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**Advances in Applied Science Research, 2015, 6(1):59-65**

ISSN: 0976-8610
CODEN (USA): AASRFC

# Effects of viscous dissipation and thermal stratification on chemical reacting fluid flow over a vertical stretching surface with heat source

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## ABSTRACT

*This paper aims to investigate the influence of chemical reaction and thermal stratification of the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface in the presence of viscous dissipation. A magnetic field is applied transversely to the direction of the flow. The basic equations governing the flow, heat transfer, and concentration are reduced to a set of non linear ordinary differential equations by using appropriate transformation for variables. The non linear ordinary differential equations are first linearised using Quasi-linearization and solved numerically by an implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The effects of physical parameters on the velocity, temperature, and concentration profiles are illustrated graphically. Velocity, Temperature and concentration profiles drawn for different controlling parameters reveal that the flow field is influenced appreciably by the presence of thermal stratification, chemical reaction, magnetic field and viscous dissipation.*

**Keywords**: Chemical reaction; Magnetic field; Heat source; Thermal stratification; finite difference scheme

## INTRODUCTION

Magneto hydrodynamic flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma Flows, chemical engineering and electronics.

Raptis [1] (Raptis 1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Elabashbeshy [2] (Elabashbeshy 1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled [3] (Chamkha and Khaled 2001) investigated the problem of coupled heat and mass transfer by magneto hydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption.

In the combined heat and mass transfer processes, it is known that the thermal energy flux resulting from concentration gradients is referred to as the dufour or diffusion–thermal effect. Similarly, the soret or thermo diffusion effect is the contribution to the mass fluxes due to temperature gradients. The dufour and soret effects may be significant in the areas of geosciences and chemical engineering. Kafoussias and Williams [4] employed the finite difference method to examine the dufour and soret effects on mixed free-forced convective heat and mass transfer along a vertical surface, various other influences that have been considered include magnetic field [5], variable suction [6], and chemical reaction [7].

In many mixed flows of practical importance in nature as well as in many engineering devices, the environment is thermally stratified. The discharge of hot fluid into enclosed regions often results in a stable thermal stratification with lighter fluid overlying denser fluid.

The thermal stratification effects of heat transfer over a stretching surface is of interest in polymer extrusion processes where the object, after passing through a die, enters the fluid for cooling below a certain temperature. The rate at which such objects are cooled has an important bearing on the properties of the final product. In the process of cooling the fluids, the momentum boundary layer for linear stretching of sheet was first studied by Crane [8].

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. In order to study the thermal stratification effects over the above-mentioned problem, an attempt has been made have to analyze the nonlinear hydro magnetic flow with heat and mass transfer over a vertical stretching surface with chemical reaction and thermal stratification effects.

In the past decades, the penetration theory of Highie 1935 had been widely applied to unsteady state diffusional problems with and without chemical reaction. As far as we can ascertain, all the solutions with chemical reaction were obtained for the case of a semi-infinite body of liquid, although physical absorption into a finite film was considered. Among some of the interesting problems which were studied is the analysis of laminar forced convection mass transfer with homogeneous chemical reaction, [9]. The effect of different values of Prandtl number of the fluid along the surface was analyzed by Gebhart [10].

Viscous dissipation which, appears as a source term in the fluid flow generates appreciable temperature, gives the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume. However in the existing convective heat transfer literature on the non-Newtonian fluids, the effect of the viscous dissipation has been generally disregarded. Gnaneswara and Bhasker Reddy [11] have studied the effects of soret and dufour on steady MHD free convection flow in a porous medium with viscous dissipation. Kishan and Shashidar Reddy [12] have studied the MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and Viscous Dissipation.

**MATHEMATICAL FORMULATION**

Let us consider two dimensional laminar boundary layer flows over a stretching plate in an incompressible electrically conducting fluid, where the x-axis is along the stretching plate and y-axis perpendicular to it, the applied magnetic field $B_0$ is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{--- (1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma\beta_0^2 u}{\rho} \qquad \text{--- (2)}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \propto\frac{\partial^2 T}{\partial y^2} + Q(T_\infty - T) + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{--- (3)}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1 C \qquad \text{--- (4)}$$

The boundary conditions are

$$u = U(x) = ax,\ v = 0,\ T = T_w(x),\ C = C_w(x) \text{ at } y = 0 \qquad \text{--- (5)}$$

$$u = 0, T = T_\infty(x) = (1 - n)T_0 + nT_w(x), C = C_\infty \text{ as } y \to \infty \qquad \text{--- (6)}$$

