpha f_{23} y^* sinh(\alpha y^*) + f_{23} cosh(\alpha y^*)$$
$$+\alpha f_{24} y^* cosh(\alpha y^*) + f_{24} sinh(\alpha y^*) + \alpha f_{25} sinh(\alpha y^*) + \alpha f_{26} cosh(\alpha y^*) +$$
$$4 f_{27} y^{*3} + 3 f_{28} y^{*2} + 2 f_{29} y^*$$

$$z_6 = f_{21} cosh(2\alpha) + f_{22} sinh(2\alpha) + f_{23} cosh(\alpha) + f_{24} sinh(\alpha) + f_{25} cosh(\alpha) +$$
$$f_{26} sinh(\alpha) + f_{27} + f_{28} + f_{29}$$

$$z_7 = f_{30} cosh(2\alpha) + f_{31} sinh(2\alpha) + f_{32} cosh(\alpha) + f_{33} sinh(\alpha) + f_{34} cosh(\alpha) +$$
$$f_{35} sinh(\alpha) + f_{36} + f_{37} + f_{38} + f_{39} + f_{40}$$

$$z_8 = f_{30} cosh(2\alpha y^*) + f_{31} sinh(2\alpha y^*) + f_{32} y^{*2} cosh(\alpha y^*) + f_{33} y^{*2} sinh(\alpha y^*) +$$
$$f_{34} y^* cosh(\alpha y^*) + f_{35} y^* sinh(\alpha y^*) + f_{36} y^{*4} + f_{37} y^{*3} + f_{38} y^{*2} + f_{39} y^* + f_{40}$$

$$c_5 = z_5 - z_4 - c_7$$
$$c_6 = - f_1 - f_5$$
$$c_7 = z_6 - c_8$$
$$c_8 = z_5 y^* + z_2 - z_4 y^* - f_1 - f_5 - z_3$$
$$d_5 = - f_{10} - f_{20}$$

$$d_6 = -d_5 coth\left(\alpha y^*\right) - z_1 cosech\left(\alpha y^*\right)$$

$$d_7 = -d_8 tanh\left(\alpha\right) - z_7 sech\left(\alpha\right)$$

$$d_8 = \frac{z_8 cosh\left(\alpha\right) - z_7 cosh\left(\alpha y^*\right)}{sinh\left(\alpha\right) cosh\left(\alpha y^*\right) - sinh\left(\alpha y^*\right) cosh\left(\alpha\right)}$$