Where a is dimensional constant and n is a constant which is the thermal stratification parameter and is such that $0 \le n \le 1$. The n defined as thermal stratification parameter is equal to $\frac{m_1}{m_1+1}$ of Nakayama and koyama [13] where $m_1$ is constant. $T_0$ is constant reference temperature say, $T_\infty(0)$. The suffixes w and $\infty$ denote surface and ambient conditions

As in Acharya et al. [14] the following change of variables are introduced:

$$\Psi = \left(\nu x U(x)\right)^{1/2} f(\eta)$$
$$\eta = (U(x) / \nu x)^{1/2}\, y \qquad \text{--- (7)}$$

The velocity components are given by
$$u = \frac{\partial\Psi}{\partial y}, v = -\frac{\partial\Psi}{\partial x} \qquad \text{--- (8)}$$

It can be easily verified that the continuity eq. (1) is identically satisfied. Similarity solutions exist if we assume that $U(x) = ax$ and introduce the non dimensional form of temperature and concentration as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$
$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$Re_x = \frac{Ux}{\nu}$ is Reynolds number

$Gr_x = \frac{\nu g\beta(T_w - T_\infty)}{U^3}$ is Grashof number

$Gc_x = \frac{\nu g\beta^*(C_w - C_\infty)}{U^3}$ is modified Grashof number

$P_r = \frac{\mu C_p}{K}$ is Prandtl number

$S_c = \frac{\nu}{D}$ is Schmidt number

$M^2 = \frac{\sigma\beta_0^2}{\rho a}$ is magnetic parameter

$\gamma = \frac{\nu k_1}{U^2}$ is Chemical reaction parameter

$S = \frac{XQ}{U}$ is Heat source parameter

$E_c = \frac{U^2}{C_p(T_w - T_\infty)}$ is Eckert number

In this work, temperature variation of the surface is taken into account and is also given by the power-law temperature, $T_w - T_\infty = Nx^n$ where N and n are constants. Also concentration variation is given by $C_w - C_\infty = N_1 x^{n_1}$ where $N_1$ and $n_1$ are constants. The nonlinear equations and boundary conditions are obtained as

$$f''' + Gc_x Re_x \varphi + Gr_x Re_x \theta - (f')^2 - \left(\frac{M^2}{Re_x}\right) f' + ff'' = 0 \qquad \text{--- (9)}$$

$$\theta'' - P_r f'\left(\theta + \frac{n}{(1-n)}\right) + P_r f\theta' - P_r S\theta + P_r E_c (f'')^2 = 0 \qquad \text{--- (10)}$$

$$\varphi'' - S_c(\varphi\gamma Re_x + f'\varphi) + S_c f\varphi' = 0 \qquad \text{--- (11)}$$

The boundary conditions are given by
$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \varphi(0) = 1, f'(\infty) = 0,$$
$$\theta(\infty) = 0, \varphi(\infty) = 0 \qquad \text{--- (12)}$$

To solve the system of transformed governing equations (9)-(11) with the boundary conditions (12) we first linearised equation (9) by Quasi-linearization teqnique

Then equation (9) transformed to

$$f''' + f''(F) + f'\left(-2F - \frac{M^2}{Re_x}\right) + fF'' + Gc_x Re_x \varphi + Gr_x Re_x \theta - (F')^2 - FF'' = 0 \qquad \text{--- (13)}$$

Where F is assumed to be known function and the above equation can written as

$$A_0 f''' + A_1 f'' + A_2 f' + A_3 f + A_4 = 0 \qquad \text{--- (14)}$$

Where

$A_0[i] = 1$

$A_1[i] = F$

$A_2[i] = -2F - \frac{M^2}{Re_x}$

$A_3[i] = F''$

$A_4[i] = Gc_x Re_x \varphi + Gr_x Re_x \theta - (F')^2 - FF''$

Equation (10) can be expressed as

$$C_0 \theta'' + C_1 \theta' + C_2 \theta + C_3 = 0 \qquad \text{--- (15)}$$

Where

$C_0[i] = 1$

$C_1[i] = P_r f$

$C_2[i] = P_r f' - SP_r$

$C_3[i] = P_r f'\left(\frac{n}{(1-n)}\right) + P_r E_c (F'')^2$

And equation (11) can be expressed as

$$E_0 \varphi'' + E_1 \varphi' + E_2 \varphi = 0 \qquad \text{--- (16)}$$

Where

$E_0[i] = 1$

$E_1[i] = S_c f$

$E_2[i] = -S_c(\gamma Re_x + f')$

Using implicit finite difference formulae, the equations (14), (15) and (16) are transformed to

$$B_0[i]f[i+2] + B_1[i]f[i+1] + B_2[i]f[i] + B_3[i]f[i-1] + B_4[i] = 0 \qquad \text{--- (17)}$$

Where

$B_0[i] = 2A_0[i], B_1[i] = -6A_0[i] + 2hA_1[i] + h^2A_2[i]$

$B_2[i] = 6A_0[i] - 4hA_1[i] + 2h^3A_3[i],\ B_3[i] = -2A_0[i] + 2hA_1[i] - h^2A_2[i]$

$B_4[i] = 2h^3A_4[i]$

$$D_0[i]g[i+1] + D_1[i]g[i] + D_2[i]g[i-1] + D_3[i] = 0 \qquad \text{--- (18)}$$

Where

$D_0[i] = C_0[i] + hC_1[i], D_1[i] = -2C_0[i] + h^2C_2[i] - hC_1[i]$

$D_2[i] = C_0[i], D_3[i] = h^2C_3[i]$

$$H_0[i]\varphi[i+1] + H_1[i]\varphi[i] + H_2[i]\varphi[i-1] = 0 \qquad \text{--- (19)}$$

Where

$H_0[i] = E_0[i] + hE_1[i], H_1[i] = -2E_0[i] + h^2E_2[i] - hE_1[i]$

$H_2[i] = E_0[i]$

Here h represents the mesh size in η direction.

Equations (17), (18) and (19) are solved under the boundary conditions (12) by Thomas algorithm for various parameters entering into the problem and computations were carried out by using C programming.

## RESULTS AND DISCUSSION

A parametric study is performed to explore the effects of magnetic field parameter, Thermal stratification, Chemical reaction and Eckert number. In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The effect of magnetic field parameter on dimensionless velocity profiles with constant chemical reaction parameter, Eckert number and thermal Stratification parameters are presented in Fig. 1. It is observed that the velocity of the fluid decreases with the increase in magnetic field parameter. The dimensionless concentration profiles for different values of magnetic field with constant chemical reaction parameter and thermal stratification parameter are demonstrated in Fig. 2. It is seen that the concentration of the fluid rises with the increase of magnetic parameter. Fig. 3 depicts the dimensionless velocity profiles for different values of thermal stratification parameter with constant Eckert number, chemical reaction parameter and the uniform magnetic field. It is observed that the velocity of the fluid decreases with the increase of thermal stratification parameter. Fig. 4 demonstrates the dimensionless concentration profiles for different values of thermal stratification parameter with constant chemical reaction parameter and the uniform magnetic field. It is seen that the concentration increases with the increase of thermal stratification parameter. The dimensionless velocity profiles for different values of chemical reaction parameter with uniform magnetic field, constant thermal stratification parameter and Eckert number are depicted in Fig. 5. It is observed that the velocity of the fluid decreases with the increase of chemical reaction parameter. The concentration of the fluid decreases with the increase of chemical reaction parameter and this is noted through Fig. 6.The dimensionless temperature profiles for different values of thermal stratification parameter with constant Eckert number, chemical reaction parameter and the uniform magnetic field are shown in Fig. 7. It is clear that the temperature of the fluid decreases with the increase of thermal stratification parameter. The dimensionless temperature profiles for different values of chemical reaction parameter with uniform magnetic field and constant Eckert number, thermal stratification parameter are displayed in Fig. 8. It is seen that the temperature of the fluid increases with the increase of chemical reaction parameter. The dimensionless temperature profiles for different values of Eckert number with uniform magnetic field, constant chemical reaction parameter and thermal stratification parameter are demonstrated in Fig. 9. It is seen that the temperature of the fluid rises with the increase of Eckert number. The dimensionless temperature profiles for different values of magnetic field with constant chemical reaction parameter, Eckert number and thermal stratification parameter are demonstrated in Fig. 10. It is seen that the temperature of the fluid rises with the increase of magnetic parameter.