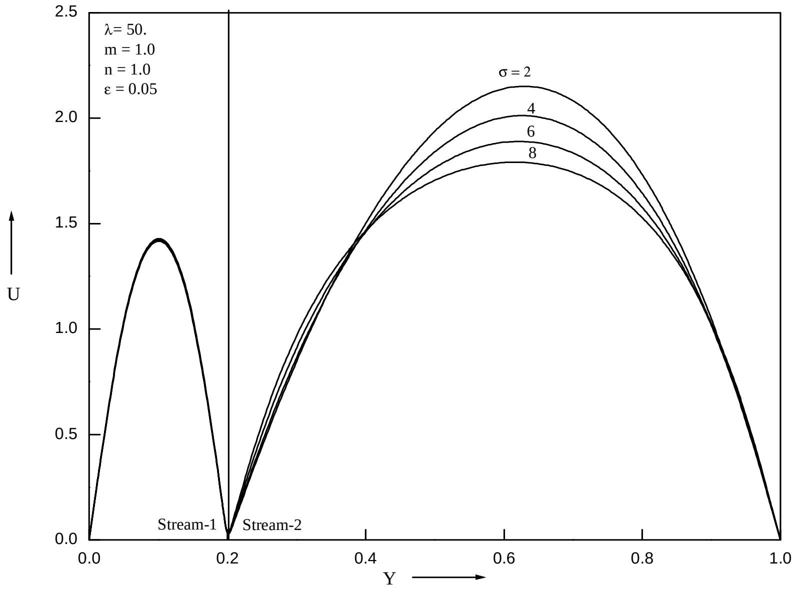

Figure.2. Velocity for different values of porous parameter σ

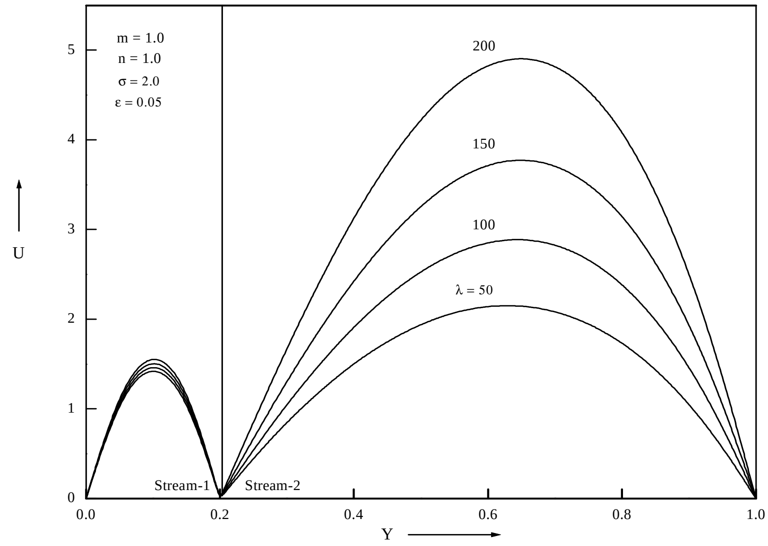

Figure.3. Velocity for diffrent values of mixed convection λ

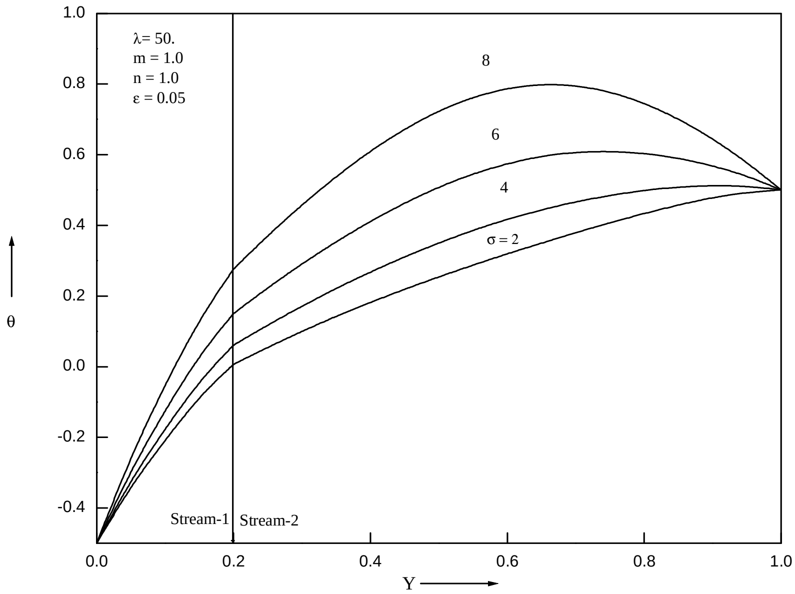

Figure.4. Temperature for different values of porous parameter σ

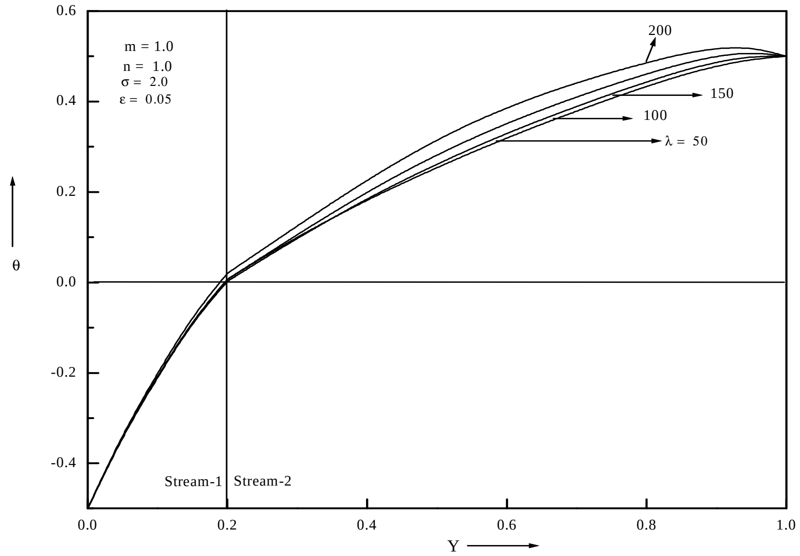

Figure.5. Temperature for different values of mixed convection λ

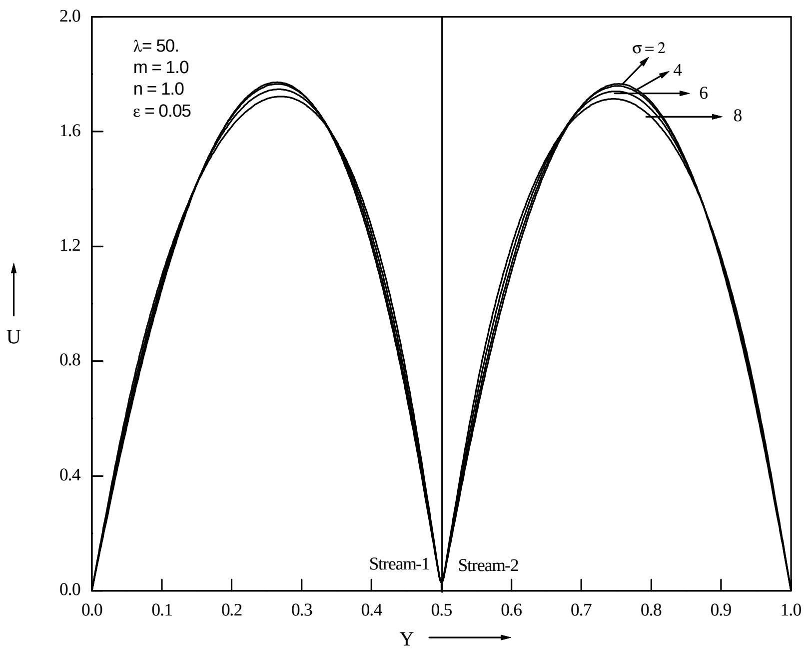

Figure.6. Velocity for different values of porous parameter σ

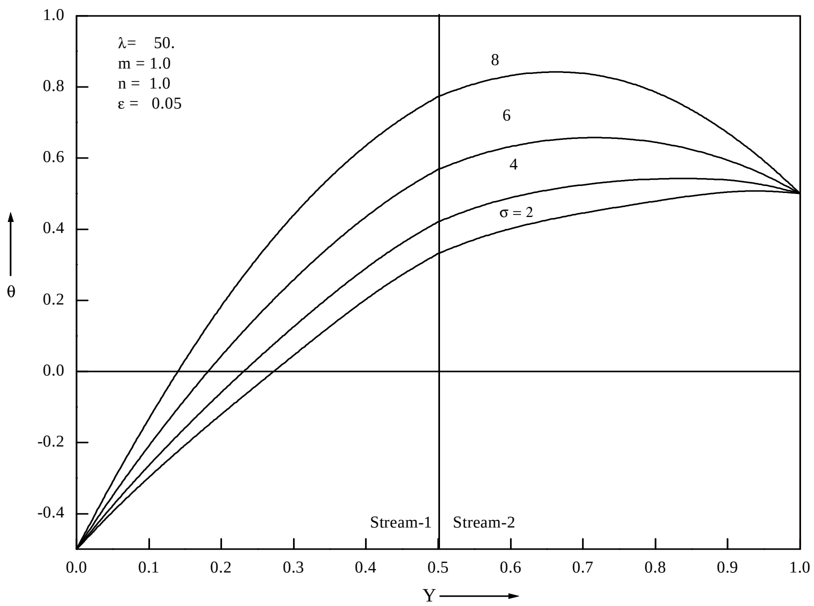

Figure.7. Temperature for different values of porous parameter σ

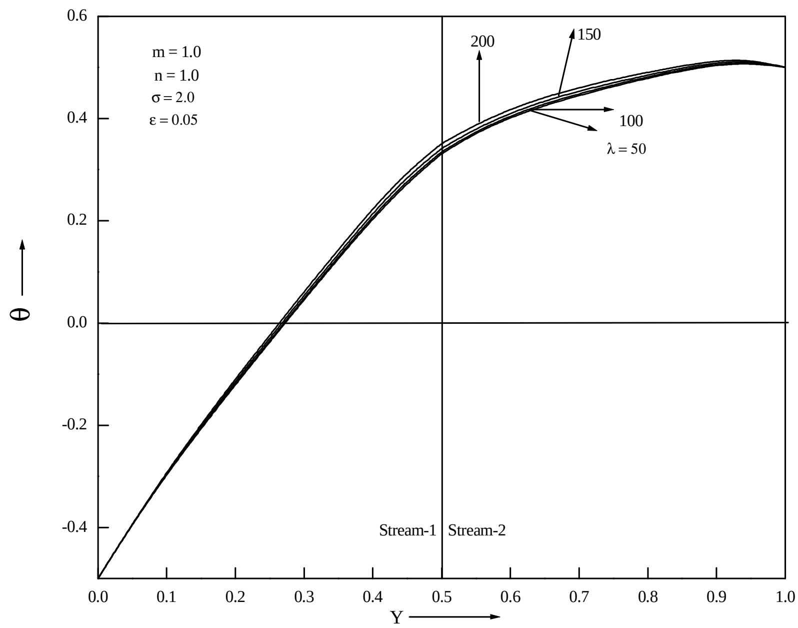

Figure.8.Temperature for diffrent values of mixed convection λ

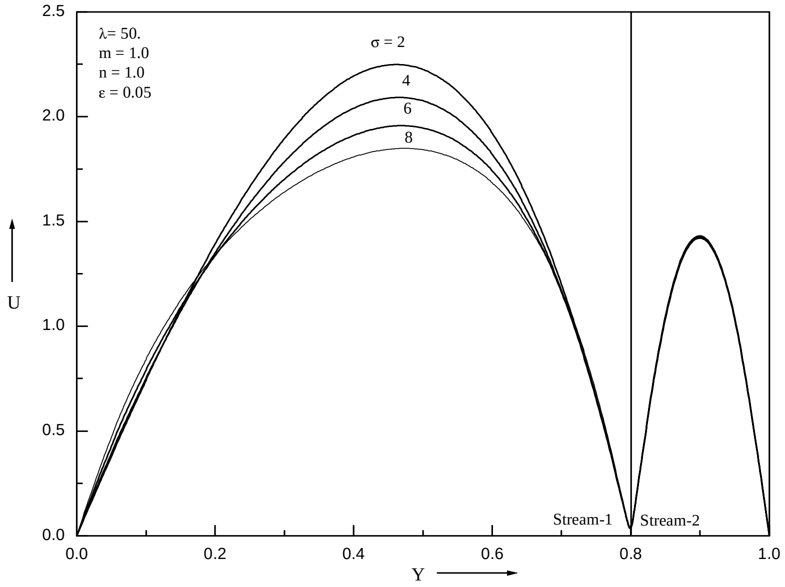

Figure.9. Velocity for different values of porous parameter σ