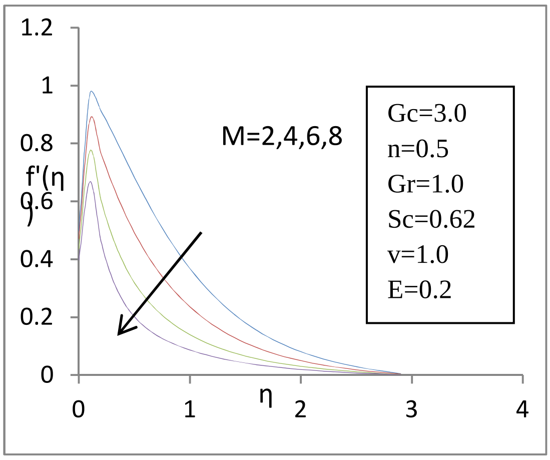


**Fig 1: Velocity profiles for different magnetic parameter**

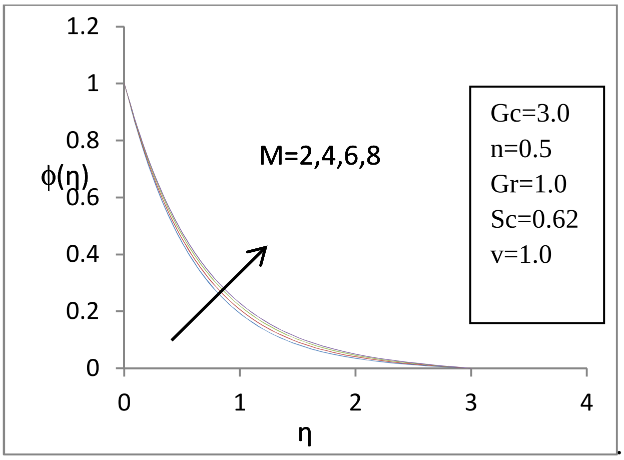


**Fig 2: Magnetic effect over concentration profiles**

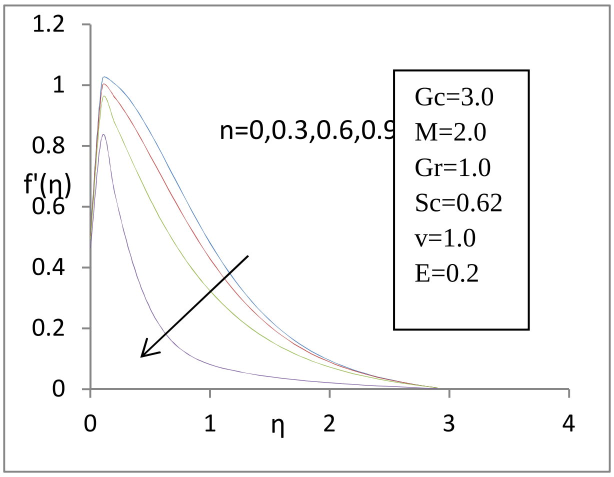


**Fig 3: Thermal Stratification effects over the velocity profiles**

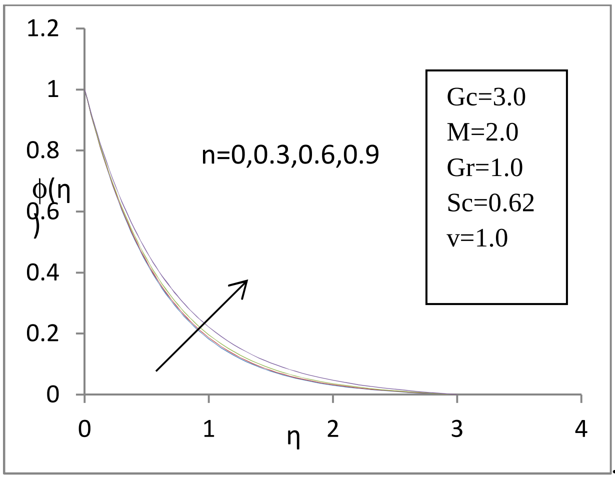


**Fig 4: Thermal Stratification effects over concentration profiles**

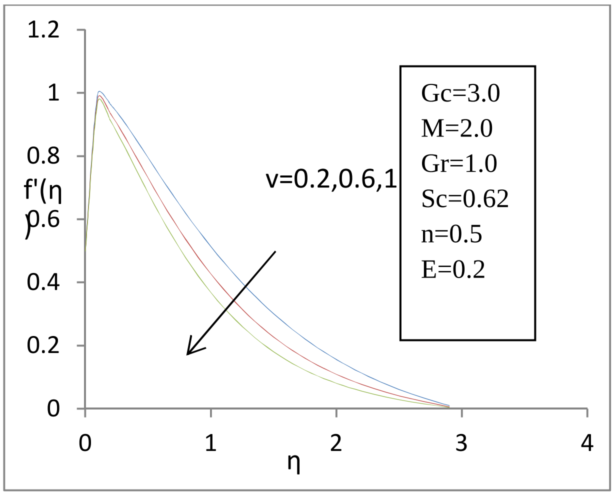


**Fig 5: Chemical reaction effects over the velocity profiles**

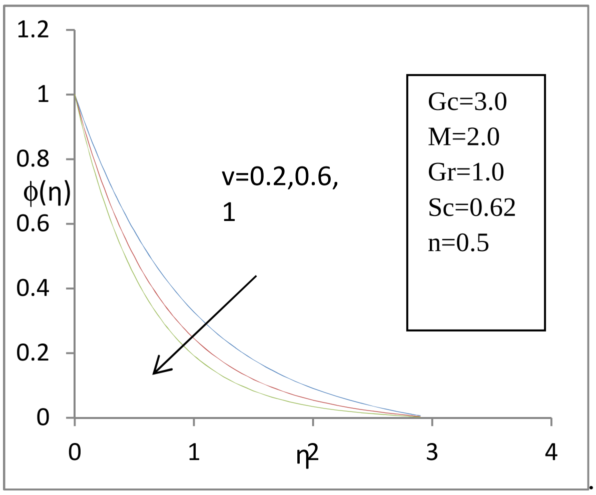


**Fig 6: Chemical reaction effects over the Concentration profiles**

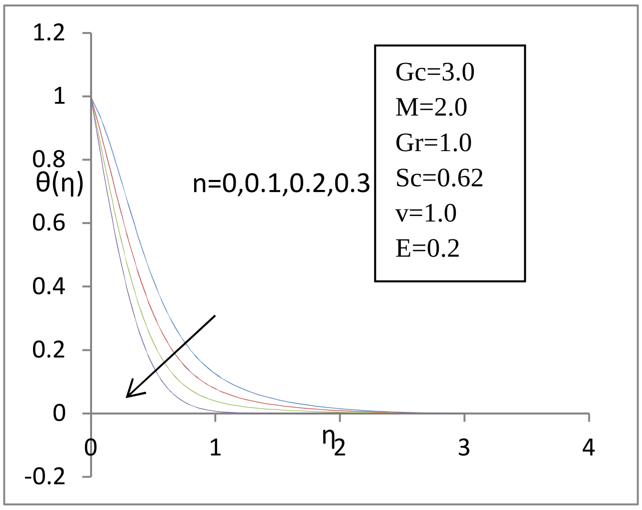


**Fig 7: Thermal Stratification effects over the Temperature profiles**

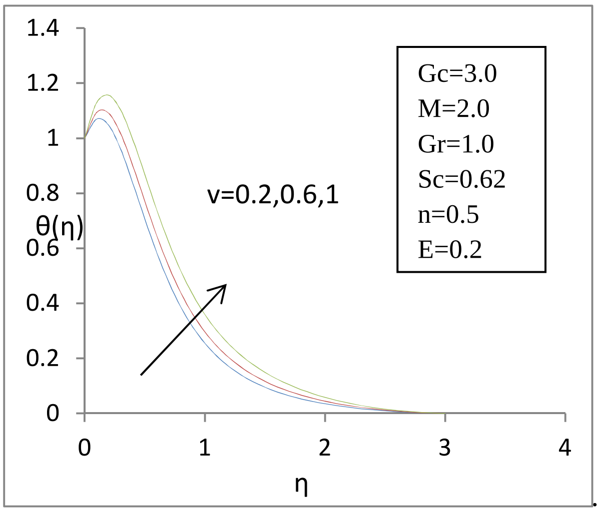


**Fig 8: Chemical reaction over the Temperature profiles**

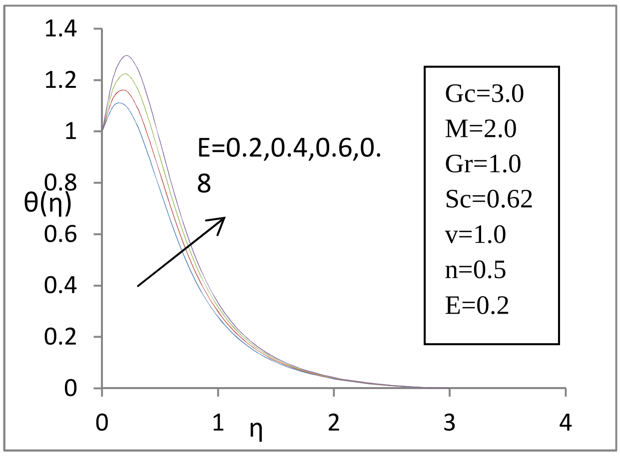


**Fig 9: Viscous dissipation effects over the Temperature profiles**

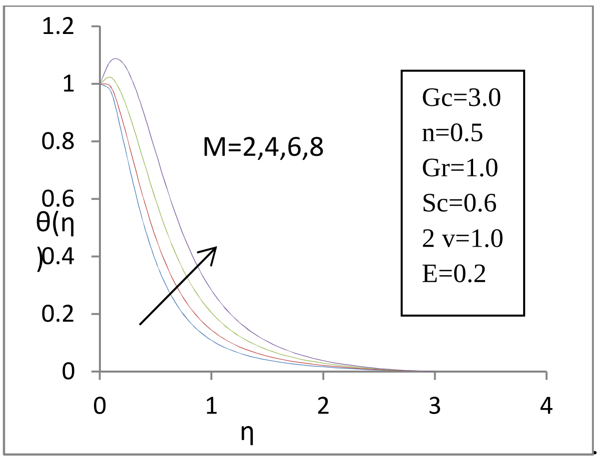


**Fig 10: Magnetic effects over the Temperature profiles**

## CONCLUSION

Due to the uniform magnetic field and thermal stratification parameter, the velocity and concentration of the fluid decrease and the