REFERENCES

[1] Bodia. J. R and Osterle. J. F. *J. Heat Transfer***,** 84, **1962**, 49-44.
[2] Aung. W**.** *Int. J of Heat and mass transfer***,** 15, **1972**, 1577-1580
[3] Aung. W. and Worku. G**.** *ASME. J .Heat Transfer***,** 10892**, 1968**, 485-488.
[4**]** Nelson D.J and Wood B.D. *Int. J. Heat mass Transfer***,** 32, **1989**, 1789-1792.
[5] K.Vafai and C.l.Tein**.** *Int. J. Heat Mass Transfer***,** 24, **1981**, 195-203.
[6] S.C. Tien and K.Vafai. *Numer .Heat Transfer, part* A, 31, **1997**, 235-254.
[7] P. Chung and W.J. Minkowycz, *J. Geophysics*, 82, **1977**, 2041-2044
[8] P. Chung and C.T. Hsu. *J. Heat Transfer*, 106, **1984**, 143-151.
[9] Candra, P.R., Niland, M.E., Han, J.C, *J. Turbomach*, 119, **1986**, 374-380
[10] Han, J.C., Zhang, Y.M., Lee, C.P**.** *J. Heat Transfer***.** 3, **1991**, 590-596.
[11] Beitelmal , a. H., Salad, M.A., Patel, C.D**.**, *Int.J.Heat Fluid flow*, 2, **2000**, 156-163.
[12] Dutta, P., Hossain**,** A**.** *Int. J. Heat Fluid flow***,** 2, **2005**, 223-232.
[13] Guo, Z. Y., Li, D. Y., Wang, B. X. *Int. J. Heat mass Transfer*, 14, **1998**, 2221-2225.
[14] Mostafa M. Salah El-Din' Laminar fully developed mixed convection with viscous dissipation in a uniformly heated vertical double passage channel", 11, **2007**, 27-41
[15] Boulama, K ,Galanis, N., "Analytical solutions for Developed mixed convection between parallel vertical plates with heat and mass trnsfer,3, **2004,** 381-388.
[16] Salah El-Din, M. M., "Effect of thermal and mass buoyancy forces on the development of laminar mixed convection between vertical parallel plates with uniform wall heat and mass fluxes" 5, **2003**, 447-453.