temperature of the fluid increases with the increase of chemical reaction parameter. In the case of constant chemical reaction and thermal stratification parameter, the velocity of the fluid decreases and the temperature and concentration of the fluid increase with the increase of magnetic parameter. Due to uniform magnetic field with constant chemical reaction parameter, the velocity and the temperature of the fluid decrease and the concentration of the fluid increases with the increase of thermal stratification parameter. Under the uniform magnetic field with constant chemical reaction and thermal stratification the temperature of fluid increases with increase in the Eckert number. A comparison of velocity profiles shows that the velocity increases near the plate and thereafter decreases. It is to note that an increase in magnetic field leads to a rise in temperature at slow rate in comparison to the velocity profiles.

It is hoped that the present investigation of the study of physics of flow over a vertical surface can be utilized as the basis for many scientific and engineering applications and for studying more complex vertical problems. The findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

## REFERENCES

[1] Raptis, A. *Int. J.Energy Res.*, **1986**, vol.10, pp. 97-101.
[2] Elabashbeshy, E.M.A. *Int. J. Eng. Sci.*, **1997,** vol.34, pp. 515-522.
[3] Chamkha, A.J., Khaled, A.R.A. *Heat Mass Transfer*, **2001**, vol.37, pp.117-123.
[4] Kafoussias.N.G., Williams, E.W. *International Journal of Engineering Science*, **1995,** vol. 33(9), pp.1369-1384.
[5] Postelnicu, A *International journal of Heat and Mass Transfer*, **2004**,vol. 47, pp.1467-1472.
[6] Alam, M.S., Rahman, M.M. *Nonlinear Analysis: Modeling and control*, **2006**, vol. 11(1), pp. 3-12.
[7] Postelnicu, A. *Heat Mass Transfer*, **2007**, vol. 43, pp. 595602.
[8] L.J. Crane, Z. *Angew. Math. Phys*. 21 **1970**, 641-647.
[9] J.D. Goddard, A. Acrivos, *Quart. J. Mech. Appl. Math*. 20 **1967**, 473-496.
[10] B. Gebhart, Heat transfer, second ed., McGrew Hill Inc., New York, **1971**, p.641.
[11] Gnaneswara Reddy,M., Bhasker Reddy, N. *Int. J. of Appl. Math and Mech.* **2010,**Vol. 6(1), pp.1-12.
[12] Kishan, N., Shashidar Reddy, B. *International journal of Applied mechanics and engineering,* **2012**, Vol. 1(4), pp.425-445.
[13] A. Nakayama, H. Koyama, *Appl. Sci. Res*.46 **1989** 309-332.
[14] M. Acharya, L.P. Singh, G.C. Dash, *Int. J. Engng. Sci*. 37 **1999**, 189-195